

## Long-Term Changes in the Periods of SX Phe

H. Landes<sup>A</sup>, K. R. Bamberg<sup>A,B</sup>, D. W. Coates<sup>A</sup> and K. Thompson<sup>A</sup>

<sup>A</sup> School of Physics, Building 27, Monash University, Melbourne VIC 3800, Australia

<sup>B</sup> Corresponding author. Email: keith.bamberg@sci.monash.edu.au

Received 2006 October 24, accepted 2007 March 29

**Abstract:** We have used times of maximum light for SX Phe, obtained by ourselves and other workers over 55 years to study the behaviour of the fundamental and first overtone radial pulsation modes of the star. We find  $(1/P_0)dP_0/dt$  to be  $(+2.53 \pm 0.05) \times 10^{-8} \text{ yr}^{-1}$  and  $(1/P_1)dP_1/dt$  to be  $(-1.60 \pm 0.03) \times 10^{-7} \text{ yr}^{-1}$ , which differ significantly from the value  $+1.9 \times 10^{-9} \text{ yr}^{-1}$  expected if the changes are due to standard evolution of the star. The residuals in O–C from a quadratic fit cannot be explained by a light–time effect in a binary. There is some evidence that the amplitudes of the two modes change slowly with time.

**Keywords:** stars: oscillations — stars: Population II — stars: variables: Delta Scuti — stars: individual: SX Phe

### 1 Introduction

SX Phe is the prototype of a class of high-amplitude Population II pulsators, the SX Phe stars. It oscillates in the fundamental radial mode ( $P_0 \sim 0.05496 \text{ d}$ ) and the first overtone radial mode ( $P_1 \sim 0.04277 \text{ d}$ ) with a marked beating effect in the light curve. Many of the properties of SX Phe have been successfully modelled by Petersen & Christensen-Dalsgaard (1996) who find that the observed period ratio of 0.7782, taken with the OPAL opacities (Iglesias, Rogers & Wilson 1992) constrain  $Z$  to be 0.001. Petersen & Christensen-Dalsgaard (1996) show that standard evolutionary models for  $Z = 0.001$  and the measured range  $T_{\text{eff}} = (7850 \pm 200) \text{ K}$  lead to a mass of 1.0 solar mass,  $M_{\text{bol}} = 2.68$ , and age 4.07 Gyr. The predicted parallax of 12 mas agrees closely with the subsequently published HIPPARCOS value of  $12.91 \pm 0.78 \text{ mas}$  (Perryman et al. 1997).

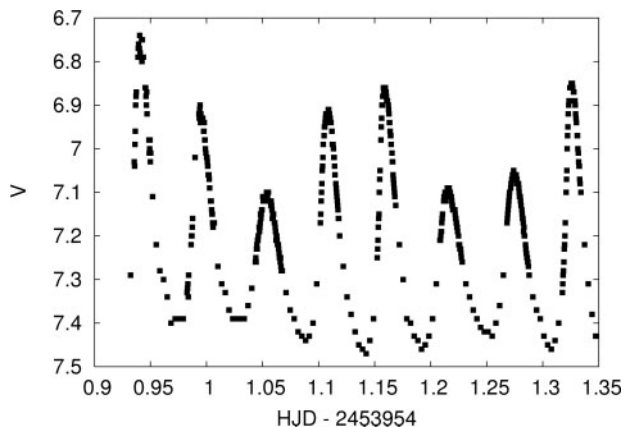
Measurements of the periods of SX Phe and their rates of change have been reported by several authors, for example Stock et al. (1972), Elst (1973), Coates et al. (1979), Coates, Halprin & Thompson (1982) and Thompson & Coates (1991). The derived long-term rates of change of these periods from quadratic fits to O–C data appear not to be constant, but this may be due to a constant long-term period increase or decrease being masked by much larger short-term changes. Garrido & Rodriguez (1996) provide evidence for two other modes of oscillation for SX Phe, a radial mode with amplitude 4.4 mmag in  $y$  at  $43.83 \text{ d}^{-1}$  or  $42.83 \text{ d}^{-1}$  and a non-radial mode with amplitude 3.3 mmag in  $y$  at  $16.32 \text{ d}^{-1}$  or  $17.32 \text{ d}^{-1}$ . These amplitudes are much smaller than those of the fundamental radial mode (205.9 mmag in  $y$ ) and the first overtone radial mode (74.2 mmag in  $y$ ) and so are unlikely to affect the beat-curve analysis we describe later.

Breger & Pamyatnykh (1998) review the relationship between period changes of Delta Scuti stars and stellar evolution, concluding that for most SX Phe stars the changes are not evolutionary in origin but are more likely caused by nonlinear effects in pulsation, the theory of which is not currently well understood. Some observed period changes are possibly due to a light–time effect in a binary, as may be the case for CY Aqr (Fu & Sterken 2003). Previous workers have concluded that most SX Phe stars show sudden jumps in period of the order  $\Delta P/P \sim 10^{-6}$ , with nearly constant periods in between.

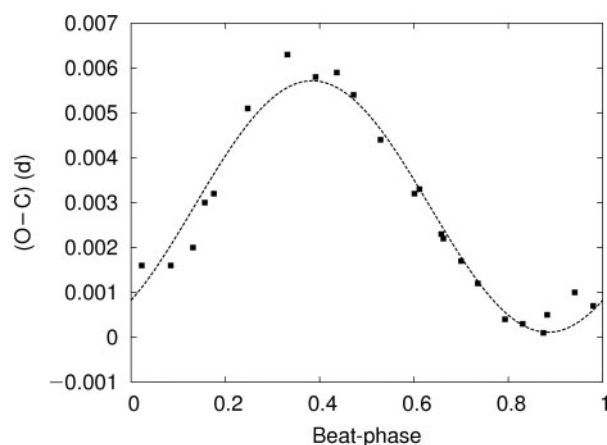
Because of beating between the fundamental and overtone oscillations, the magnitudes and times of maximum light for SX Phe differ from those they would have if the star only had one mode of oscillation. To a good approximation the discrepancy  $\Delta t$  in the time of maximum light varies sinusoidally with respect to the phase of the beat envelope at which the fundamental maximum falls. One may construct beat curves in the form

$$\Delta t = B + A \sin 2\pi(\phi_B + \delta) \quad (1)$$

where  $\phi_B = EP_0/P_{\text{Beat}}$ ,  $P_{\text{Beat}}$  is the beat period and  $E$  is the cycle number since epoch. Full details of this beat-curve analysis are given in Coates et al. (1979); Coates, Halprin & Thompson (1982), but in summary, changes in the mean level and phase of the sinusoidal curve reflect changes in the fundamental and beat periods of the star, from which changes in the overtone period can be derived. In this paper we have collected times of maximum light for SX Phe spanning 55 years, constructed beat curves, and hence deduced  $P_0$  and  $P_1$  at epoch and their rates of change. The amplitude of the beat curves is proportional to the ratio of the amplitudes of the overtone and fundamental



**Figure 1** Light-curve for the night of 2006 August 7/8.



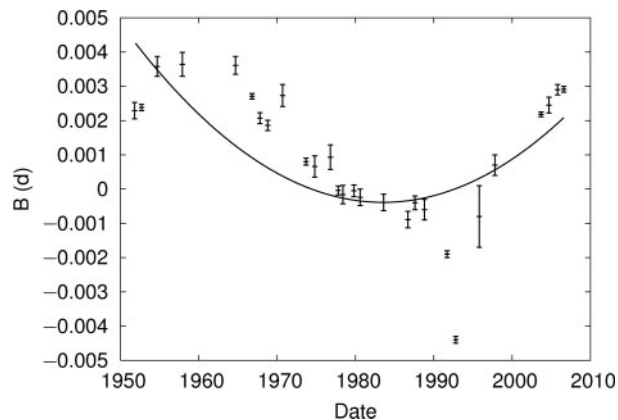
**Figure 2** Beat-curve for the season of 2006.

oscillations, so we also check for changes in the beat-curve amplitudes.

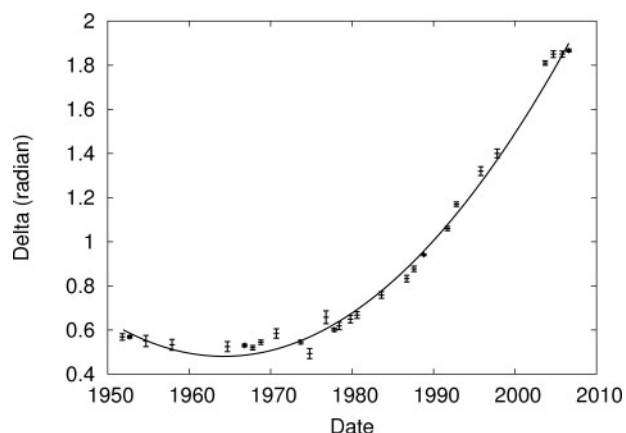
## 2 Experimental Methods

Some of the more recent data we use in our analysis were obtained with the newly commissioned remote-access 0.3-m Schmidt–Cassegrain telescope (Landes 2004) of the School of Physics at Monash University, equipped with a Santa Barbara Instrument Group (SBIG) ST7 CCD detector. 10-s exposures were made, and instrumental magnitudes were obtained using IRAF *qphot* photometry (Massey & Davis 1992) after flat-field and dark-count correction. In 2003 the target was observed using this instrument on five nights, yielding nine times of maximum light. Concurrently, as part of a Monash Third-Year project, Turnbull (2003) also made independent observations of maximum light in the *V* band for SX Phe using a photoelectric photometer at the Newtonian Focus of Monash University's 0.45-m Jeffree Telescope with output to a chart recorder via a DC amplifier. In 2004, 2005 and 2006 the remote-access instrument alone was used on a total of 24 nights to yield 19, 24 and 23 further times of maximum light respectively.

CCD data were obtained using the SBIG *G* filter. A satisfactory calibration to Johnson *V* was made using



**Figure 3** Data for beat-curve mean levels versus time, together with the quadratic fit.



**Figure 4** Data for beat-curve phase-shifts  $\delta$ , and quadratic fit.

observations of E-region standards. Same-frame differential photometry of the double-star system HD 134330 ( $V = 7.57$ ) and HD 134331 ( $V = 7.00$ ) indicate that the precision of the instrument for same-frame photometry is 0.01-mag for stars of magnitude 7 (Landes 2004).

On the night of 2006 August 7/8, prior to running the SX Phe image-acquisition program, six images were obtained of the Standard Star 44 (HD 156274) ( $V = 5.474$ ,  $B - V = 0.764$ ) in the region E7. This permitted us to calibrate the minimum and maximum *V* magnitudes for SX Phe that night to be  $6.76 \pm 0.02$  and  $7.46 \pm 0.02$ . This is in good agreement with the range 6.76 to 7.53 reported in the Combined General Catalogue of Variable Stars (Samus & Durlevich 2004). The light curve for this night is given in Figure 1.

Times of maximum light were determined by fitting polynomials to the data near maximum light. The precision varied between approximately 30 s and 100 s, depending on the quality of the night and the shape of the peak. The average precision is estimated to be 40 s.

## 3 Results and Discussion

References to the sources of data up to 1990 which we have used are given in Thompson & Coates (1991). References for data taken post-1990 are: Rush (1991), Stankov et al.

**Table 1. Fitting coefficients**

Coeff.	Mean level ( $B$ )
$a_B$	$(4.6 \pm 0.1) \times 10^{-6} \text{ d yr}^{-2}$
$b_B$	$(-2.95 \pm 0.06) \times 10^{-4} \text{ d yr}^{-1}$
$c_B$	$(4.29 \pm 0.08) \times 10^{-3} \text{ d}$
Coeff.	Phase shift ( $\delta$ )
$a_\delta$	$(7.91 \pm 0.07) \times 10^{-4} \text{ yr}^{-2}$
$b_\delta$	$(-1.97 \pm 0.04) \times 10^{-2} \text{ yr}^{-1}$
$c_\delta$	$(0.603 \pm 0.005)$

**Table 2. Calculated periods and rates of change**

	Periods (d) at epoch
$P_0$	0.054964432(1)
$P_1$	0.04277282(3)
$P_{\text{Beat}}$	0.19283613(4)
	Rates of change of periods ( $\text{d yr}^{-1}$ )
$dP_0/dt$	$(+1.39 \pm 0.04) \times 10^{-9}$
$dP_1/dt$	$(-6.8 \pm 0.1) \times 10^{-9}$
$dP_{\text{Beat}}/dt$	$(-1.56 \pm 0.03) \times 10^{-7}$

(2002) and Turnbull (2003). Innis, Lajus & Joner (private communication) provided us with unpublished data for SX Phe. The remaining data were taken using the 0.3-m remote-access telescope at Monash University by one of us (HL). All of the times of maximum light available to our knowledge are available online<sup>1</sup>.

For each annual block of data, we created a beat curve as in equation (1), using the method described in Coates et al. (1979) with time origin HJD 2433923.9596 and assumed periods at epoch,  $P_0(\text{ass}) = 0.054964477 \text{ d}$ ,  $P_{\text{Beat}}(\text{ass}) = 0.19283428 \text{ d}$  as given in Thompson & Coates (1991). As an example, the beat curve for the 2006 season is given in Figure 2, for which the RMS deviation between the data and the fitted curve is 0.00037 d (32 s).

Figures 3 and 4 show graphs of the mean level ( $B$ ) and phase shift ( $\delta$ ) versus time, together with quadratic fits to the data made using singular value decomposition (SVD) with weighted points, as described by Press et al. (1992). The error bars in Figures 3 and 4 represent formal statistical errors.

The fitting parameters for the quadratics are given in Table 1, as  $a_B, b_B, c_B$  in  $B = a_B t^2 + b_B t + c_B$  and  $a_\delta, b_\delta, c_\delta$  in  $\delta = a_\delta t^2 + b_\delta t + c_\delta$ , where  $t$  is time (yr) since epoch (1951.8). These parameters enable us to calculate the best estimates of  $P_0$  and  $P_1$  at epoch, and the rates of change (assumed constant) of these periods, using the methods described in Coates, Halprin & Thompson (1982), which we now summarise.

We let  $\alpha_0$  and  $\alpha_1$  be the true fundamental and overtone periods at epoch, and  $\beta_0$  and  $\beta_1$  be their constant rates of change. Then the periods at any time  $t$  since epoch will be

$$P_0(t) = \alpha_0 + \beta_0 t \tag{2}$$

$$P_1(t) = \alpha_1 + \beta_1 t \tag{3}$$

The beat period at any time,  $P_{\text{Beat}}(t)$ , can be shown, to a good approximation, to be

$$P_{\text{Beat}}(t) = \alpha_{\text{Beat}} + \beta_{\text{Beat}} t \tag{4}$$

where

$$\alpha_{\text{Beat}} = \frac{\alpha_0 \alpha_1}{(\alpha_0 - \alpha_1)}, \quad \beta_{\text{Beat}} = \frac{\alpha_0^2 \beta_1 - \alpha_1^2 \beta_0}{(\alpha_0 - \alpha_1)^2} \tag{5}$$

The true periods at epoch,  $\alpha_0$  and  $\alpha_{\text{Beat}}$ , and hence  $\alpha_1$ , are found from the gradients of the fitted functions to  $B$  and  $\delta$ , evaluated at epoch:

$$\frac{dB}{dt} = 2a_B + b_B = b_B \text{ at epoch} \tag{6}$$

$$\frac{d\delta}{dt} = 2a_\delta + b_\delta = b_\delta \text{ at epoch} \tag{7}$$

Now, for the periods  $P_0(t)$  and  $P_{\text{Beat}}(t)$  at any time, we have

$$P_0(t) = P_0(\text{ass}) \left[ 1 + \frac{dB/dt}{365.2563} \right] \tag{8}$$

$$P_{\text{Beat}}(t) = P_{\text{Beat}}(\text{ass}) \left[ 1 + \frac{dB/dt - P_{\text{Beat}}(\text{ass})d\delta/dt}{365.2563} \right] \tag{9}$$

where  $P_0(\text{ass})$  and  $P_{\text{Beat}}(\text{ass})$  take the assumed values 0.054964477 d and 0.19283428 d as given earlier.

Substituting from equations (6) and (7) into (8) and (9), then differentiating with respect to time, we have

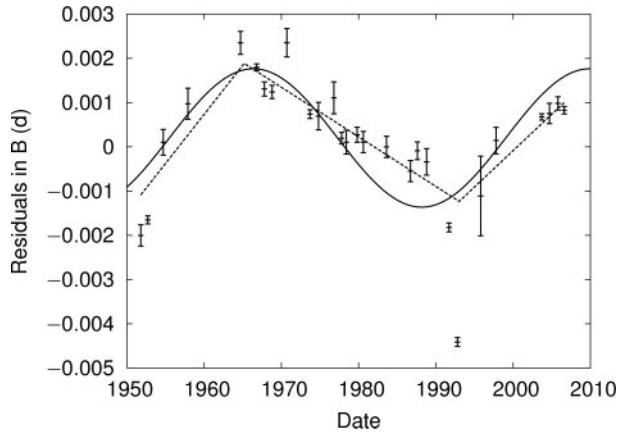
$$\frac{dP_0}{dt} = \frac{2a_B P_0(\text{ass})}{365.2563} \tag{10}$$

$$\frac{dP_{\text{Beat}}}{dt} = 2P_{\text{Beat}}(\text{ass}) \left[ \frac{a_B - a_\delta P_{\text{Beat}}(\text{ass})}{365.2563} \right] \tag{11}$$

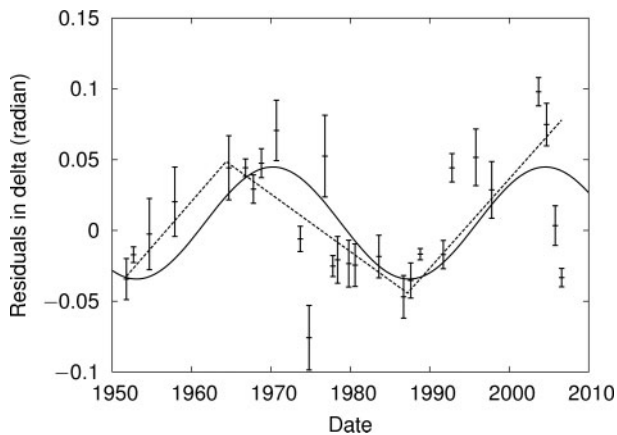
Equations (10) and (11) yield  $\beta_0$  and  $\beta_{\text{Beat}}$ , and having obtained  $\alpha_0$  and  $\alpha_1$  previously, we can calculate  $\beta_1$  using equations (5). Note that the calculated rates of change of the periods will be in  $\text{d yr}^{-1}$  if the periods are substituted in d,  $a_B$  in  $\text{d yr}^{-2}$  and  $b_\delta$  in  $\text{yr}^{-2}$ . The results are given in Table 2, which lists the calculated periods at epoch (1951.8) and their rates of change, based on the assumption that the rates of change are constant.

We can compare the values of  $(1/P)dP/dt$  with that expected from standard evolutionary theory. Petersen (private communication) kindly supplied us with an evolutionary model computed by J. Christensen-Dalsgaard for their successful fitting of the major properties of SX Phe (Petersen & Christensen-Dalsgaard 1996). This model yields the result  $(1/P)dP/dt = +1.9 \times 10^{-9} \text{ yr}^{-1}$ ,

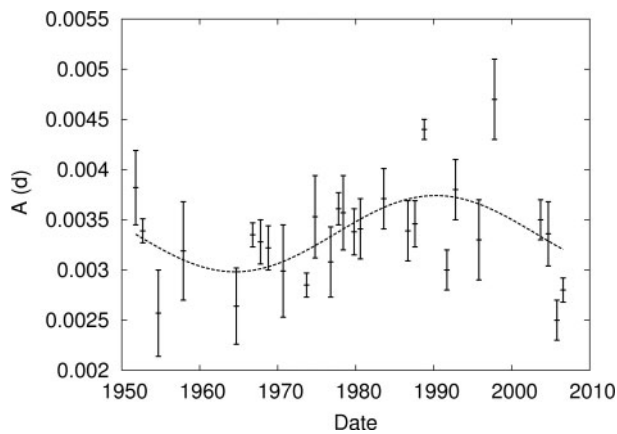
<sup>1</sup> <http://www.physics.monash.edu.au/research/SXPhe.zip>



**Figure 5** Residuals in beat-curve mean values ( $B$ ), with piecewise linear and sinusoidal fits.



**Figure 6** Residuals in beat-curve phase-shifts ( $\delta$ ), with piecewise linear and sinusoidal fits.



**Figure 7** Beat-curve amplitudes ( $A$ ), with sinusoidal fit.

compared with the values  $(+2.53 \pm 0.05) \times 10^{-8} \text{ yr}^{-1}$  for the fundamental period, and  $(-1.60 \pm 0.03) \times 10^{-7} \text{ yr}^{-1}$  for the first overtone from our quadratic fits to the data for SX Phe. The large discrepancies may be explained by small real underlying constant rates of change being masked by much larger short-term changes.

The residuals between the quadratic fits to the mean level ( $B$ ) and phase shift ( $\delta$ ) and the data are shown

in Figures 5 and 6. Residuals in O–C ( $B$ ) similar in form, but of approximately twice the amplitude, for the single-mode SX Phe star CY Aqr were modelled as a light-time effect in a binary by Fu & Sterken (2003). We are able to test whether a binary explanation is possible for SX Phe, because the behaviour of both the fundamental period and the overtone must be consistent with the same light-time effect. A light-time shift of  $\Delta t$  will increase  $B$  (O–C for the fundamental mode) by  $\Delta t$  and decrease  $\delta$  by  $\Delta t/P_{\text{Beat}}$ . Thus the residuals in  $B$  and the residuals in  $\delta$  must be of the same form but *out of phase*, if they are due to a light-time effect in a binary. In fact Figures 5 and 6 show that these residuals are indeed similar in form (and so are probably related) but that they are *in phase*. In addition, the residuals have the wrong relative amplitudes for the binary case. From the sinusoidal fit to the residuals in Figure 5 we may deduce  $\Delta t \sim 0.0016 \text{ d}$ . This would give an expected amplitude in the phase term residuals of  $0.0016/0.1928 \sim 0.083$ , whereas the observed amplitude of those residuals is  $\sim 0.04$ . We therefore conclude that these residuals are not due to a light-time effect in a binary.

The residuals are also not to be explained by the offset of approximately 35 s which has accumulated between UT(C) and ephemeris (or atomic) time in the period between 1951 and 2005. We have applied a correction for this (on the assumption that all the observations we have used are based on UT) and find negligible changes in the above results.

The beat curve for data taken in 1992 yields a value of  $B$  which has high statistical precision but a large negative residual. This could of course represent the real behaviour of the star at this time, but we investigated whether our findings would change significantly if this value of  $B$  were removed, and find that they do not.

As can be seen from Figures 5 and 6, both piece-wise linear and sinusoidal fits describe the residuals in  $B$  and  $\delta$  adequately. It is not clear whether either of the fits truly describes the data, but each would have physical meaning if it were a true description. The linear fits would correspond to sudden jumps in the periods, in the case of the fundamental by  $\Delta P/P \sim 9 \times 10^{-7}$  (positive in approximately 1965, negative in approximately 1993). This figure is typical for SX Phe stars which exhibit sudden jumps in period (Breger & Pamyatnykh 1998). The sinusoidal fits would represent cyclic changes in the periods, on a timescale of  $43 \pm 10 \text{ yr}$ . Interestingly, although the data are scattered, a sinusoidal fit with a similar timescale to this (53 yr) may be cautiously suggested for the amplitudes of the beat-curves (Figure 7).

It can be shown that the amplitude of the beat curve is proportional to the ratio  $m_1/m_0$ , where  $m_1$  is the amplitude of the overtone signal and  $m_0$  is the amplitude of the fundamental signal. Thus we may have evidence that the relative amplitudes of the pulsation modes may be changing on a similar timescale to the possible changes in their periods. At the moment one cannot say whether both of the amplitudes are changing, or only one of them. An important caveat is that we cannot be sure yet whether

truly cyclic variations are occurring at all, given that the star has only been observed for a time close to the putative timescale of the variations.

#### 4 Conclusions

It is possible that the pulsation properties of SX Phe are undergoing cyclical changes. To test this suggestion will require good measurements of times of maximum light over the next few years.

Other SX Phe stars may also undergo cyclical changes, perhaps on a similar timescale, so that continuing observations of all SX Phe stars are quite important.

It is also important that high-precision photometry be carried out for SX Phe itself, to confirm or otherwise the existence of the two periods for which Garrido & Rodriguez (1996) found evidence.

#### Acknowledgments

We thank M. D. Joner, E. F. Lajus and J. Innis for access to photometric data for SX Phe. The staff of the Electronic and Mechanical Workshops of the School of Physics, Monash University, particularly Alan Holland and Nino Benci, provided invaluable assistance in the commissioning of the 0.3-m telescope for remote-access observing. We used the IRAF package, from the US National Optical Astronomical Observatories in our data reduction. Thanks are also due to the Australian Research Council for the award of a Small Grant which financed the original purchase of the telescope and a CCD camera. D. Coates thanks the Faculty of Science, Monash University, for the provision of an Honorary Research Fellowship.

#### References

- Breger, M. & Pamyatnykh, A. A., 1998, *A&A*, 332, 958  
Coates, D. W., Dale, M., Halprin, L., Robinson, J. & Thompson, K., 1979, *MNRAS*, 187, 83  
Coates, D. W., Halprin, L. & Thompson, K., 1982, *MNRAS*, 199, 135  
Elst, E. W., 1973, *A&A*, 23, 285  
Fu, J. N. & Sterken, C., 2003, *A&A*, 405, 685  
Garrido, R. & Rodriguez, E., 1996, *MNRAS*, 281, 696  
Iglesias, C. A., Rogers, F. A. & Wilson, B. G., 1992, *ApJ*, 397, 717  
Landes, H., 2004, MAppSci thesis, Monash University  
Massey, P. & Davis, L. E., 1992, *A User's Guide to Stellar CCD Photometry with IRAF*  
Perryman, M. A. C., et al., 1997, *A&A*, 323L, 49  
Petersen, J. O. & Christensen-Dalsgaard, J., 1996, *A&A*, 312, 463  
Press, W. H., Teukolsky, S. A., Vetterling, W. T. & Flannery, B. P., 1992, *Numerical Recipes in Fortran* (Cambridge: Cambridge University Press)  
Rush, R., 1991, BSc Hons report, Monash University  
Samus, N. N. & Durlevich, O. V., 2004, *Combined General Catalogue of Variable Stars*, Vizier Online Data Catalogue  
Stankov, A., Sinachopoulos, D., Elst, E. & Breger, M., 2002, *CoAst*, 141, 72  
Stock, J., Kunkel, W. E., Hesser, J. E. & Lasker, B. M., 1972, *A&A*, 21, 249  
Thompson, K. & Coates, D. W., 1991, *PASA*, 9, 281  
Turnbull, R., 2003, BSc Project report, Monash University