DYNAMO–ACTION IN ACCRETION DISKS

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Abstract. Accretion disks are approximated by thin tori and the generation of magnetic fields by torus–dynamos is investigated. Solutions for the general $\alpha^2\omega$–dynamo embedded into vacuum and into an ideally conducting medium are presented. Whereas the former solutions are qualitatively similar to those obtained for thin disks, there is a mode for the latter peculiar to torus–geometry. Excitation conditions for torus–dynamos are compared to those realized in accretion disks in cataclysmic variables, around T Tauri stars and in AGN's.

Key words: Mean–Field Electrodynamics – Dynamo Theory – Accretion Disks

1. Introduction

There is no direct observational evidence for magnetic fields in accretion disks. Nevertheless, the question of dynamo–action is not purely academic. If magnetic fields could be produced, they were a very effective means for the transport of angular momentum, for swing–amplification or for jet collimation. We shall try to find out whether the generation of suitable magnetic fields is possible by presenting the properties of axisymmetric torus dynamos (Sect. 2) and by comparing the resulting excitation conditions with those realized in accretion disks in cataclysmic variables, around young stellar objects and in AGN's (Sect. 3).

2. Torus–Dynamo

If the magnetic field strength is split into a poloidal and toroidal part — in the case of axial symmetry according to

$$\mathbf{B} = \mathrm{rot} A\mathbf{e}_\varphi + B\mathbf{e}_\varphi$$

($\mathbf{e}_\varphi$ unit vector in azimuthal direction) — the corresponding dynamo equations are

$$\frac{\partial A}{\partial t} + D(\mathrm{rotrot} A\mathbf{e}_\varphi)\cdot\mathbf{e}_\varphi = \alpha B$$

$$\frac{\partial B}{\partial t} + D(\mathrm{rotrot} B\mathbf{e}_\varphi)\cdot\mathbf{e}_\varphi = \mathrm{rot}(\mathbf{v}\times\mathrm{rot} A\mathbf{e}_\varphi + \alpha A\mathbf{e}_\varphi)\cdot\mathbf{e}_\varphi$$

(D effective magnetic diffusivity). A torus with equatorial radius a and with meridional radius ρ may conveniently be desribed by torus coordinates η, θ, φ, where the surface of the torus is given by $\eta = \eta_0 = \sinh^{-1}(a/\rho), -\pi \le \theta \le +\pi$ and $0 \le \varphi \le 2\pi$. For details of the formalism and for the determination of the decay modes see Grosser (1988). The induction effects driving the dynamo are:
(i) a velocity field due to differential rotation

$$\mathbf{v} = \omega(r) r\mathbf{e}_\varphi$$

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(r distance to the axis of rotation). For the present purpose Kepler disks have been considered and the following first order expansion

$$\omega = \omega_0 + \omega_0'(r - a)$$

with

$$\omega_0 = \left(\frac{GM}{a^3}\right)^{1/2}, \qquad \omega_0' = -\frac{3}{2}\frac{\omega_0}{a}$$

has been used;

(ii) α–effect due to turbulent motion within the rotating disk according to

$$\alpha = \alpha_0 \sin\theta;$$

an order of magnitude estimate for α_0 is

$$\alpha_0 = \frac{\omega_0 l^2}{\rho}$$

(l correlation length).

The effective magnetic diffusivity of a turbulent medium with mean velocity v_{T} is assumend to be

$$D = \frac{1}{3} v_{\mathrm{T}} l.$$

With these quantities the induction effects may be written in dimensionless form by two characteristic Reynolds numbers

$$R_\omega = \frac{\omega_0' a^3}{D}, \qquad R_\alpha = \frac{\alpha_0 a}{D}.$$

The torus containing the dynamo–active plasma is embedded either into vacuum or into a field–free ideally conducting medium, whence suitable boundary conditions may be formulated for the field variables A and B. In satisfying them a boundary value problem is obtained with the eigenvalue Ω defined by $A, B \sim \exp(\Omega t)$. It is solved by expanding the space–dependent parts of A and B into a complete set of functions, each of which is satisfying the boundary conditions separately (see Grosser 1988). The expansion coefficients are determined by a system of linear homogeneous equations and Ω follows from its vanishing secular determinant. As for kinematic linear dynamos in other geometries this system of linear equations splits into two independent parts describing fields symmetric ("quadrupolar") and antisymmetric ("dipolar") with respect to the equatorial plane.

The excitation properties of torus–dynamos may be inferred quite generally by-considering the different Ω–paths in the complex plane as R_ω and R_α are varied. In case of $|R_\omega| \ll |R_\alpha|$ these properties depend of $R_{\alpha\omega} = R_\alpha R_\omega$ only ("$\alpha\omega$–dynamo"), but also the more general $\alpha^2\omega$–dynamo is investigated for what follows. Whenever such a path — originating at Ω for some decay–mode somewhere along the negative Ω_{R}–axis — crosses the Ω_{I}–axis, a dynamo is excited. The values of R_α and R_ω corresponding to such situations are called critical dynamo numbers. In Figures 1 and 2 Ω–paths are presented for an $\alpha\omega$–dynamo embedded into vacuum. For details see Schmitt (1990) and Deinzer et al. (1993). In Table I critical dynamo numbers are listed for general $\alpha^2\omega$–dynamos. These numbers are strongly dependent on the aspect ratio $(a/\rho) = \sinh \eta_0$. This strong dependence almost disappears, however, if the numbers

$$R_\omega^\rho = \frac{\omega_0' a \rho^2}{D} = R_\omega \left(\frac{\rho}{a}\right)^2, \qquad R_\alpha^\rho = \frac{\alpha\rho}{D} = R_\alpha \left(\frac{\rho}{a}\right)$$

are evaluated instead. Thus the thickness, ρ, rather than the distance from the axis, a, is the decisive length scale. In this respect the behaviour of torus–dynamos is quite similar to dynamos in infinitely extended disks (see Ruzmaikin et al. 1988, Stepinski & Levy 1991). According to Table I the $\alpha\omega$–dynamo excited most easily is of quadrupolar symmetry. It evolves out of the decay mode with the slowest decay–rate. Surprisingly, for dipolar symmetry the analogous decay mode never becomes excited although its decay–rate is still slower. Its magnetic surfaces $A \sinh \eta / c =$ const., however, are over a wide range of almost cylindrical shape and are thus nearly coinciding with the surfaces of constant ω; hence differential rotation cannot achieve the necessary induction. This conjecture is confirmed by the behaviour of the more general $\alpha^2\omega$–dynamo (see Kehrer 1991), where for small values of $|R_\omega / R_\alpha|$ the dipolar mode is excited easier indeed — see Table I.

Now, the dynamo–active plasma may not always be embedded into vacuum. To get a feeling as to the importance of the conductivity of the surrounding medium, tori embedded into an ideally conducting medium were investigated, too (see Sonja Richter 1992). Results for α^2– and $\alpha\omega$–dynamos are compared to those embedded into vacuum in Table II. In all cases the former are excited easier. The dynamo excited most easily is of quadrupolar symmetry throughout, which is growing out of the decay–mode with the slowest decay–rate indeed. Attention should be paid to its very singular field configuration: it consists of a predominantly toroidal part of constant field strength. This is true already for the decay–mode which (due the very slow decay–rate of $\Omega = -a^2/D$) needs only marginal induction to become excited: $R_\alpha^2 = 8$ independent of R_ω. This mode remains excited, however, only for $R_\omega > 0$; for $R_\omega < 0$ (as realized in Keplerian accretion disks) differential rotation seems to act destructively and its excitation is only marginal for the $\alpha\omega$–dynamo. Yet, the quadrupolar mode coming next is excited under less favourable conditions than the first dipolar mode — see Table II. Hence for $R_\omega < 0$ the mode excited most easily will be considered to be of dipolar symmetry in what follows (which in turn is less easily excited than the quadrupolar mode of an $\alpha\omega$–dynamo embedded into vacuum.

3. Application to Accretion Disks

To compare critical dynamo numbers with actual physical situations, the order of magnitude expressions for ω_0' and D are substituted into the above formulae for R_ω and R_α

$$R_\omega = \frac{\omega_0' a^3}{D} = -4.5 \left(\frac{\omega_0 \rho}{v_T}\right) \left(\frac{\rho}{l}\right) \left(\frac{a}{\rho}\right)^2 < 0,$$

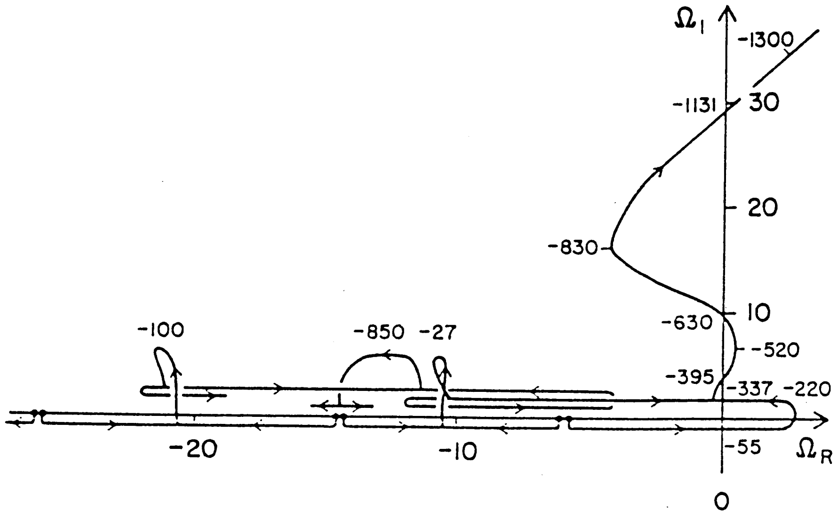

Fig. 1. Evolution of the eigenvalues Ω of the quadrupolar symmetry in the complex (Ω_R, Ω_I)-plane for varying dynamo number $R^{\rho}_{\alpha\omega} = R^{\rho}_{\alpha} R^{\rho}_{\omega} < 0$. A torus with $\eta_0 = 1.5$ was chosen. $|R^{\rho}_{\alpha\omega}|$ increases along the curves as indicated by the arrowheads. Due to the position of Ω in the diagram the dynamo mode is growing ($\Omega_R > 0$) or decaying ($\Omega_R < 0$), non-oscillating ($\Omega_I = 0$)) or oscillating ($\Omega_I > 0$). The Ω's are given in units of D/ρ^2. The thick dots on the real axis give the position of the decay modes ($R^{\rho}_{\alpha\omega} = 0$).

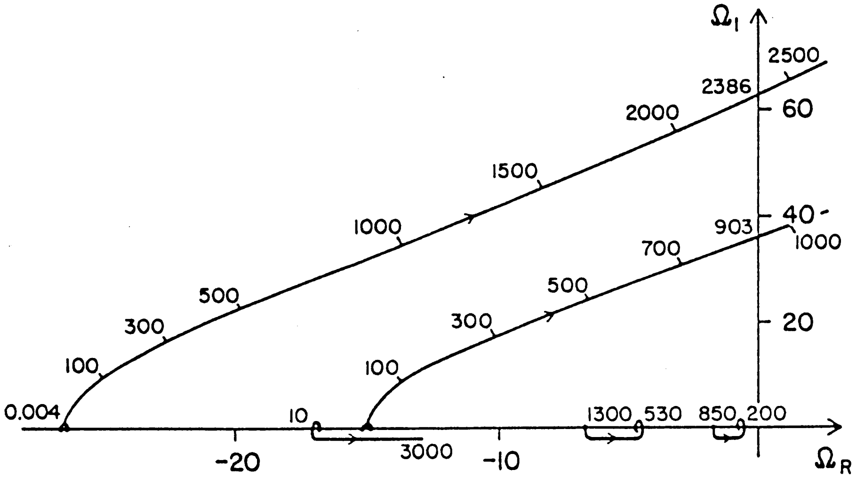

Fig. 2. Same as Fig. 1 for dipolar symmetry and positive dynamo number

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TABLE I

Critical Dynamo Numbers for Torus $\eta_0 = 1.5$ Embedded in Vacuum (* not oscillating)

R_ω/R_α	R_α^2	
	Dipole	Quadrupole
-100	61 ($R_{\alpha\omega} = -7423$)	5.1* ($R_{\alpha\omega} = -527$*)
-10	220	37*
-1	108*	116*
0	104*	175*
+1	98*	192
+10	81*	158
+100	72 ($R_{\alpha\omega} = 8713$)	39 ($R_{\alpha\omega} = 4572$)

TABLE II

Selected Critical Dynamo Numbers for a Torus $\eta_0 = 3$ Embedded in Vacuum and an Ideal Conducting Medium

	α^2–Dynamo: R_α^2		$\alpha\omega$–Dynamo: $R_{\alpha\omega}$			
	Vacuum	Id. Cond.	Vacuum		Ideal Conductor	
Dipole	2146	1033	-766450	866053	-185600	20600
Quadrupole	3838	8	-47000	444500	[$R_\alpha^2 = 8$]	$R_\alpha^2 = 8$
					-307900	186600

$$R_\alpha = \frac{\alpha_0 a}{D} = 3\left(\frac{\omega_0 \rho}{v_T}\right)\left(\frac{l}{\rho}\right)\left(\frac{a}{\rho}\right).$$

Thus, besides geometrical ratios a characteristic Rossby–number is determining the excitation conditions. Since for $l \ll \rho$ and $\rho < a$ the ratio

$$\frac{R_\omega}{R_\alpha} = -1.5\left(\frac{\rho}{l}\right)^2\left(\frac{a}{\rho}\right) \ll -1$$

assumes very large values we shall be dealing with $\alpha\omega$–dynamos in the following, characterized by the dynamo number

$$R_{\alpha\omega} = R_\alpha R_\omega = -13.5\left(\frac{\omega_0 \rho}{v_T}\right)^2\left(\frac{a}{\rho}\right)^3.$$

According to Tables I and II an $\alpha\omega$–dynamo is excited if

$$R_{\alpha\omega} \leq -47\left(\frac{a}{\rho}\right)^3$$

for outside vacuum — the mode has quadrupolar symmetry — and if

$$R_{\alpha\omega} \leq -185\left(\frac{a}{\rho}\right)^3$$

for outside medium of infinite conductivity — the mode has dipolar symmetry. For want of anything better

$$v_T \sim 0.1 \cdot c_s$$

(c_s velocity of sound) has been assumed for computing the Rossby–number.

3.1. Accretion Disks in Cataclysmic Variables

Characteristic parameters for a corresponding binary system with a white dwarf as the accreting component are

$$M = 0.5 M_\odot = 10^{33}\text{g}$$

$$a = 0.6 R_\odot = 4.2 \times 10^{10}\text{cm}$$

$$\rho = 10^8\text{cm}$$

$$v_T = 0.1 \cdot c_s = 0.1(1.7 \times 10^4 T^{1/2}) = 5.4 \times 10^4\text{cm/sec} \qquad \text{for} \qquad T = 10^3\text{K}$$

which yield

$$R_{\alpha\omega} = -41 \left(\frac{a}{\rho}\right)^3 .$$

Comparing this with the numbers above, a quadrupolar mode is likely to be excited in a dynamo embedded into vacuum.

3.2. Accretion Disks Around Young Stellar Objects

Characteristic parameters for accretion disks around T Tauri stars are

$$M = 2 M_\odot$$

$$10^{12}\text{cm} < a < 10^{17}\text{cm}$$

$$\rho = R_\odot$$

$$v_T = 0.1 \cdot c_s = 1.7 \times 10^4\text{cm/sec} \qquad \text{for} \qquad T = 100\text{K}$$

which yield

$$-10^{-5} \left(\frac{a}{\rho}\right)^3 > -13.5 \left(\frac{\omega_0 \rho}{v_T}\right) \left(\frac{a}{\rho}\right)^3 > -2 \times 10^3 \left(\frac{a}{\rho}\right)^3 .$$

Hence excitation is well possible depending on where the dynamo–active region is loacated within the disk. A dipolar field would be desirable for producing a central cavity in the disk by swing amplification (see Tagger et al. 1990) as a possible interpretation of the spectra of weak line T Tauri stars (see Montmerle 1990).

3.3. Accretion Disks in Active Galactic Nuclei

A hypothetical model for AGN's is: a massive black hole ($10^8 M_\odot$) is surrounded by an accretion disk of torus shape with an equatorial radius of several Schwarzschild radii.

Here the velocity of sound, c_s, may be gathered from vertical hydrostatic equilibrium

$$-\frac{1}{\rho_m}\frac{\partial P}{\partial z} = \frac{GM}{r^2}\frac{z}{r}$$

(P gas pressure, ρ_m density), which is estimated to be

$$\frac{c_s^2}{H_p} = \omega_0^2 a \frac{H_p}{a} \qquad \text{or} \qquad c_s = \omega_0 H_p$$

(H_p pressure scale height). With this estimate the dynamo number becomes very simple

$$R_{\alpha\omega} = -13.5\left(\frac{\omega_0 \rho}{v_T}\right)^2\left(\frac{a}{\rho}\right)^3 = -13.5\left(\frac{c_s}{v_T}\frac{\rho}{H_p}\right)^2\left(\frac{a}{\rho}\right)^3 .$$

A Rossby–number between 1 and 10^4 is plausible and excitation is well possible. A quadrupolar field with field lines leaving the disk at very small inclination would be desirable for the collimation of the bipolar jet–outflow (see Blandford & Payne 1982).

4. Conclusion

In choosing a torus for investigating dynamo–action in accretion disks we were guided by the idea of treating a configuration of finite but non–vanishing distance from the axis of symmetry which is nevertheless simple enough to allow for the solution as an eigenvalue problem and thus providing the full Ω–spectrum at one glance. As the above results are indicating this special geometrical shape is of no great importance if embedded into vaccum; the results are not dramatically different from those obtained for thin disks for example. If embedded into an ideally conducting medium, however, the easiest excited mode is peculiar to torus geometry and does not occur in ususal disks. Further work on its relevance is in progress.

The knowledge of the Ω–spectrum is very important for starting initial–value computations, as carried out presently for torus–dynamos by Susanne Bellmer (1993). The expansion into a suitable set of functions and the solution of an eigenvalue problem is presently extended by M. Foth (1993) to non–axisymmetric torus–dynamos.

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