THE MODERN THEORY OF AEROFOILS AND ITS APPLICATION TO AEROPLANE DESIGN.

Paper read by Captain W. H. Sayers (Honours Member) before the Institution in the Lecture Room of the Junior Institution of Engineers, 39, Victoria Street, Westminster, S.W.1. on 8th June, 1926, Mr. Lawrence Wingfield in the Chair.

It is a matter of common knowledge that during the past few years very great progress has been made towards the development of a satisfactory theory of aerofoils. In Germany, where the main work of developing the " Vortex " theory has been carried on, the theory has rapidly been applied to practical purposes, and modern text books on aircraft design are to be found showing the application of that theory.

In this country, despite the fact that the Vortex theory was actually originated by Mr. F. W. Lanchester as far back as 1908, and that the general principles of its application to aircraft design were laid down by him in his volumes on Aerial Flight (1911), I have not been able to discover any published material dealing with the practical application of modern knowledge to the design of aircraft.

This does not of course imply that British designers fail to use that knowledge—the contrary is the case—but it does seem to indicate that a simplified statement of the methods of using the theory in its practical applications might prove useful and interesting.

I have no intention of making this paper a general guide to the theory itself. Those who desire to study it for its own sake must be referred to the fairly voluminous literature on the subject already in existence. At the same time some elementary outline does seem essential, but it must necessarily be of a very sketchy and incomplete nature.

The Vortex theory itself does not account for all the qualities of an aerofoil. It deals only with effects into which the viscosity of the air does not enter. In other words it accounts only for such part of the behaviour of aerofoils as would occur in the hypothetical inviscid fluid of hydrodynamics. Other effects which result from viscous forces occur with the real aerofoil, but the theory permits of the separation of these two classes of effects.

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For many years hydrodynamic theory failed dismally to account for any resistance phenomena except such as could be traced to viscous forces. The idea of a circulatory flow round an aerofoil or other body as a mechanism which would produce a lateral force of the "lift" type has long been familiar, but it appeared that such a lift would not be accompanied by resistance or drag

It has now been shown that this is true only for the case of a lifting element of infinite span and that if the conditions at the ends of a real lifting element—such as a wing—are considered, a definite drag must accompany this lift. The drag is not dependent on viscosity—it would occur in the ideal fluid of hydrodynamics.

The "circulation" round the wing which leads to the development of lift cannot cease suddenly at the tip of the wing. Actually it forms part of a vortex system which is completed by a pair of trailing vortices passing off from the tips down stream—to infinity in the case of a fluid possessing no viscosity, or to decay gradually in the real fluid. In either case the wing may be regarded as a vortex producer producing two feet of vortex for every foot which it advances through the air.

A vortex is fluid in motion of a very definite type. It therefore represents kinetic energy, and if the length of vortex produced per unit of time and the amount of motion involved in the vortex are both known, the rate at which energy in this form is being dissipated is also known. Energy so dissipated necessarily implies resistance, and this resistance is known if the energy rate is known.

The lift of a wing per unit of span is proportional to the product of the air speed and the "strength" of the circulation or vortex system associated with it. (That the lift of a given wing at a given incidence varies as the square of the air speed results from the fact that all other conditions being the same the circulation is proportional to the speed.) The "strength" of the circulation is a measure of the amount of motion in unit length of vortex, therefore of the energy loss per unit of motion of the wing.

Thus any wing carrying the same load per unit of span at the same speed has the same strength of circulation and the same total energy loss, and consequently the same drag due to this energy loss. This drag—which has been termed the "induced" drag—is therefore entirely independent of the actual dimensions of the wing or of its section—it depends only on the loading per unit of span.

This would be strictly true if the circulation over the whole span of any wing were uniform and the whole of the energy lost were carried off in a pair of single vortices from each wing tip. Actually the circulation, and consequently the lift, is not uniform over the span. The circulation alters from point to point, and there is a sheet of vortices along the whole trailing edge. The total energy loss, and consequently the induced drag, depends on the distribution of these vortices along the span, and this in turn governs the lift distribution along the span.

Provided that the lift distribution is always similar, however, the above statement holds, and fortunately for normal rectangular wings the lift distribution varies so little that the rule may be taken to hold for all practical purposes, and the induced drag of all similarly shaped wings with equal loadings per unit of span has the same value at the same speed.

Given the total weight and the span, therefore, one can calculate for a given machine the value of induced drag at any speed, and this drag is entirely independent of such questions as the actual area or section of the wing employed. (The induced drag will vary with certain other qualities—it will not be the same for a biplane as for a monoplane—and for a tapered or twisted wing will differ from the induced drag of a straight rectangular wing, but as will be shown later, for most practical cases the induced drag can be written down straight away for any type of machine once the span and weight, etc., are known.)

Over and above this "induced" drag a real aerofoil in real air has an additional resistance which is due to viscous forces. The drag—known as "profile" drag—can easily be evaluated from the results of ordinary model tests by calculating the induced drag, and subtracting from the total drag. "Profile" drag, unlike "induced drag," is characteristic of particular wing sections. It can properly be expressed in terms of a profile drag coefficient, and the value of this profile drag coefficient, for any section, is independent of span, or air speed. That is, for the given section and at the same lift coefficient, total profile drag varies with wing area and the square of the speed.

If therefore one takes the normal test results of total drag coefficient (Kd), calculates for the appropriate conditions the induced drag coefficient (Kdi), and subtracts this from Kd, one is left with a profile drag coefficient (Kdp).

Knowing the total weight and the desired landing speed of any aeroplane, the wing area necessary (for a given wing section) can be calculated from the maximum lift coefficient. Then, using the values of profile drag coefficient obtained by the above method, the profile drag curve for the wings can at once be obtained. This profile drag will not be affected by span, aspect ratio, or whether the machine is a mono, bi or tri-plane. It depends only on total area, wing section and speed.

The induced drag, for any span or wing arrangement, can be calculated separately and without reference to the section which is used.

The sum of these two drags is the total drag of the wings.

In this way one separates wing drag into two components—one of which is purely and entirely governed by the qualities of the section employed, and the other by the details of the wing arrangement (particularly the span) employed.

The method is much simpler and more elastic than the method of computing wing resistance from the usual total drag coefficient, for the value of this coefficient is largely dependent on aspect ratio. If one compares two wings of markedly different maximum lift coefficient on the basis of equal aspect ratio, say from the direct results of N.P.L. model tests, the wing with the higher maximum lift will appear to have a much higher drag. If one were to use these wings at constant aspect ratio, the smaller surface needed with the high lift wing would involve a smaller span. In fact it is the small span which accounts for the high drag, and usually speaking one will increase the aspect ratio of the higher lift wing, which will reduce the induced drag. Dealing with total drag coefficients, it will be necessary to recompute these for each

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wing for every value of aspect ratio which may have to be considered.

The alternative method involves the reduction of the model figures for drag to figures for profile drag alone, once and for all. These figures are a true index of the characteristic of the section itself and can be used for that section irrespective of span, aspect ratio or wing arrangement.

The precise mechanism by which the energy dissipated in the trailing vortices is converted into a drag force is both interesting and important. In fact the whole wing works in the field of these vortices and the effect is to produce a downward inclination of the airstream at the wing itself. The "lift" force produced on the wing is at right angles to the stream direction at the wing, and, owing to the "downwash," this direction is inclined downwards and backwards. The lift force is inclined to the same extent, and therefore has a horiontal component—which is the induced drag.

It is obvious that the effect of this downwash is to reduce the real incidence of the wing. The incidence given in the model test results is measured from the horizontal (or the direction of the undisturbed air stream), whereas the wing is actually working in a down current, and its real incidence is less than the apparent incidence by the angle of downwash. The lift coefficient of a particular section depends on the real incidence, and as the angle of downwash at a given lift coefficient varies with aspect ratio, two wings of the same section but different aspect ratio give the same lift coefficients at different apparent incidences.

The curve of lift of a given section plotted against apparent incidence therefore changes in slope with change of aspect ratio, and this is important in practical design as it affects tail settings and so forth.

The real incidence, however, is constant for the same Kl for any one section. The downwash angle depends only on total lift, span and speed (or on Kl and aspect ratio) and the value of this angle for any aspect ratio can be computed as a function of Kl once and for all. This angle is then subtracted from the apparent incidence of the model wing to give the real incidence for that section or to plot a lift against the incidence curve, which is characteristic of that section.

When the span and general disposition of the wing have been settled, the incidence corrections for that wing can once more be calculated from figures for span and weight, and the apparent angle for each lift coefficient is obtained from the curve by simple addition.

The alternative method of working is that of correcting for aspect ratio and drawing new Kl against incidence curves for each wing section and each aspect ratio—a much more laborious process than the one I have suggested.

When dealing with rectangular monoplanes, the estimation of total wing drag thus becomes exceedingly simple.

In the case of a biplane the trailing vortices of one wing produce a downwash at the other wing and vice versa. The induced drag of a biplane is therefore not the same as that of a monoplane of the same span, load and speed being equal. The effect of the difference depends practically entirely on the ratio between gap and span, but the correction to both induced drag

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and angle of incidence for a given Kl to within limits of accuracy sufficient for practical purposes is a very simple matter. In the case of the biplane also the interference of one wing with another modifies the C.P. or the moment coefficient characteristics of the section.

The case of tapered or twisted wings involves a more complicated procedure, as the distribution of lift along the span will vary in each case. It is, however, quite possible to calculate with a high order of accuracy the aerodynamic qualities of a tapered or twisted wing from the result of tests on rectangular models of the section or sections used.

For triplanes or other multiplanes, methods analogous to those applicable to the biplane could be applied, but so far as I am aware, no computations of the various corrections have yet been published. In general the induced drag of a multiplane is less than that of a monoplane by an amount increasing with increasing number of planes, but the improvement possible in practice must be of decreasing value with increase in the number of wings.

In the appendix, details of the methods which may be used for practical purposes with some illustrations of their application are given, and I hope that these will be found sufficiently simple and straightforward for everyday use.

APPENDIX.

List of Symbols.

Symbol.	Meaning.	Units or dimensions.
A	Total Wing Area	sq. ft.
a	Aspect ratio	$\frac{\text{Span}^2}{\text{Area.}}$
C	Chord	ft.
D	Drag	lbs.
Di	Induced drag	lbs.
Dp	Profile drag	lbs.
F	Biplane correction for induced downwash and drag	——— no dimension
g	Gap	ft.
i	Angle of incidence	radians.
i°	Angle of incidence	degrees
$i°_0$	Angle of incidence at infinite aspect ratio	degrees
L	Lift	lbs.
M	Pitching moment	lbs., ft
N	Downwash and induced drag correction factor for rectangular wings ...	No dimensions.
S	Span	feet.
V	Air speed	ft. p. sec. or m.p.h.
W	Weight	lbs.
α	Angle of induced downwash	Radians.
$\alpha°$	Angle of induced downwash	Degrees.
β	Biplane incidence correction	Radians.
$\beta°$	Biplane incidence correction	Degrees.

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COEFFICIENTS :—

Kd = Coefficient of total drag ... ... = $\dfrac{D}{\rho AV^2}$

Kdi = Coefficient of induced drag ... ... = $\dfrac{Di}{\rho AV^2}$

Kdp = Coefficient of profile drag... ... ... = $\dfrac{Dp}{\rho AV^2}$

Kl = Coefficient of Lift ... ... = $\dfrac{L}{\rho AV^2}$

Km = Coefficient of pitching moment ... ... = $\dfrac{M}{\rho AV^2} \quad \dfrac{1}{C}$

EQUATIONS.

(A) *Fundamental forms.*

For monoplanes elliptically loaded :—

$$\alpha = \frac{2}{\pi} \quad \frac{1}{\rho} \quad \frac{L}{S^2V^2} \qquad 1$$

$$Di = \frac{2}{\pi} \quad \frac{1}{\rho} \quad \frac{L^2}{S^2V^2} \qquad 2$$

also —

$$\alpha = \frac{2}{\pi} \quad Kl \quad \frac{1}{a} \qquad 3$$

$$Kdi = \frac{2}{\pi} \quad Kl^2 \quad \frac{1}{a} \qquad 4$$

Hence —

$$\alpha = \frac{Di}{L} = \frac{Kdi}{Kl} \qquad 5$$

For monoplanes not elliptically loaded :—

$$\alpha = N \quad \frac{2}{\pi} \quad \frac{1}{\rho} \quad \frac{L}{S^2V^2} \qquad 1a$$

$$Di = N \quad \frac{2}{\pi} \quad \frac{1}{\rho} \quad \frac{L^2}{S^2V^2} \qquad 2a$$

$$\alpha = N \quad \frac{2}{\pi} \quad Kl \quad \frac{1}{a} \qquad 3a$$

$$Kdi = N \quad \frac{2}{\pi} \quad Kl^2 \quad \frac{1}{a} \qquad 4a$$

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Biplanes.

Wings of equal loading :—

$$\alpha = F \frac{2}{\pi} \frac{1}{\rho} \frac{L}{S^2V^2} \qquad \text{1b}$$

$$Di = F \frac{2}{\pi} \frac{1}{\rho} \frac{L^2}{S^2V^2} \qquad \text{2b}$$

$$\alpha = F \frac{2}{\pi} Kl \frac{1}{a} \qquad \text{3b}$$

$$Kdi = F \frac{2}{\pi} Kl^2 \frac{1}{a} \qquad \text{4b}$$

Wings of unequal loading :—

Let the wings have spans S_1 and S_2, carrying loads L_1 and L_2, then:—

$$\alpha = \frac{2}{\pi} \frac{1}{\rho} \left\{ \frac{\left(\frac{L_1}{S_1}\right)^2 + (4F-2)\left(\frac{L_1L_2}{S_1\,S_2}\right) + \left(\frac{L_2}{S_2}\right)^2}{L_1 + L_2} \right\} \frac{1}{V^2} \qquad \text{(1c)}$$

$$Di = \frac{2}{\pi} \frac{1}{\rho} \left\{ \left(\frac{L_1}{S_1}\right)^2 + (4F-2)\left(\frac{L_1L}{S_1\,S_2}\right) + \left(\frac{L_2}{S_2}\right)^2 \right\} \frac{1}{V^2} \qquad \text{(2c)}$$

Incidence correction for biplanes :—

Total incidence ... $i = i_o + \alpha + \beta$ (6)

where :

$$\beta = \frac{1}{8\pi} \left(\frac{C}{G}\right)^2 Kl \qquad (7)$$

Moment correction for biplanes :—

$$\Delta Km = \frac{\pi}{4} \beta \qquad (8)$$

Expressions for Practical Use.

In these angles are given in degrees, and the numerical values of coefficients are given for the direct application to British units.

For the reduction of model results :—

Elliptically loaded wings :—

$$\alpha^\circ = 36{\cdot}5\ Kl \frac{1}{a} \qquad \text{(3c)}$$

$$Kdi = {\cdot}637\ Kl^2 \frac{1}{a} \qquad \text{(4c)}$$

Rectangular wings :—

Values of N, the correction factor for rectangular wings are given in Table I. for aspect ratios from 4 to 10. From expressions (3) and (3*a*) above, it is obvious that the difference between the downwash of a rectangular and of an elliptically loaded wing may be written :—

$$\Delta\alpha = \frac{N-1}{a} \frac{2}{\pi} Kl$$

Similarly from (4) and (4a)....

$$\Delta Kdi = \frac{N-1}{a} \frac{2}{\pi} Kl^2$$

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TABLE I.

Correction Factor for Induced Downwash and Drag Rectangular Wings.

Aspect Ratio = a.	N.	$\frac{N-1}{a}$
4	1·032	·008
5	1·042	·0084
6	1·053	·0088
7	1·065	·0093
8	1·075	·0094
9	1·085	·0095
10	1·092	·0092

Values of $\frac{N-1}{a}$ given in Table I. vary only between ·008 and ·0095 between a = 4 & a = 10. The effect of this variation is to alter the value of Δ Kdi by less than the probable error of measurement of Kd in the channel. It is therefore permissible to assume for practical purposes $\frac{N-1}{a}$ constant and equal to ·0088. Expressions (3a) and (4a) can then be written.

$$\alpha = \frac{2}{\pi} \text{ Kl } \frac{1}{a} + \cdot 0088 \frac{2}{\pi} \text{ Kl}$$

$$\text{Kdi} = \frac{2}{\pi} \text{ Kl}^2 \frac{1}{a} + \cdot 0088 \frac{2}{\pi} \text{ Kl}^2$$

These become for practical purposes—

$$\alpha^\circ = 36 \cdot 5 \text{ Kl } \frac{1}{a} + \cdot 322 \text{ Kl} \qquad (3d)$$

$$\text{Kdi} = \cdot 637 \text{ Kl}^2 \frac{1}{a} + \cdot 0056 \text{ Kl}^2 \qquad (4d)$$

As $\Delta\alpha^\circ = \cdot 301$ Kl and ΔKdi = ·0056 Kl² for rectangular wings are independent of aspect ratio, the change in α and Kdi, with change of aspect ratio is the same as that for elliptic wings.

$$i^\circ_0 = i^\circ - \alpha^\circ \qquad (9)$$

$$\text{Kdp} = \text{Kd} - \text{Kdi} \qquad (10)$$

For computing Downwash angle and Induced Drag.

Rectangular monoplanes :—

$$\alpha^\circ = 7162 \frac{W}{S^2} \frac{1}{V^2} \quad \text{for V = M.P.H.} \qquad (1d)$$

$$\alpha^\circ = 15356 \frac{W}{S^2} \frac{1}{V^2} \quad \text{for V = ft. P.S.} \qquad (1e)$$

$$\text{Di} = 125 \left(\frac{W}{S}\right)^2 \frac{1}{V^2} \quad \text{for V = M.P.H.} \qquad (2d)$$

$$\text{Di} = 268 \left(\frac{W}{S}\right)^2 \frac{1}{V^2} \quad \text{for V = ft. P.S.} \qquad (2e)$$

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also :—

$$\alpha^\circ = 57{\cdot}3\frac{Di}{L} \qquad (5a)$$

$$i^\circ = i^\circ_o + \alpha^\circ \qquad (11)$$

$$D = Dp + Di \qquad (12)$$

[*Note.*—(1d), (1e) and (2d), (2e), are strictly correct for elliptical loading only. But as pointed out above the change of α and Di with span or aspect ratio is the same for rectangular and elliptically loaded wings. If $\Delta\alpha$ and Δ Kdi have been neglected in reducing model figures, use of these expressoins leads to no error. If $\Delta\alpha$ and Δ Kdi have been used to reduce model results, it is best to add $\Delta\alpha$ to i°_o and Δ Kdi to Kdp as though they were section characteristics, otherwise the process of computation is unnecessarily complicated. Thus (11) becomes $i^\circ = (i^\circ_o + \Delta\alpha^\circ) + \alpha$ and (12) $D = (Dp + \Delta Di) + Di$.

Rectangular Biplane :—

Equally or nearly equally loaded wings :—

$$Di = 125 \quad F\left\{\frac{W}{\frac{1}{2}(S_1+S_2)}\right\}^2 \frac{1}{V^2} \text{ for } V = \text{M.P.H.} \qquad (2f)$$

$$Di = 268 \quad F\left\{\frac{W}{\frac{1}{2}(S_1+S_2)}\right\}^2 \frac{1}{V^2} \text{ for } V = \text{ft.P.S.} \qquad (2g)$$

Where S_1 and S_2 are the spans of the two wings :

$$\alpha^\circ = 57{\cdot}3\frac{Di}{L} \qquad (5a)$$

Unequally loaded biplane.

$$Di = 125 \quad \left\{\left(\frac{L_1}{S_1}\right)^2 + (4F-2)\left(\frac{L_1L_2}{S_1S_2}\right) + \left(\frac{L_2}{S_2}\right)^2\right\}\frac{1}{V^2} \qquad (2h)$$

$$\text{for } V = \text{M.P.H.}$$

$$Di = 268 \quad \left\{\left(\frac{L_1}{S_1}\right)^2 + (4F-2)\frac{L_1L_2}{S_1S^2} + \left(\frac{L_2}{S^2}\right)^2\right\}\frac{1}{V^2} \qquad (2i)$$

$$\text{for } V = \text{Ft.P.Sec.}$$

Where L_1 is load on wing of Span S_1,

L_2 is load on wing of Span S_2,

F is factor obtained from Fig. 5 for the appropriate values of,

$$\frac{S_1}{S^2} \text{ and } \frac{G}{\frac{1}{2}(S_1+S)}$$

$$\alpha^\circ = 57{\cdot}3 \frac{Di}{L} \qquad (5a)$$

Incidence correction for biplane :—

$$I^\circ = (I^\circ + \alpha^\circ + \beta^\circ) \qquad (6a)$$

$$\beta^\circ = \frac{57{\cdot}3}{8\pi}\left(\frac{C}{G}\right)^2 Kl$$

$$= 2{\cdot}28 \left(\frac{C}{G}\right)^2 Kl \qquad (7a)$$

Fig. 6 also gives values of $\frac{\beta^\circ}{Kl}$ in terms of $\frac{G}{C}$.

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Moment correction for biplane :—

$$\Delta Km = \frac{1}{57.3}\frac{\pi}{4}\beta^\circ = .0137\,\beta^\circ \qquad (8a)$$

APPLICATION TO PRACTICAL PURPOSES.

Reduction of Model Results.

Model results are given in terms of Kl and Kd against incidence, generally for a monoplane, and for a definite aspect ratio. From these we proceed to extract the values of i_o and Kdp. for each value of Kl, thus obtaining the essential characteristics of the section, independent of the induced drag and downwash which depend on aspect ratio. For this purpose tabulated values of Δ and Kdi at standard aspect ratios may be computed once and for all. This has been done in Tables II. and III., using expressions (3d) and (4d).

TABLE II.

α° = Induced Downwash angle for monoplane model test reduction.

Kl	(1) $\Delta\alpha^\circ$ = 322 Kl.	(2) 36·5 Kl.	(3) $= \frac{(2)}{a}$ α° for elliptic loading. a = 5.	a = 6.	(4) = (3) + (1) α° for rectangular wings. a = 5.	a = 6.
	°	°	°		°	°
·05	·016	1·82	0·36	0·30	0·38	0·32
·10	·032	3·65	0·73	0·61	0·76	0·64
·15	·048	5·47	1·09	0·91	1·14	0·96
·20	·064	7·30	1·46	1·22	1·52	1·28
·25	·080	9·12	1·82	1·52	1·90	1·60
·30	·096	10·95	2·19	1·82	2·29	1·92
·35	·112	12·77	2·55	2·13	2·66	2·24
·40	·128	14·60	2·92	2·43	3·05	2·56
·45	·144	16·42	3·28	2·73	3·42	2·87
·50	·161	18·25	3·65	3·04	3·81	3·20
·55	·176	20·07	4·01	3·34	4·19	3·52
·60	·193	21·90	4·38	3·65	4·57	3·84
·65	·209	23·72	4·74	3·95	4·95	4·16
·70	·225	25·55	5·11	4·26	5·33	4·48
·75	·241	27·37	5·47	4·56	5·71	4·80
·80	·257	29·20	5·84	4·87	6·10	5·13

In Table II., Column 1 gives values of the correction to α° for rectangular wings. This may be neglected if the results are afterwards only to be used for rectangular wings.

Column 2 (36·2 Kl) gives the value of α° for aspect ratio 1 and elliptic lift distribution. For any other aspect ratio, divide column 2 by the aspect ratio in question. In column 3 this has been done for a = 5 and a = 6.

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These figures may be used to compute change of incidence due to aspect ratio for rectangular wings, without reference to column 1. To arrive at true values of i°_0 and Kdp for research purposes, or to compute tapered, etc., wings, Column 1 must be added to the elliptic loading values. This has been done in Column 3 for $a = 5$ and $a = 6$ again.

TABLE III.

Kdi = Induced Drag Coefficients for Monoplane for Model test reductions.

Kl.	Kl².	(1) ·ΔKdi= ·0056 Kl	(2) ·637 Kl²	(3)=(2)+$\frac{1}{a}$= Kdi, elliptic.		(4)=(1)+(3)= Kdi rectangular.	
				a = 5	a = 6	a = 5	a = 6
·05	·0025	·000014	·00159	·00032	·00026	·00033	·00028
·10	·010	·000056	·00637	·00127	·0011	·00133	·00112
·15	·0225	·000126	·01432	·0029	·0024	·0030	·0025
·20	·040	·000224	·0255	·0051	·0042	·0053	·0045
·25	·0625	·00035	·0398	·0080	·0066	·0083	·0070
·30	·09	·000504	·0573	·0115	·00955	·0120	·0100
·35	·1225	·00069	·0780	·0156	·0130	·0163	·0140
·40	·16	·00090	·1019	·0204	·0170	·0213	·0180
·45	·2025	·00134	·1275	·0255	·0212	·0266	·0224
·50	·25	·00140	·1591	·0318	·0265	·0333	·0280
·55	·3025	·00170	·1911	·0382	·0319	·0399	·0335
·60	·36	·0020	·2291	·0458	·0382	·0478	·0400
·65	·4225	·0024	·2675	·0535	·0449	·0560	·0470
·70	·49	·0027	·3129	·0626	·0522	·0653	·0550
·75	·5625	·00315	·3566	·0713	·0594	·0745	·0630
·80	·64	·0036	·4074	·0815	·0679	·0851	·0715

In Table III., Column I is again the correction for rectangular wings and may be neglected if only rectangular wings are to be considered. Column 2 is Kdi for aspect ratio = 1 and elliptic distribution and divided by the aspect ratio gives Kdi for that aspect ratio. Column 3 gives Kdi for $a = 5$ and 6, for elliptic loading, and Column 4 for $a = 5$ and 6 corrected for rectangular wings.

To reduce a given set of model results, subtract from i°, the model incidence, at each value of Kl the value of α° at the same Kl for the correct aspect ratio —using Column 2 divided by a (or Column 3 if $a = 5$ or 6). If accurate values of α° are necessary, Column 1 should be added. This gives at each value of Kl the value of i°_0 which is the "real" incidence of the section for that Kl.

At each of the same values of Kl, compute Kdi for the aspect ratio of the model, using Table III. precisely as above and subtract Kdi from the model Kd at the correct value of Kl giving values of Kdp against Kl.

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The most convenient method of performing these operations is graphical. Plot α° and Kdi against Kl for the model aspect ratio. On the same chart plot angle of incidence and Kd from the model. Measure $(i^\circ - \alpha^\circ) = i^\circ_0$ and replot to give the "real" or infinite aspect ratio lift incidence curve. Measure Kd—Kdi and replot to give Kdp against Kl. This process has been carried out (Figs. 2 and 3) for RAF 15 using the results for VL = 100 of R. & M. 888. The results may alternatively be tabulated as Table IV.

TABLE IV.

R.A.F. 15. Infinite Aspect Ratio Characteristics.

From R & M 888, Vl = 100.

	Kl.	i°_0.	Kdp.	C.P.
	·05	— 1·4°	·0052	·90
	·10	— 0·22°	·0052	·41
	·15	+ 0·82°	·0053	·36
	·20	1·9°	0053	·33
	·25	2·5	·0054	·315
	·30	4·1°	·0054	·305
	·35	5·2°	·0060	·30
	·40	6·2°	·0065	·296
	·45	7·3°	·0075	·286
	·50	8·4°	·009	·282
	·55	9·6°	·0125	·278
Max.	·575	10·7°	—	—

The lift-incidence and profile drag-lift curves, together with the curve of Km (or of C.P.) against lift are the essential characteristics of the section and no other information concerning the qualities of a section are required—except information which in many cases is only available as the result of trial and error concerning the so-called "scale effect" on maximum lift, etc.

APPLICATION OF REDUCED SECTION CHARACTERISTICS.

Subject to structural considerations—which may be very largely influenced by such matters as the range of C.P. movement and the possibility of including spars of adequate depth within the section the designer's chief interest in any section is centred in its drag, and "profile" drag alone is affected by the section used.

In the most general case the total weight of machine and the maximum permissible landing speed are among the initial data of design.

For any section of known characteristics the total wing area can at once be computed from the value of Kl max., and thence the total profile drag using that section can be computed for all speeds above the fixed minimum. As an example the profile drag for R.A.F. 15 section, for a total weight of 5,000 lbs. and a landing speed of 50 m.p.h. has been computed in Table V. The results apply to the stated conditions entirely irrespective of how the total wing area may be arranged. Similar profile drag figures may be computed for any other

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section, and the section which gives the lowest profile drag will also give the lowest total drag if a similar wing arrangement of the same span, gap, etc., is used in each case.

TABLE V.

Computation of Profile Drag.

Weight loaded, 5,000 lbs. Landing speed (V. min.), 50 m.p.h.

Section :—R.A.F. 15, Kl Max. = ·575.

$$\text{Wing loading} = (\text{V min.})^2 \times \text{Kl max.} \times \rho.$$
$$= 50^2 \times \cdot575 \times \cdot0051 = 7\cdot34 \text{ lbs./sq. ft.}$$

$$\text{Total area} \therefore = \frac{5000}{7\cdot34} = 682 \text{ sq. ft.}$$

$$\text{At any speed V, Kl} = \text{Kl max.} \left(\frac{\text{V min}}{\text{V}}\right)^2$$
$$= \cdot575 \left(\frac{50}{\text{V}}\right)^2 \quad \text{(a)}$$

$$\text{Dp} = \rho.\text{A.V.}^2\,\text{Kdp} = \cdot0051 \times 682 \times (\text{V}^2\,\text{Kdp}) \text{ or}$$
$$\text{Dp} = 5{,}000\,\frac{\text{Kdp}}{\text{KL}} \quad \text{(b).}$$

V. m.p.h.	Kl (from (a))	Kdp (1).	Kdp / Kl	Dp (from (b))
				lbs.
170	·05	·0052	0·104	520
150	·064	·0052	0·0813	406
130	·087	·0052	0·0597	300
110	·119	·0053	0·0446	223
90	·177	·0055	0·0322	161
70	·294	·0059	0·0201	100
50	·575	·023*	0·044	218*

(1) Kdp here has been taken from Fig. 3, but Δ Kdi of Fig. 1 has been added as suggested in the text on the assumption that only rectangular wings are to be considered, and that only change in induced drag due to span therefore need be treated as induced.

*These figures for drag at Kl max. are not reliable.

THE INDUCED DRAG.

To fix the induced drag for a given load and speed it is necessary first of all to fix all the dimensions of the wing system which can be seen in an outline front elevation. For a monoplane this involves span only, but for a biplane, both span and gap are required. (In the case of biplanes chord and stagger produce secondary effects of minor and negligible size so far as drag is concerned, and may usually be neglected. In the case of wings with marked taper, plan form does have an effect, and heavily tapered wings together with wings of compound or twisted section require special treatment. For

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practical purposes wings with rounded, or normally tapered, tips can be treated as rectangular.)

THE MONOPLANE.

For monoplanes expression (1d) or (1e) may be applied, and it is obvious that the effect of variation in span over any possible range is very easily computed. In Table VI. the induced drag in lbs. for a machine of 5,000 lbs. weight have been computed for speeds from 50 to 170 m.p.h., on spans of 50, 60 and 70 ft. If these figures are combined with those of Table V. for the profile drag of R.A.F. 15, we have total wing drag for R.A.F. 15 carrying 5,000 lbs. load, landing at 50 m.p.h. for each of these spans. Moreover, these induced drag characteristics can be combined with the profile drag characteristics of any other section.

TABLE VI.

Di = Induced Drag for Monoplane.
Weight, 5,000 lbs.
Span Case (1) 50 ft.
,, (2) 60 ft.
,, (3) 70 ft.

$$Di = 126{\cdot}4 \left(\frac{W}{S}\right)^2 \frac{1}{V^2} \qquad (2d)$$

For case (1) $$Di = 126{\cdot}4 \left(\frac{5000}{50}\right)^2 \frac{1}{V^2} = \frac{1{,}264{,}000}{V^2}$$

,, (II.) $$Di = 126{\cdot}4 \left(\frac{5000}{60}\right)^2 \frac{1}{V^2} = \frac{877{,}216}{V^2}$$

,, (III.) $$Di = 126{\cdot}4 \left(\frac{5000}{70}\right)^2 \frac{1}{V^2} = \frac{644{,}640}{V^2}$$

[NOTE. The numerical coeficient 126·4 used above in the expression for Di is a slide rule approximation and a more accurate value is 124·9. In the text the round figure 125 has been used and it is suggested that this should be used in practice. The tables (VI, VII, VIII) however were computed using the first named figure, and as the error so introduced (about 1%) in no way detracts from their value for illustrative purposes it has not been thought necessary to alter them.]

V. (m.p.h.)	Di (lbs.)		
	Case I.	Case II.	Case III.
170	43·5	30·2	22·2
150	56	39	28·5
130	75	52	38·0
110	104	72	53
90	156	108	80
70	268	186	137
50	506	351	258

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THE BIPLANE.

R.A.F. 15 is not really a suitable section for a 5,000 lb. monoplane. To arrive at the total drag characteristics for a R.A.F. 15 biplane for the load and landing speed already assumed we have no further concern with the specific characteristics of the section; we have only to compute induced drags for an appropriate biplane arrangement and add to these the same profile drag results as have already been computed. For this purpose we assume suitable span and gap figures and refer to figure 5 for the appropriate value of F, the biplane factor for induced drag. If the biplane has wings of equal span, $S_1 = S_2$ and the upper curve is used at the appropate value of $\frac{G}{\frac{1}{2}(S_1+S_2)} = \frac{G}{S}$

Then the induced drag is the induced drag of a monoplane of equal span multiplied by F.

For our 5,000 lb. case we may take 60 ft. span 6 ft. gap. (This with 682 sq. ft. which we need in R.A.F. 15 gives a chord of 5·7 ft., so with this section we shall have a more or less normal gap-chord ratio). In this case we have $S_1 = S_2$, $\frac{G}{S} = \cdot 1$, and from Fig. 5 F = ·8775.

The induced drag is, therefore, ·8775 of the induced drag of a monoplane of equal span (as shown in 2f and 2g.). (Specially note that as the area is fixed by the section characteristic of Kl max., and the landing speed, the "aspect ratio" of the biplane as usually understood is twice as great as that of the monoplane of equal span. By computing induced drag on the $\left(\frac{L}{S}\right)^2 \frac{1}{V^2}$ base we are enabled to treat it independently of section characteristics. If we consider induced drag in terms of Kdi, each section—even on the same span—will have a different aspect ratio and it will be necessary to compute Di for each case separately.)

Expressions 2f and 2g are rigorously accurate only if the loading per unit span of both wings is the same, but where the two spans are nearly equal quite large variations in the loading have but very little effect on the total induced drag, and this simple expression may safely be used.

TABLE VII.

Di = Induced drag for equal winged biplane
Wt = 5,000 lbs.
Span = 60 ft. $\qquad \frac{G}{S} = 0{\cdot}1$
Gap = 6 ft. $\qquad$ F. = ·8775

$$Di = \cdot 8775 \times 126{\cdot}4 \left(\frac{5000}{60}\right)^2 \frac{1}{V^2}$$

(That is, Di is Di for 60 ft. span monoplane × ·8775.)

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V. m.p.h.	Di monoplane. (From Table VI.) Case II.	Di, lbs.
170	30·2	26·5
150	39	34·2
130	52	45·6
110	72	63
90	108	95
70	186	163
50	351	309

The computation of Di for the case of 5,000 lbs. weight 60 ft. span and 6 ft. gap by this method has been carried out in Table VII. It may be well to indicate the legitimacy of using the simplified expressions for this case, by assuming that the loading on the lower wing is in fact only 80% of that on the top wing, and applying the more accurate expression (2h) or (2g) to this case. On these assumptions we have $\frac{S_1}{S_2} = 1$, $\frac{G}{\frac{1}{2}(S_1+S_2)} = \cdot 1$, and $F = \cdot 8775$ as before. Also $L_1 = 2{,}222$ lbs. $L_2 = 2{,}778$ lbs.

Instead of $$F \cdot \left(\frac{L}{S}\right)^2 = \cdot 8775\left(\frac{5000}{60}\right)^2 = 6093$$

we substitute $$\left(\frac{L_1}{S_1}\right)^2 + (4F-2)\left(\frac{L_1 L_2}{S_1 S_2}\right) + \left(\frac{L_2}{S_2}\right)^2$$

$$= \left(\frac{2222}{60}\right)^2 + 1{\cdot}51 \frac{2222 \times 2778}{60 \times 60} + \left(\frac{2778}{60}\right)^2 = 6151$$

The variation in Di is proportioned to the variation between these two quantities, and the simpler expression thus gives an induced drag $\frac{6093}{6151}$ or about ·99 of the more accurate value.

If either spans or values of $\frac{L}{S}$ for the two separate wings differ widely, it is desirable to use the more accurate expression. The difficulty here is to determine appropriate values to give to $\frac{L_1}{S_1}$ and $\frac{L_2}{S_2}$ but fortunately the total induced drag changes relatively slowly for changes in these values. And, consequently, quite a rough approximation to the relative loadings is sufficient to give fairly close estimates of Di.

If for the same load we substitute for an equal winged biplane of 60 ft. span one with $S_1 = 45$ ft, $S_2 = 75$ ft., $\frac{S_1}{S_2} = \cdot 6$. If the gap is still 6 ft. $\frac{G}{\frac{1}{2}(S_1+S_2)}$ is $= \cdot 1$ but, from the curve Fig 5 $\left(\frac{S_1}{S_2} = \cdot 6\right)$ we have $F = \cdot 742$. If we apply the simple equation 2h or 2g assuming equal $\frac{L}{S}$ for the two wings, the value of $F\left(\frac{L}{S}\right)^2$ becomes $\cdot742\left(\frac{5000}{60}\right)^2$ or 5,153. If the more accurate factor $\left(\frac{L_1}{S_1}\right)^2 + (4F-2)\left(\frac{L_1L_2}{S_1S_2}\right) + \left(\frac{L_2}{S_2}\right)^2$ is used and it is assumed that $\frac{L_1}{S_1} = \cdot8\frac{L_2}{S_2}$ it will be seen that $L_1 = 1{,}632$ lbs., $L_2 = 3{,}378$ lbs. and we have $\left(\frac{1632}{45}\right)^2 + \cdot968\left(\frac{1632 \times 3378}{45 \times 75}\right) + \left(\frac{3378}{75}\right)^2$ which is 4,876. The simpler expresion thus over-estimates the induced drag for this case by a little over $4\frac{1}{2}\%$. But even the assumption that the lower wing carried no load, which is equivalent to treating the machine as a monoplane of 75 ft. span, only gives a value for Di about 10% too low, so that extreme accuracy in the apportionment of loading is not essential.

Di for this last case of loading and span has been computed in Table VIII.

TABLE VIII.

Di for Biplane with unequal Wings unequally loaded.

Wt. = 5,000 lbs

S_1 = 45 ft. $\qquad L_1$ = 1,532 lbs.

S_2 = 75 ft. $\qquad L_2$ = 3,378 lbs.

G = 6 ft.

$$\frac{S_1}{S_2} = 0\cdot6. \quad \frac{G}{\frac{1}{2}(S_1+S_2)} = 0\cdot1. \quad F = 0\cdot742.$$

$$\begin{aligned} Di &= 126\cdot4 \left\{\left(\frac{L_1}{S_1}\right)^2 + (4F-2)\left(\frac{L_1L_2}{S_1S_2}\right) + \left(\frac{L_2}{S_2}\right)^2\right\}\frac{1}{V^2} \\ &= 126\cdot4 \ \{4920\}\frac{1}{V^2} \\ &= 621{,}888 \ \frac{1}{V^2} \end{aligned}$$

V. m.p.h.	Di, lbs.
170	21·5
150	27·5
130	36·8
110	51·4
90	77
70	127
50	249

We have now seen how to compute profile drag for any section and the induced drag of any arrangement of rectangular monoplane or biplane. The case of the triplane has not so far as I can discover yet been reduced to tractable terms. Tapered, warped or compound wings can be computed from the model characteristics of the sections composing them by a process of integration. For this I refer you to R. & M. Nos. 806 and 824. Such wings cannot be dealt with by the general method outlined above.

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INCIDENCE CORRECTIONS.

The reduced model results for profile characteristics give the values of i°_o for each value of Kl. The total incidence of the real wing will be greater than i°_o by α° the induced downwash angle. The designer is generally more directly interested in the first place by drag than by incidence variations. so that he will usually have the induced drag figures for the particular arrangement of wings to be used before he need concern himself with this matter, Therefore the simplest method of arriving at α° is to use the relation $\alpha = \frac{Di}{L}$ or $\alpha^\circ = 57{\cdot}3\ \frac{Di}{L}$ (5a).

This gives α° for each value of speed, and to correct the incidence curve of a given section speed must be expressed in terms of Kl for that section and the loading employed. This has, however, necessarily been done for the computation of profile drag. The method of computing $i^\circ = (i^\circ_o + \alpha^\circ)$ is shown in Table IX. for a monoplane, and here the whole wing characteristics for the particular case have been tabulated.

TABLE IX.

Total Wing characteristics.
Monoplane, Wt. = 5,000 lbs. Span = 50 ft.
Section, R.A.F. 15. 682 sq. ft. Landing speed 50 m.p.h.

V. m.p.h.	Kl.	(1) i°_o.	(2) α°.	$i^\circ = i^\circ_o + \alpha^\circ$	(3) C.P.	(4) Dp. lbs.	(5) Di. lbs.	Total Drag lbs.
170	·05	— 1·4°	+ ·50°	— ·90°	·59	520	43·5	563
150	·064	— 1·1°	+ ·64°	— ·46°	·50	406	56	462
130	·087	— 0·1	+ ·86°	+ ·76°	·44	300	75	375
110	·119	+ 0·2°	+ 1·19°	+ 1·39°	·39	223	104	327
90	·177	+ 1·4°	+ 1·79°	+ 3·19°	·34	161	156	317
70	·294	+ 3·9°	+ 3·08°	+ 6·98°	·30	100	268	368
50	·575	+10·8°*	+ 5·82°	16·62°	·28	218*	506	724*

(1) From Section characteristics. Fig 2.

(2) From Kl and Table II., or from $\alpha^\circ = 57{\cdot}3\ \frac{Di}{L}$ (5a).

(3) From Section characteristics, Fig. 3.
(4) From Table V.
(5) From Table VI. case (1).
* These figures at Kl max. are not very reliable.

For a biplane α° is computed in the same way using (5a) above, but the total incidence is not $i^\circ_o + \alpha^\circ$ for this case. This is because the flow round each wing curves the streamlines passing the other wing, and has precisely the same effect as would a slight reduction in the curvature of the section. Strictly speaking this means that the characteristics of the section change to those of one of a slightly reduced camber. The most important effect of a change of camber is to increase the angle of no lift, and as the slope of the lift curve remains constant, the value of i° for every value of Kl is increased. There will also be a small change in profile drag characteristics, but this is for practical conditions very small indeed and can be neglected. The only corrections that need be considered as the result of the change in effective camber are the increase of the total incidence by a correction β and a reduction in the range of C.P. travel, or of the value of Km. This latter is not very important and could usually safely be neglected.

Thus the total incidence of a biplane is $i^\circ_o + \alpha^\circ + \beta^\circ$. β° may be computed from 7a, or the value of β°/Kl taken from Fig. 7. It will be noticed that β depends on the ratio of chord to gap and therefore its value in a particular case depends on the section used. The computation of total incidence for a biplane is given in Table X.

TABLE X.

Total Wing Characteristics, Biplane.

Weight = 5,000 lbs. Span = 60 ft. Gap = 6·0 ft. Chord 5·7 ft.
Wings, R.A.F. 15, 682 sq. ft.

V. m.p.h.	Kl.	(1) i°_o.	(2) α°.	(3) β°.	$i^\circ = i^\circ_o + \alpha^\circ + \beta^\circ$	Dp. (4) lbs.	Di. (5) lbs.	Total Drag.	Km(mono) (6)	Δ Km. (7)	Km.(bip)	C.P(8)
170	·05	— 1·4°	·30°	·11°	—0·99°	520	26·5	546	·029	·0015	·028	·56
150	·064	— 1·1°	·39°	·14°	—0·57°	406	34·2	440	·032	·0019	·030	·47
130	·087	—0·1	·52°	·19°	+0·61°	300	45·6	346	·037	·0026	·034	·39
110	·119	+0·2	·72°	·26°	+1·18°	223	63	285	·044	·0035	·040	·34
90	·177	+1·4	1·09°	·38	+2·87°	161	95	256	·059	·0052	·054	·30
70	·295	+3·9	1·87°	·64°	+6·41°	100	163	263	·089	·0088	·080	·27
50	·575	+10·8*	3·55°	1·24°	15·59°*	218*	309	527*	·16*	·017	·143	·25

(1) From Characteristic, Fig. 2.

(2) From $\alpha^\circ = 57{\cdot}3 \dfrac{\mathrm{Di}}{\mathrm{L}}$ (5a).

(3) $\beta^\circ = 2{\cdot}16\ \mathrm{Kl}.$ for $\dfrac{\mathrm{G}}{\mathrm{C}} = 1{\cdot}05$.

(4) From Table V.

(5) From Table VII.

(6) Section Characteristic, Fig. 3.

(7) $\Delta\mathrm{Km.} = {\cdot}0137\ \beta^\circ$.

(8) C.P. = $\dfrac{\mathrm{Km}}{\mathrm{Kl}}$

* Unreliable.

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The correction to C.P. in the case of a biplane is best affected by the use of Km, the moment coefficient. The value of Kmo (Km at Kl = O) is unaltered and is taken directly from the model figures.

The value of ΔKm. = ·0137 $\beta°$ (5a) is computed at a fairly high value of Kl say for R.A.F. 15 at Kl = ·5. Δ Km. is then subtracted from the monoplane value of Km at the chosen value of Kl, and a line is drawn from Kmo through to the new value of Km, and produced if necessary. Those who prefer to deal in C.P. as a fraction of chord can then obtain C.P. from

$$\text{C.P.} = \frac{\text{Km}}{\text{Kl}}$$

For the case of the 60 ft. span biplane dealt with in Table X. $\frac{G}{C} = 1{\cdot}05$ and $\beta° = 2{\cdot}16$ Kl. At Kl. = ·5, $\beta°$ = 1·08, and Δ Km. = ·014. From Fig. 3 Km. for R.A.F. 15 monoplane at Kl = ·5 is ·139. Km. at Kl = ·5 for this biplane is therefore (·139 — ·014) or ·125. Kmo monoplane is ·017 and remains so. A new Km curve is drawn by joining Kmo at ·017 with Km. = ·125 at Kl = 0.5.

The values of $\beta°$ given here apply strictly only to an equally loaded biplane of zero stagger. The effect of departure from these conditions is not large, and the corrections themselves are relatively small, and for practical purposes it is safe to treat all biplanes of normal stagger as unstaggered, and of normal loading inequality as though they were equally loaded using mean values for span, loading, and Kl.

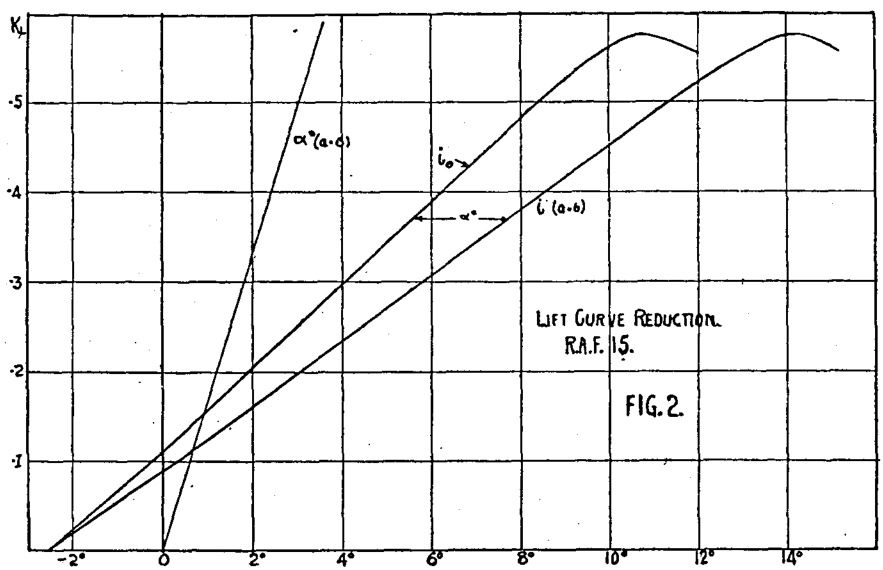

FIG. 2.

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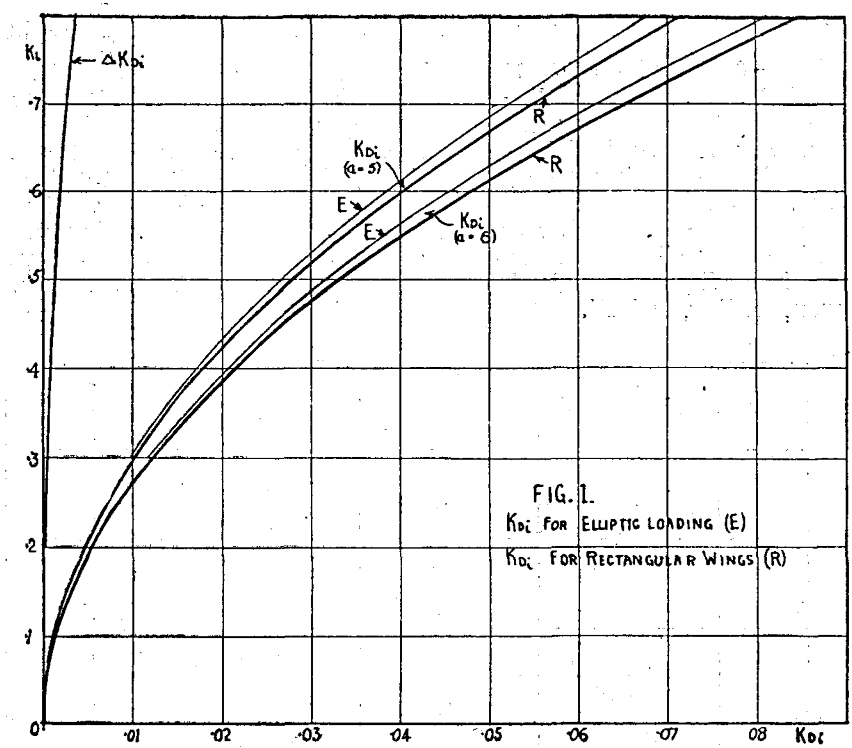

FIG. 1.
K_{Di} FOR ELLIPTIC LOADING (E)
K_{Di} FOR RECTANGULAR WINGS (R)

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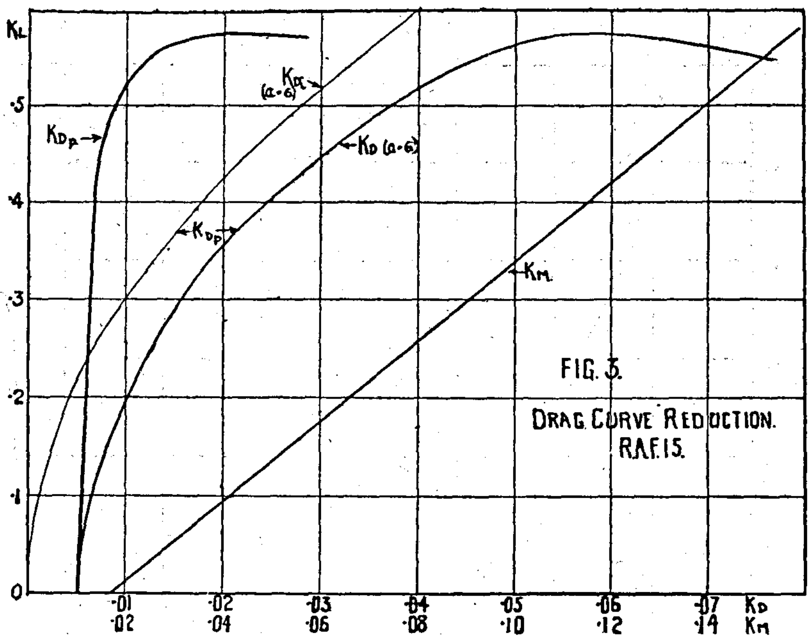
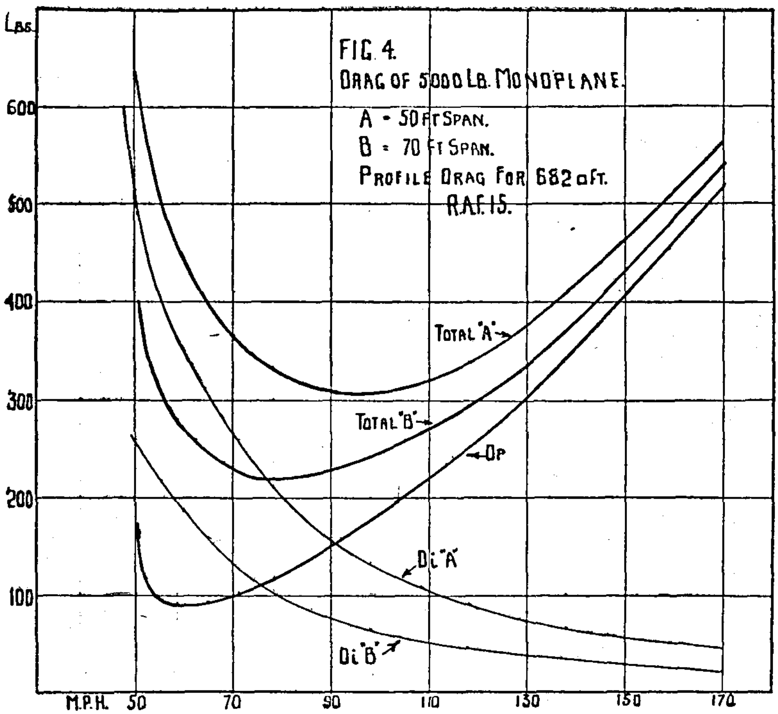


FIG. 3.

DRAG CURVE REDUCTION.
R.A.F. 15.



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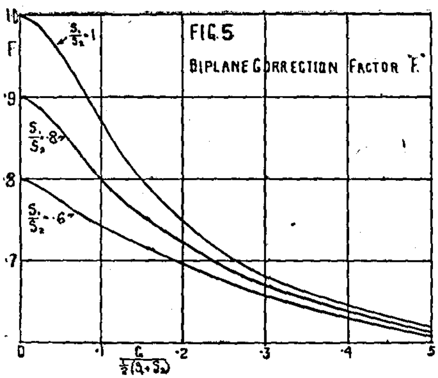

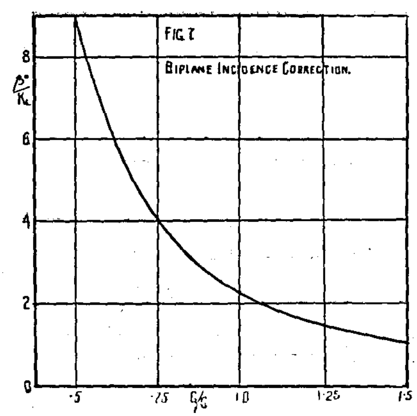

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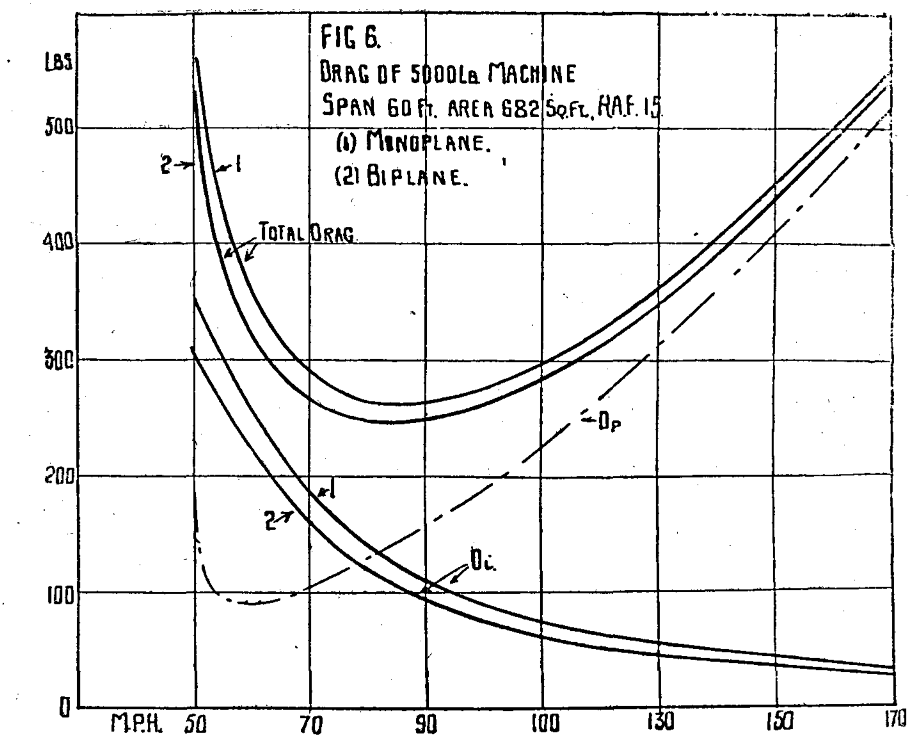

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