CORRESPONDENCE.

POLICY-VALUES.

To the Editor of the Transactions of the Faculty of Actuaries.

SIR,—The following simple demonstration showing the relation between the values of a policy subject to a yearly premium and one subject to premiums payable m times a year, or, in symbols, that ${}_nV_x^{(m)} = {}_nV_x\left(1+\frac{m-1}{2m}\pi_x\right)$, may interest some of your readers.

We have

$$\begin{aligned} {}_nV_x^{(m)} - {}_nV_x &= A_{x+n} - \pi_x^{(m)}\mathbf{a}_{x+n}^{(m)} - A_{x+n} + \pi_x\mathbf{a}_{x+n} \\ &= \pi_x\mathbf{a}_{x+n} - \pi_x^{(m)}\mathbf{a}_{x+n}^{(m)} \\ &= \frac{A_x}{\mathbf{a}_x}\cdot\mathbf{a}_{x+n} - \frac{A_x}{\mathbf{a}_x^{(m)}}\cdot\mathbf{a}_{x+n}^{(m)} \\ &= A_x\left\{\frac{\mathbf{a}_{x+n}\mathbf{a}_x^{(m)} - \mathbf{a}_x\mathbf{a}_{x+n}^{m}}{\mathbf{a}_x\mathbf{a}_x^{(m)}}\right\} \\ &= \frac{A_x}{\mathbf{a}_x^{(m)}}\left\{\frac{\mathbf{a}_{x+n}\left(\mathbf{a}_x - \frac{m-1}{2m}\right) - \mathbf{a}_x\left(\mathbf{a}_{x+n} - \frac{m-1}{2m}\right)}{\mathbf{a}_x}\right\} \\ &= \pi_x^{(m)}\cdot\frac{m-1}{2m}\cdot\frac{\mathbf{a}_x - \mathbf{a}_{x+n}}{\mathbf{a}_x} \\ &= \frac{m-1}{2m}\cdot\pi_x^{(m)}\cdot{}_nV_x \end{aligned}$$

$$\therefore\ {}_nV_x^{(m)} = {}_nV_x\left(1+\frac{m-1}{2m}\pi_x^{(m)}\right)$$

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It will be observed that the annuities-due, payable yearly and fractionally have been used throughout in the above demonstration, and that the ordinary correction $\frac{m-1}{2m}$ in the well-known approximation $a_x^{(m)} = a_x + \frac{m-1}{2m}$, is retained in the form $\mathbf{a}_x^{(m)} = \mathbf{a}_x - \frac{m-1}{2m}$; with the result that the proof appears to be easier to follow than that given in the *Institute of Actuaries' Text-Book*, Part II, page 345.

The annuity-due might with advantage be oftener used in such demonstrations; and, as it is so closely connected with the calculation of premiums, it is, I think, a cause for regret that, in the monetary tables based upon the mortality experiences, the values of the annuity-due and the corresponding logarithms are not tabulated, in preference to the values of the annuity payable at the end of the year. If this were done, the risk of error would be reduced, and time saved, in calculations where at present it is necessary to interpolate to obtain the annuity-due, either whole-life or temporary, for a fractional age.

Yours faithfully,

M. M. LEES.

EDINBURGH, 14 *March* 1902.