COLUMN ACCRETION ON TO WHITE DWARFS

A. R. King
Astronomy Department, University of Leicester,
Leicester LE1 7RH, England.

The problem of column accretion on to white dwarfs in the AM Her and DQ Her systems is reviewed. Particular attention is paid to recent progress in explaining the large soft X-ray - EUV fluxes observed in these systems in terms of nonlocal electron energy transport into the white dwarf photosphere.

1. INTRODUCTION

In the AM Her and DQ Her systems ('polars' and 'intermediate polars' respectively) the magnetic field of the white dwarf is thought to be strong enough to channel the accretion flow from the secondary star radially on to the magnetic polecaps in a column-like configuration (Fig. 1). Because most of the accretion energy must be released near the white dwarf surface an understanding of column accretion is basic to any discussion of these systems. Moreover, from a theoretical point of view an accretion column represents almost the simplest accretion problem one can pose, being essentially one-dimensional. We might therefore hope for a more rapid progress in our understanding here than in the study of more complicated accretion flows. For these reasons the study of column accretion on to white dwarfs has received growing attention in recent years.

Figure 1 gives a basic picture of an accretion column (not to scale). The cool, freely-falling (and thus highly supersonic) accretion flow is channelled by the magnetic field on to a small fraction f ($\lesssim$ one percent) of the white dwarf surface area. For accretion rates relevant to the AM Her and DQ Her systems the accreting ions cannot penetrate to the white dwarf photosphere before being stopped in the atmosphere. Thus there must be some kind of standing shock above the polecaps in which the kinetic energy of the accretion flow (residing mostly in the ions) is thermalized. The precise nature of this shock and the 'settling' flow beneath it are the main things one hopes to learn from a study of column structure; given these one can for example predict where in the electromagnetic spectrum the accretion luminosity will ultimately be radiated.

M. Livio and G. Shaviv (eds.), Cataclysmic Variables and Related Objects, 181–197.

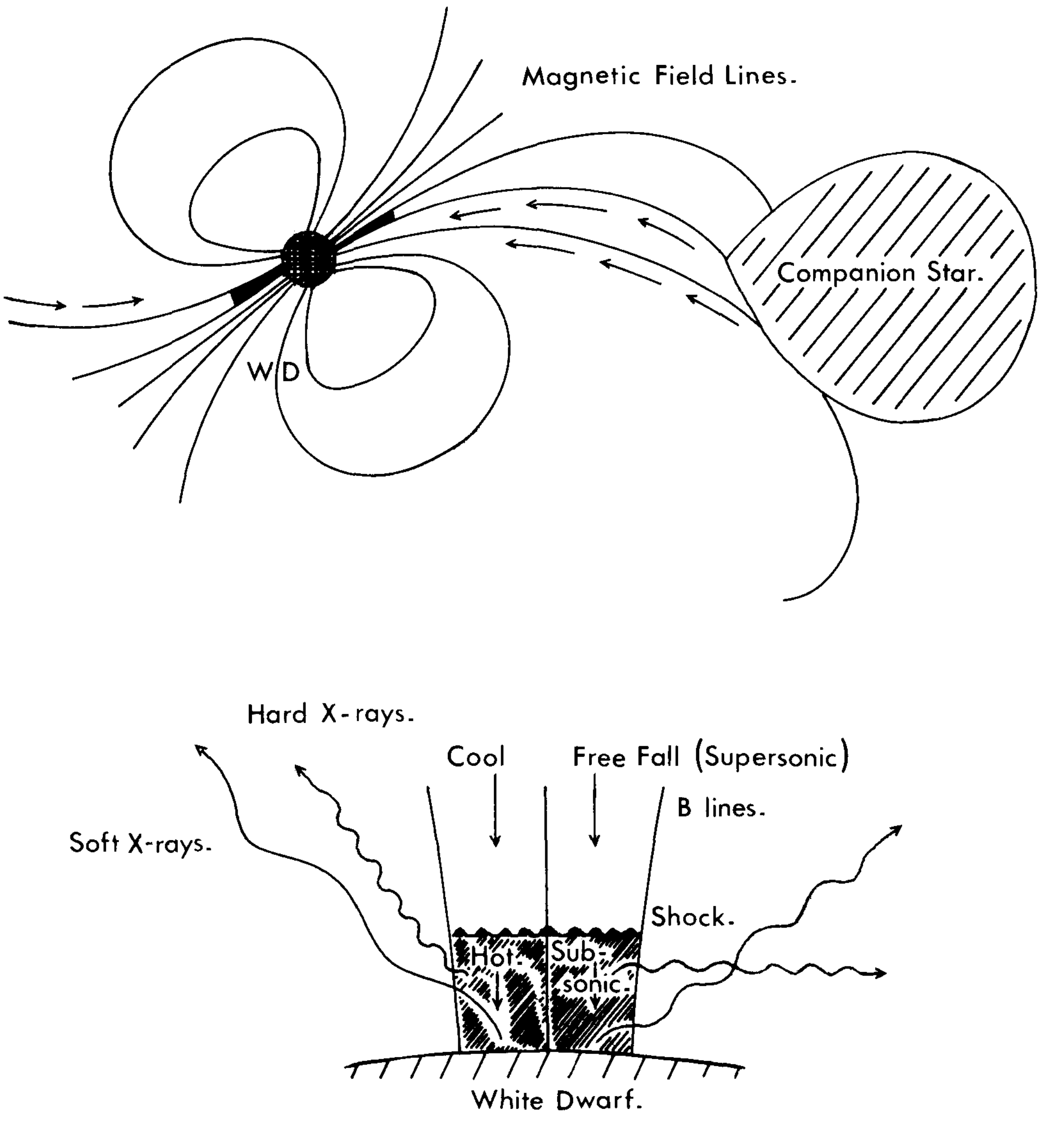

Figure 1. Accretion columns in an AM Her system. In a DQ Her ('intermediate polar') system the upper picture would be replaced by one in which the accretion flow from the companion star initially orbits the white dwarf in a disc, the disc being disrupted by the white dwarf magnetic field at some radius, whereupon the matter flows down field-lines to the magnetic poles. The two types of system are similar in the flow near the polecaps shown in the lower picture (not to scale).

In a steady state the accretion energy must be removed from the post-shock flow at the same rate at which it is supplied: it is the manner in which this removal is carried out, for example by direct radiation or particle transport processes, which determines the column structure. Note that this leaves open the possibility of distinct self-consistent column solutions for the same white dwarf parameters and accretion rate. We shall see examples of this in what follows, i.e. radiation-dominated column structures in which transport processes are negligible, and vice versa. The question of which solution actually occurs in a given case is then presumably decided either by stability considerations or by the history of the system. Langer *et al.* (1981, 1982) and Chevalier and Imamura (1982) have discussed the stability of radiative columns.

2. THE SOFT X-RAY PROBLEM

Historically the first type of solution to be discussed was the radiative one, in which the postshock flow cools by radiating the accretion luminosity directly. Since in this case the gas is characterized by the adiabatic shock temperature

$$T_s = \frac{3}{8}\frac{GM\mu m_p}{kR} = 3.7 \times 10^8 \text{ K } M_1 R_9^{-1} \qquad (1)$$

(where $M = M_1 M_\odot$, $R = 10^9\ R_9$ cm are the white dwarf mass and radius, and the other symbols have their usual meanings) most of this 'primary' emission is in the form of bremsstrahlung X-rays in the 10 - 50 keV region.[1] Because most of the AM Her stars are hard X-ray sources this was seen as encouraging, and a large number of models, of varying degrees of elaboration, were constructed on this basis both for spherical accretion (Hoshi, 1973; Aizu, 1973; Katz, 1977; Lamb and Masters, 1979; Kylafis and Lamb, 1979; Imamura *et al.*, 1979, Wada *et al.*, 1980) and, more relevantly for the AM Her and DQ Her stars, for column accretion (Fabian *et al.*, 1976; King and Lasota, 1979).[2] However, all such models were found wanting when soft X-ray and UV observations of AM Herculis became available.

For radiative columns with realistic accretion rates the shock height D << white dwarf radius R, so that one half of the primary hard X-radiation from the shocked gas is intercepted by the white dwarf surface. Of this some fraction a_X is elastically scattered, while the remaining X-rays are absorbed by the white dwarf envelope and re-emitted as blackbody radiation with characteristic temperature

$$T_b = 1.2 \times 10^5 \text{ K } \dot{M}_{16}^{\frac{1}{4}} f_{-2}^{-\frac{1}{4}} M_1^{\frac{1}{4}} R_9^{-\frac{3}{4}} f_{soft}^{\frac{1}{4}} \qquad (2)$$

Here $\dot{M}_{16}$ is the accretion rate in units of 10^{16} g s^{-1} ≃ 1.6×10^{-10} $M_\odot$ y^{-1}, $f_{-2} = 10^2 f$ and $f_{soft} = \frac{1}{2}(1-a_X)$ is the fraction of the total accretion luminosity.

$$L_{acc} = 1.33 \times 10^{33}\ \dot{M}_{16} M_1 R_9^{-1} \text{ erg s}^{-1} \qquad (3)$$

https://doi.org/10.1017/S025292110009031X Published online by Cambridge University Press

which is re-emitted. Hence radiative column models predict that the luminosities L_X in hard X-rays (characterized by T_S) and L_{soft} in soft X-rays - EUV (characterized by T_b) should be in the ratio

$$\frac{L_X}{L_{soft}} = \frac{\frac{1}{2}(1 + a_X)}{\frac{1}{2}(1 - a_X)} \sim 2 \quad (4)$$

for a typical albedo $a_X \sim 0.3$ (Felsteiner and Opher, 1976). Early soft X-ray measurements already showed a spectrum rising steeply towards low energies, which if interpreted as the Wien tail of a blackbody spectrum violated (4) by large factors. Recognizing this King and Lasota (1979) attempted to salvage the existing radiative column models by adopting a non-blackbody interpretation of the soft X-ray spectrum. However the door was firmly shut on all such attempts by the phase-dependent UV observations of Raymond *et al.* (1979) using the IUE satellite. These showed that the component of the 3000 Å - 1200 Å flux eclipsed at the same binary phase as the hard and soft X-rays had a Rayleigh-Jeans distribution, with flux levels consistent with a single blackbody interpretation of soft X-rays and UV. The situation is shown in Fig. 2, where recent observations of Fabbiano *et al.* (1981) and Rothschild *et al.* (1981) are plotted. Clearly the total luminosity in the 'soft' component is very uncertain: the characteristic temperature T_b means that most of this component is hidden by interstellar absorption in the 13.6 eV - 100 eV range. From Fig. 2 we have

$$\left.\frac{L_X}{L_{soft}}\right|_{observed} \lesssim 0.1 \ - \ few \times 10^{-3} \quad (5)$$

The value 0.1 results from a very conservative interpretation between 10 and 100 eV in Fig. 2, and the small value from the blackbody interpretation. The large discrepancy between (4) and (5) implies that for AM Her simple radiative column models fail to account for rather more than 90% of the luminosity. Wherever sufficiently good data exist for other sources to make possible a comparison with theory a similar discrepancy exists. Further, the X-ray reprocessing which seems to be required to explain the beat periodicities observed in DQ Her systems like H2252-037 also calls for large unseen fluxes, presumably in soft X-rays, exceeding the hard X-ray flux by factors $\sim 10^2$ (e.g. Hassall *et al.*, 1981).

3. REMEDIES

There have been three main attempts to explain the large soft X-ray excesses of these systems exemplified by (5).

(i) Steady Nuclear Burning

The first approach is to assume that the radiative column models described above are broadly correct, but that the accreted material under-

https://doi.org/10.1017/S025292110009031X Published online by Cambridge University Press

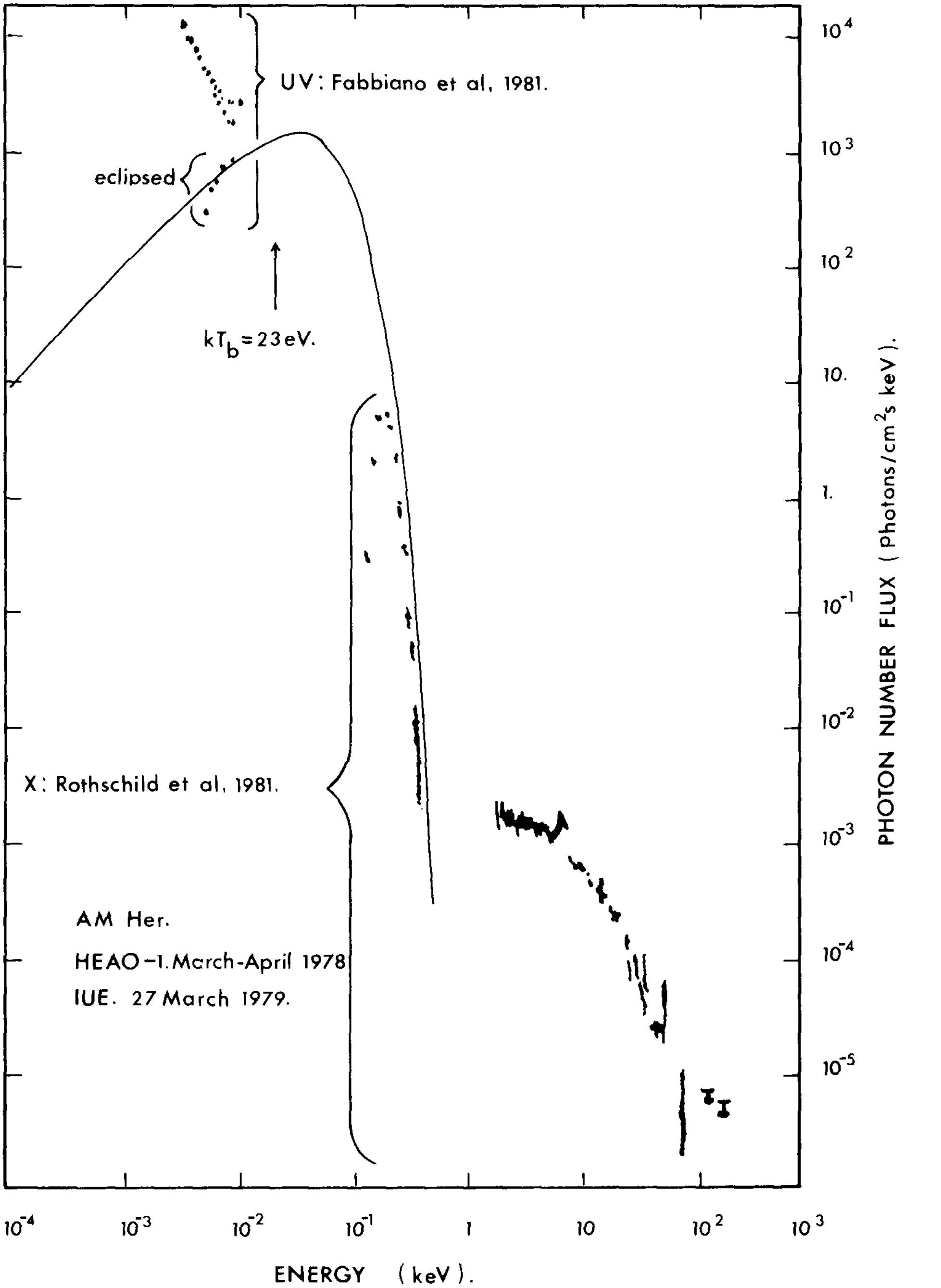

Figure 2. UV, soft and hard X-ray observations of AM Her (Fabbiano *et al.*, 1981; Rothschild *et al.*, 1981) showing the very large soft X-ray - EUV component of the eclipsed flux. The solid curve shows a 23 eV blackbody. The region between 13.6 eV and ∿ 0.1KeV is inaccessible to observation because of interstellar absorption.

https://doi.org/10.1017/S025292110009031X Published online by Cambridge University Press

goes steady nuclear burning at the base of the column, providing a large blackbody component. The main difficulty here is that steady (as opposed to runaway) burning does not seem to be possible at the required luminosities. The calculations of Papaloizou *et al.* (1982) imply an upper limit

$$L_{burn} \lesssim 2 \times 10^{32} \text{ f erg s}^{-1}$$

which is far too low to give the blackbody luminosity ($\sim 10^{34}$ erg s^{-1}, Fabbiano *et al.*, 1981) inferred for AM Her even if f = 1, which is very unlikely. There is a possible further stable burning regime if the accretion rate per unit surface area can be made to exceed the equivalent of 10^{-7} $M_{\odot}$ y^{-1} over the whole white dwarf surface (e.g. Paczynski and Zytkow, 1978), implying a lower limit

$$L_{burn} \gtrsim 4 \times 10^{37} \text{ f erg s}^{-1} ,$$

which is much too high for the AM Her and DQ Her systems unless f is considerably smaller than currently believed.

(ii) Unsteady Flow

In a recent preprint, Kuijpers and Pringle (1982) have pointed out that if the accretion flow is sufficiently inhomogeneous and unsteady the bulk of the accreting matter could penetrate to large optical depths in the white dwarf envelope before releasing its energy, thus giving a large blackbody flux. At the time of writing it is unclear whether the required (high) degree of inhomogeneity (due to cooling instabilities in the accretion flow) can be realized.

(iii) Non-Radiative Transport into the White Dwarf Photosphere

In this approach one notes that the prediction of similar hard and soft X-ray fluxes (4) arises because of the assumption that all the losses from the shocked column are radiative. If most of the energy from this region could be transported under the white dwarf photosphere before being radiated, a ratio L_X/L_{soft} closer to the observed value (5) would be predicted. We follow here the simple analysis of King and Lasota (1980), who investigated the possibility mentioned by Fabian *et al.* (1976) that column solutions might exist which were dominated by electron thermal conduction. Such a solution requires a conductive flux

$$F_{cond} \sim L_{acc}/4\pi R^2 f \qquad (6)$$

into the white dwarf. In the usual diffusion approximation, F_{cond} is related to the temperature gradient dT/dz in the column by the thermal conductivity K:

$$F_{cond} = K \frac{dT}{dz} \quad . \qquad (7)$$

https://doi.org/10.1017/S025292110009031X Published online by Cambridge University Press

For the ionized postshock gas of density N and temperature T

$$K \sim \frac{Nk^2T}{m_e v_e} \lambda_{ei} \quad .$$

Here k is Boltzmann's constant, m_e is the electron mass, $v_e = (kT/m_e)^{\frac{1}{2}}$ and λ_{ei} is the mean free path. (This yields the familiar expression $K = \text{constant} \times T^{5/2}$ when evaluated.) Also at the shock we must have the approximate equality (from energy conservation)

$$L_{acc}/4\pi R^2 f \sim NvkT \quad ,$$

where $v \sim (kT/m_p)^{\frac{1}{2}}$ is the postshock gas velocity (m_p = proton mass). To estimate the shock height D for this type of column structure we set $dT/dz \sim T/D$. Using all these relations in (6) yields

$$D \sim (m_p/m_e)^{\frac{1}{2}} \lambda_{ei} = \lambda_{eq} \tag{8}$$

Here λ_{eq} is the typical lengthscale over which the ions and electrons of the shocked flow establish equipartition. Since this is much less than the radiative shock height $D_{rad} \sim vt_{ff}$, with t_{ff} the free-free cooling time of the gas, this crude picture is self-consistent in that radiative losses from the column are small; a ratio L_x/L_{soft} of the observed order (eq. (5)) is predicted. However equation (8) means that two of the assumptions made in constructing this picture are questionable: first, since $D \sim \lambda_{eq}$, it is not clear that equipartition between ions and electrons is established, so that a full two-fluid treatment is required in which the electron and ion temperatures are allowed to differ. Second, since D is so small the mean free paths of electrons in the high energy tail of the Maxwellian distribution can exceed it: that is, the high energy electrons behind the shock are not 'localized' since there is a large difference in temperature over a mean free path. In particular this means that the diffusion approximation (7) for the conductive flux is poor, since this explicitly depends on being able to represent the temperature difference over an electron mean free path as $\Delta T \sim \lambda_{ei}\, dT/dz$. We shall see later (Section 5) how these defects can be overcome by a full two-fluid treatment taking account of the energy transport by nonlocal electrons.

4. THE GLOBAL BEHAVIOUR OF ONE-FLUID ACCRETION FLOWS

Before proceeding to the two-fluid description, it is worth taking a closer look at one-fluid accretion flows. There are several reasons for this. First, any two-fluid flow will eventually achieve equipartition: after all it must join on to a slowly 'sinking' white dwarf envelope solution at large optical depths, where equipartition must certainly hold. This is seen for example in the radiative (spherical) solution of Imamura *et al.*(1979), where because of the very high white dwarf mass assumed (1.4 $M_\odot$) Compton cooling of the postshock electrons by the blackbody photon flux keeps them initially cooler than the ions, equipartition being achieved for $T \lesssim 0.3\ T_s$. For lower-mass white dwarfs Compton cooling is much less important (Frank *et al.*, 1982, eq. (13)): for $M < 0.5\ M_\odot$ it is less than 4% of free-free losses even near the shock, so that radiative columns for moderate-mass white dwarfs are effectively in equipartition throughout. Finally one would like to understand the relation of the radiative and conduction-dominated flows to each other: as these are solutions of the same set of equations the distinction must be specified by the boundary conditions.

Figure 3 (taken from Frank *et al.*, (1982) shows the global structure of all column flows in which equipartition holds and thermal conduction can be represented by the diffusion approximation (7). The variables θ ($= T/T_s$) and ϕ are dimensionless measures of the temperature and conductive flux at any point in the column: a column flow in which the matter is in equipartition throughout starts near $\theta = 1$ ($T = T_s$) at the shock and moves to very small values of $\theta \simeq T_b/T_s \sim 10^{-3}$ at the base of the column. A two-fluid solution reaching equipartition at $T < T_s$ starts at a smaller value of θ and again moves to $\theta \simeq T_b/T_s$. Positive values of ϕ represent a heat flow into the white dwarf; for $\phi < 0$ the flow is reversed. It is immediately apparent from the figure that all equipartition flows can be divided into just two families, labelled types 1 and 2; they are separated by a critical solution ($\phi = \phi_{crit}$ on the figure) which is the unique one satisfying $\phi = 0$ at $\theta = 0$. It can be shown (Frank *et al.*, 1982) that all column solutions have negligible optical depth so that optically thin radiative cooling may be assumed throughout.

Let us examine the type 2 solutions first. It is a simple matter to show from the equations describing the flow that the temperature T must have a minimum wherever the conductive flux F_{cond} vanishes. Thus the type 2 solution curves pass through $\phi = 0$ at their minimum value of θ and bend away to increasing values of θ with $\phi < 0$. (It is for this reason that a non-zero conductive flux is required behind the shock, so that by energy conservation T is slightly less than T_s - see Frank *et al.*, 1982 for details.) Because we want an accretion flow which cools to temperatures $T_b \sim 10^{-3}\ T_s$ we are only interested in solution curves which approach very close to the ϕ-axis ($\theta \simeq 0$). From the inset it can be seen that the type 2 solutions which do this run very near to the critical solution. Near the point $\phi = 0$ the type 2 solutions can be matched to a slowly sinking atmosphere in radiative equilibrium, so that the $\phi < 0$ part of the solution curve is not realized. A unique equipartition solution is then given by choosing the matching temperature so that the photospheric luminosity of the atmosphere ($\simeq$ blackbody luminosity at T_b) is just that supplied by X-ray heating from the column above. (Clearly conduction supplies nothing to the atmosphere as $\phi = 0$ near $T = T_b$.) Since the resulting solution runs so close to the critical solution and always has a small value of ϕ, this type of cooling flow is for many practical purposes indistinguishable from radiative flows where conduction is entirely neglected and the (singular) boundary conditions $v = 0$, $T = 0$ are adopted at the base of the flow (e.g. Aizu, 1973).

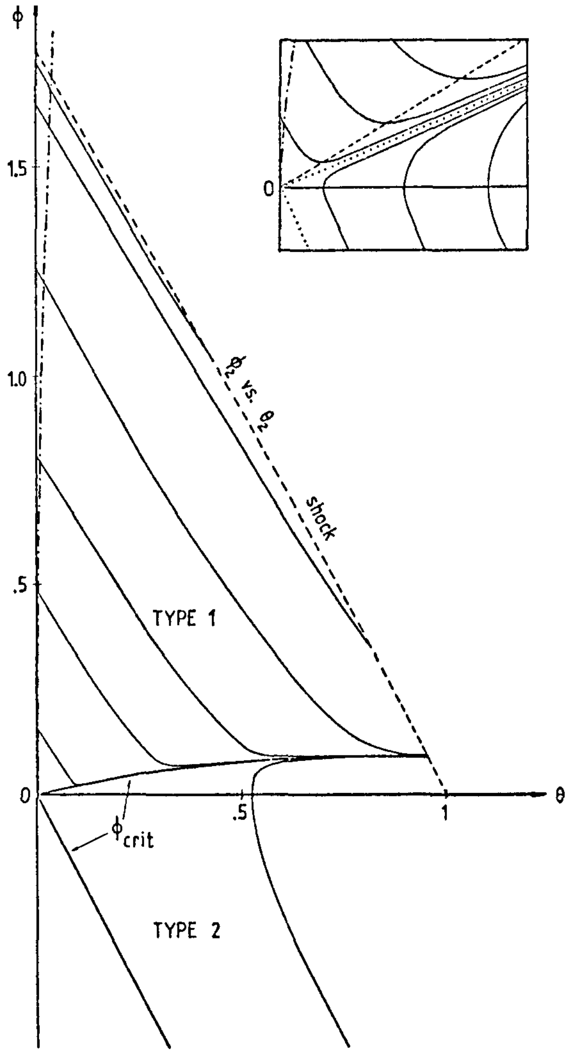

Figure 3. The global behaviour of all equipartition column solutions (Frank *et al.*, 1982). Each solution is characterized by the relation between its conductive flux and temperature, here represented by the dimensionless quantities ϕ and θ (= T/T_S). The solutions divide into two families (types 1 and 2) described in the text, corresponding to conductive- and radiative-dominated solutions respectively. The two families are separated on the ϕ - θ plane by the critical solution (labelled 'ϕ_{crit}') which is the unique one satisfying $\theta = 0$ at $\phi = 0$. The inset shows an expanded (not to scale) version of the neighbourhood of the origin. In all type 1 solutions the conductive flux ultimately 'saturates' (dot-dash curve) and the diffusion approximation breaks down (see text). The locus of minima of the type 1 solutions (dashed curve on inset) gives Aizu's (1973) solution in which conduction is entirely neglected; this runs very close to the critical solution (dotted).

https://doi.org/10.1017/S025292110009031X Published online by Cambridge University Press

The global behaviour of the type 1 solutions is quite different. As can be seen from Fig. 3 all of these reach low values of θ. All of them have minima in ϕ very close to the critical solution (this is best seen on the inset); as θ decreases further ϕ increases monotonically. Because $\phi \propto T^{5/2}\, dT/dz$ and T is decreasing, this implies that the temperature gradient dT/dz becomes very large - so large that on the dot-dash curve close to the ϕ-axis the conduction 'saturates'. This means that the temperature scale length $L = T/|dT/dz|$ has become comparable with the mean free path of the average electron, so that the diffusion approximation (7) for the conductive flux breaks down. This is just the behaviour we discovered in our crude analysis of conduction-dominated solutions in the last section. Indeed the resemblance becomes closer when one realizes that for consistency the saturation must occur at a temperature such that nonlocal electrons can transport the accretion energy under the white dwarf photosphere. This implies saturation at temperatures comparable to T_s (see below) so again it is unclear that equipartition would have been achieved.

5. TWO-FLUID ACCRETION COLUMNS WITH NONLOCAL ELECTRON ENERGY TRANSPORT

The discussion of the last two sections shows that to construct column models in which nonradiative energy transport is dominant a two-fluid treatment with nonlocal electrons is required. In this section I shall describe an approach to this problem developed by Dr. J. Frank and myself.

Let us consider first just how nonlocal the energy-transporting electrons are required to be. In order to penetrate the photosphere we require that the lengthscale λ_E over which a nonlocal electron deposits its energy in the white dwarf atmosphere should exceed the average photon mean free path $\lambda_{ph} \simeq (\kappa_b\rho_b)^{-1}$, where κ_b, ρ_b are the (Rosseland) opacity and density of the atmospheric plasma (i.e. we require the electron to penetrate to optical depth $\tau_E = \kappa_b\rho_b\lambda_E > 1$). Because for electrons with velocities $v_e >> (kT_b/m_e)^{\frac{1}{2}}$ the deflection time t_D to electron-electron collisions is shorter than the energy exchange time t_E the electron will random-walk a distance $\lambda_E = (t_E t_D)^{\frac{1}{2}}\, v_e$ into the white dwarf. Using standard expressions to relate t_E, t_D to v_e, T_b and ρ_b, and recalling that the photospheric gas pressure $\rho_b kT_b/\mu m_p$ must approximately equal the ram pressure of the accretion flow, one finds that λ_E will exceed λ_{ph} (given by Kramers' opacity) for electron velocities

$$v_e \gtrsim v_{min} = 10^{10}\ \text{cm s}^{-1}\ (\dot{M}_{16}/f_{-2})^{1/20}\ .\ M_1^{3/20}\ R_9^{-5/20} \qquad (9)$$

(Frank *et al.*, 1982). A very similar numerical value for v_{min} results when electron scattering is more appropriate than Kramers', with a similar extremely weak dependence on white dwarf parameters and accretion rate (this arises because $\lambda_E \propto v_e^5$). If we were to require an electron temperature T_e such that the average electron had $v_e > v_{min}$ we would find $T_e \simeq 0.5\ T_s$; this is the result referred to at the end of Section 4. Our simple considerations of Section 3 suggest that the height of the shocked column will be $D \lesssim \lambda_{eq}$. It is easy to check that electrons with $v_e > v_{min}$ have mean free paths exceeding D, so they are indeed nonlocal.

The main practical obstacle remaining is the question of how to represent the energy transport by the nonlocal electrons. We note that as far as the electron gas in the column is concerned this is a loss process like, for example, a radiative loss process (although it is much more efficient). Indeed one can think of the column as being 'optically thin' to these electrons, so that energy 'leaks' into the photosphere directly. Analogous situations are encountered in plasma physics and stellar dynamics; if the timescale for the energetic particles to escape the region of interest is short compared to the timescale to refill by relaxation (collisions) the Maxwellian tail of the local distribution function the standard recipe is the so-called 'loss-cone' approximation. In our case the condition for the loss-cone approximation to hold is that the time of flight down the column for the escaping electrons should be short compared to the relaxation time for refilling the Maxwellian tail. Thus the loss-cone approximation will describe the 'leak' process well provided there exists a second minimum electron velocity v'_{min} such that

$$t_{flight}\,(v'_{min}) = \frac{D}{v'_{min}} < t_{tail}\,(v'_{min}) \qquad (10)$$

where $t_{tail}\,(v'_{min})$ is the relaxation time for $v > v'_{min}$. If this holds the volume loss rate to the electron gas due to the leak process can be written

$$j_{leak} = \frac{\xi P_e}{t_{tail}} \quad . \qquad (11)$$

Here $P_e = N_e k T_e$ is the electron pressure: with $v = \max\,(v_{min}, v'_{min})$, ξ is the fraction of the local electron distribution with $v_e > v$ directed towards the white dwarf, and t_{tail} the relaxation time for $v_e > v$. Note that (10) must be used as a consistency requirement: having found a column structure using (11) we must insert the calculated value of D to check that (10) holds, so that the structure is self-consistent.

The results of a full two-fluid calculation using (11) are shown in Figure 4. As usual for plasma shocks (see e.g. Zeldovich and Raizer, 1966) there is an ion shock where the ion temperature rises abruptly to a value $\sim 2T_s$ (since the electrons are not yet taking their share of the thermalized kinetic energy). The electron gas before the ion shock is 'preheated' by classical (diffusive) conduction from the shocked region. Behind the ion shock the ions attempt to heat the electrons collisionally to bring about equipartition. This is opposed by the very efficient loss of energy from the electron gas into the white dwarf photosphere given by (11). These two terms, the heating of the electrons by the ions ($\propto N_e^2\, T_e^{-3/2}\, (T_i - T_e)$), and the leak term ($\propto \xi\, N_e^2\, T_e$, eq. (11))

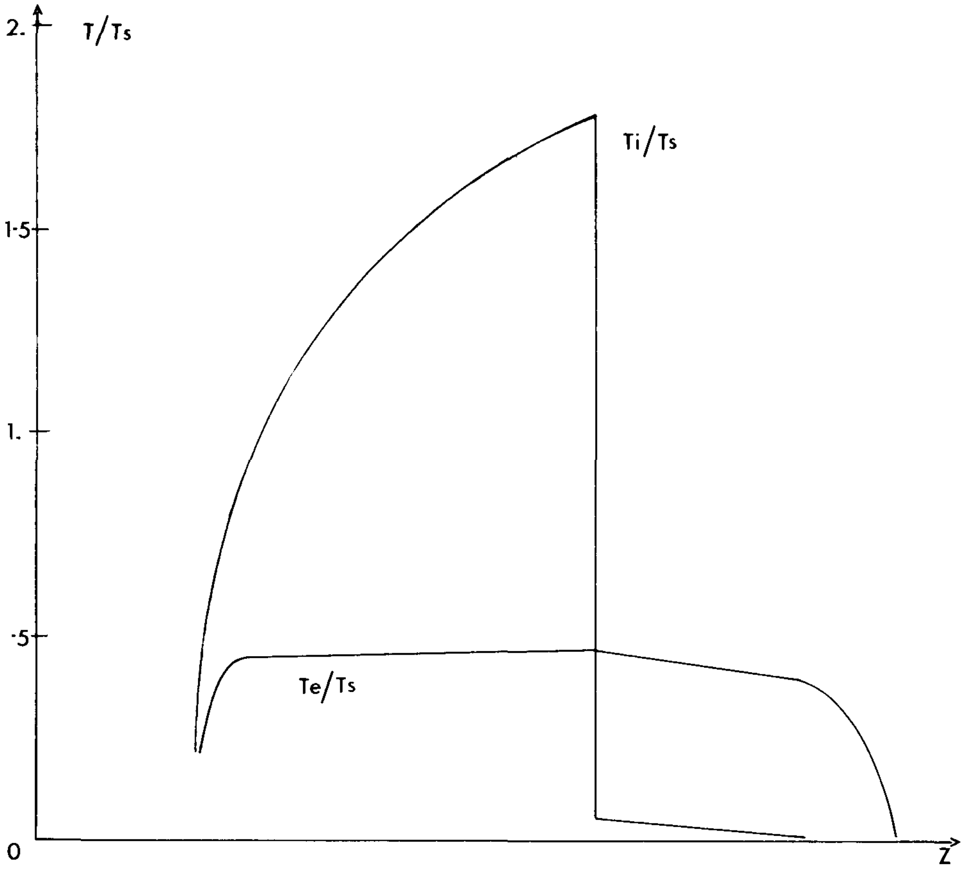

Figure 4. Electron and ion temperature profiles T_e, T_i in the accretion column model of Frank and King (1982) which is dominated by nonlocal electron energy transport. T_e, T_i are plotted as functions of vertical distance z in units of the adiabatic shock temperature T_s. The distance from the ion shock (vertical section of T_i curve) to the equipartition point ($T_i = T_e$) is of order 10^4 cm: there is very little hard X-ray emission from this region, most of the energy being transported directly under the white dwarf photosphere by energetic electrons.

dominate the electron energy equation behind the ion shock. The almost constant electron temperature profile there results from the great sensitivity of the fraction ξ in j_{leak} to T_e: although T_i gradually decreases behind the ion shock as the electrons drain energy from the ions, only a very small decrease in T_e produces a change in ξ sufficient to accommodate the change in the ion heating rate ($\propto (T_i - T_e)$). This behaviour ceases once $T_i - T_e$ is so small that ξ enters a parameter

https://doi.org/10.1017/S025292110009031X Published online by Cambridge University Press

regime where it is no longer so sensitive to T_e: this gives the sharp decrease in T_e as equipartition ($T_i = T_e$) is approached at the left of the figure. For the parameters used to produce Fig. 3 the electron temperature is almost constant at $\sim 0.5\ T_s$ behind the ion shock, dropping abruptly to $\sim 0.25\ T_s$ at equipartition. Since at this point $T_i = T_e \simeq 0.25\ T_s$, three-quarters of the accretion energy has been removed from the post-shock gas at this stage. Almost all of this energy has been transported by the high-energy electrons into the white dwarf photosphere, whence it will be radiated as blackbody emission. The length of the two-fluid region is so short ($\sim 10^4$ cm) that direct radiation by hard X-rays accounts for only a few parts in 10^3 of the losses. The situation depicted in Fig. 3 is thus smaller in scale by a factor $\sim 10^3$ (for the same accretion rate) than radiative cases where a very high assumed white dwarf mass and resultant Compton cooling cause two-fluid behaviour, despite a superficial resemblance of the temperature profiles (e.g. Imamura *et al.*, 1979, with $M = 1.4\ M_\odot$). Further, this very small shock height implies that the condition (10) for the validity of the loss-cone approximation is satisfied.

Once equipartition is reached ($T_i = T_e \simeq 0.25\ T_s$) the gas has still to release $\sim \frac{1}{4}$ of the accretion energy before joining on to the white dwarf envelope. Thus it must follow one of the one-fluid solution curves of Fig. 3. It is not a straightforward matter to tell which type (1 or 2) of solution we have here, since this is extremely sensitive to the temperature gradient the flow 'inherits' from the two-fluid region: note in Fig. 3 how close to the critical solution *all* solution curves which reach $\theta \sim 10^{-3}$ run for $\theta \sim 0.25$, despite their entirely different behaviours for very small θ. The simplest possibility is that the gas cools radiatively for $T < 0.25\ T_s$: this emission at $\sim 0.25\ T_s$ would dominate the observed hard X-rays, and by a similar argument to that giving (4), produce a ratio

$$\frac{L_X}{L_{soft}} \sim 0.16 \tag{12}$$

The other extreme possibility is a type 1 solution below $0.25\ T_s$: then the X-ray spectrum would be a mixture of $0.5\ T_s$ and $0.25\ T_s$ emission, with a very small ratio

$$\frac{L_X}{L_{soft}} \sim \text{few} \times 10^{-3} \tag{13}$$

Clearly the estimates (12) and (13) show that we have made some progress towards accounting for the observational estimate (5) of this ratio. A further desirable consequence of our treatment is the lower hard X-ray temperature; our expected range (using (1))

$$kT_X \sim (8 - 16)\ \text{keV}\ M_1\ R_9^{-1} \tag{14}$$

fits more comfortably with hard X-ray observations than the one-fluid estimate (1), which predicts $kT_X \sim 64$ keV for a $1\ M_\odot$ white dwarf.

ACKNOWLEDGEMENTS

Much of the work reported here has been carried out in collaboration with Drs. J. Frank and J.P. Lasota, with the support of the SERC. I thank the Royal Society for travel support and Dr. Frank for valuable help in the preparation of this review.

NOTES

1. Thermal cyclotron radiation, despite providing the classic observational property - phase-dependent optical polarization - of the AM Her stars, cannot be an important loss mechanism for columns producing the luminosities ($\gtrsim 10^{33}$ erg s^{-1}) observed for these systems. Cyclotron-dominated columns might be observationally important if one of the magnetic poles in an AM Her system receives a rather low accretion rate but radiates predominantly in the optical-near IR, the other pole providing most of the luminosity, or if there are systems with considerably higher magnetic fields than those (few x 10^7 G) so far found for the AM Her systems. See King and Lasota (1979) for discussion. Note that some early papers considerably overestimated the importance of cyclotron emission.

2. Spherical models can be rescaled to describe the postshock region of a column flow, but differ greatly from columns in the transfer of outgoing radiation and the ionization structure above the shock: in spherical models a photon released near the white dwarf surface must traverse the entire accretion flow before escaping, while all such photons eventually escape from the *sides* of a column. For the luminosities ($\lesssim 10^{34}$ erg s^{-1}) characteristic of the AM Her and DQ Her stars optical depth effects such as Compton degradation of the hard X-ray spectrum are negligible (King and Lasota, 1979).

REFERENCES

Aizu, K.: 1973, Prog. Theor. Phys. 49, p.1184.
Chevalier, R.A., and Imamura, J.N.: 1982, preprint.
Fabbiano, G., Hartmann, L., Raymond, J., Steiner, J., and Branduardi-Raymont, G.: 1981, Astrophys. J. 243, p.911.
Fabian, A.C., Pringle, J.E., and Rees, M.J.: 1976, Mon.Not.R.astr.Soc. 175, p.43.
Felsteiner, J., and Opher, R.: 1976, Astron. Astrophys. 46, p.189.
Frank, J., and King, A.R.: 1982, in preparation.
Frank, J., King, A.R., and Lasota, J.P.: 1982, Mon.Not.R.astr.Soc., in press.
Hassall, B.J.M., Pringle, J.E., Ward, M.J., Whelan, J.A.J., Mayo, S.K., Echevarria, J., Jones, D.H.P., Wallis, R.E., Allen, D.A., Hyland, A.R.: 1981, Mon.Not.R.astr.Soc. 197, p.275.
Hoshi, R.: 1973, Prog. Theor. Phys. 49, p.776.
Imamura, J.N., Durisen, R.H., Lamb, D.Q., and Weast, G.J.: 1979, in

https://doi.org/10.1017/S025292110009031X Published online by Cambridge University Press

'White Dwarfs and Variable Degenerate Stars', Proc. IAU Colloq. 53.
Katz, J.I.: 1977, Astrophys. J. 215, p.265.
King, A.R., and Lasota, J.P.: 1979, Mon.Not.R.astr.Soc. 188, p.653.
King, A.R., and Lasota, J.P.: 1980, Mon.Not.R.astr.Soc. 191, p.721.
Kuijpers, J., and Pringle, J.E.: 1982, preprint.
Kylafis, N.D., and Lamb, D.Q.: 1979, Astrophys. J. Lett. 228, p.L105.
Lamb, D.Q., and Masters, A.R.: 1979, Astrophys. J. Lett. 234, p.L117.
Langer, S.H., Chanmugam, G., and Shaviv, G.: 1981, Astrophys. J. Lett. 245, p.L23.
Langer, S.H., Chanmugam, G., and Shaviv, G.: 1982, Astrophys. J. 258, p.289.
Paczynski, B.E., and Zytkow, A.N.: 1978, Astrophys. J. 222, p.604.
Papaloizou, J., Pringle, J.E., and Macdonald, J.: 1982, Mon.Not.R.astr. Soc. 108, p.215.
Raymond, J.C., Black, J.H., Davis, R.J., Dupree, A.K., Gursky, H., and Hartmann, L.: 1979, Astrophys. J. Lett. 230, P.L95.
Rothschild, R.E., Gruber, D.E., Knight, F.K., Matteson, J.L., Nolan, P.L., Swank, J.H., Holt, S.S., Serlemitsos, P.J., Mason, K.O., and Tuohy, I.R.: 1981, Astrophys. J. 250, p723.
Wada, T., Shimizu, A., Suzuki, M., Kato, M., and Hoshi, R.: 1981, Prog. Theor. Phys., in press.
Zeldovich, Ya. B., and Raizer, Yu. P.: 1966, 'Physics of Shock Waves and High Temperature Hydrodynamic Phenomena' Vols. I and II (New York: Academic Press).

DISCUSSION FOLLOWING A. KING'S TALK

WILLIAMS: To what extent does this effect depend upon the presence of a magnetic field, that is, will this also serve to cut down the hard to soft X-rays in the non polars?

KING: First, it does not depend on magnetic fields in the sense that there were no magnetic fields in the calculation, but it does, in the sense that the flow was radial.

LAMB: Let me remark that the kind of one-fluid solutions you have talked about were discussed by Imamura and Durisen in a paper presented at the AAS meeting in Austin in 1978. They encountered the same problems with saturation of the conductive flux and so on. Following that meeting, I and Weast initiated two-fluid calculations. Several examples of our two-fluid calculations were published in the proceedings of IAU Colloquium No. 53 at Rochester in 1979. Since then, we have calculated an extensive series of two-fluid models for a wide range of conditions.

KING: I don't think they are quire the same. The effects that your group considered were primarily those of Comptonization which has the effect of keeping the temperatures different, but an important difference is that the losses to your gas were radiative.

LAMB: What I want to emphasize is that two-fluid calculations are necessary. When we did them, we found that the conductive flux was not important in conducting energies down to the region near the stellar surface. The diagram that you have shown is for one-fluid calculations; you can draw certain conclusions from it, but it does not apply to two-fluid calculations.

KING: It applies when the temperatures have equalized.

LAMB: No, it absolutely does not. For example in that figure you have an arbitrary choice of the temperature gradient at the shock and therefore the initial conductive flux; but when you do a two-fluid calculation, the initial conductive flux is specified.

KING: But when the temperatures equalize you have one fluid calculations. The gradient of the temperature can be anything you like, this diagram covers the whole parameter space.

LAMB: I would also like to say that suprathermal electrons cannot deposit energy deep in the photosphere in white dwarfs because the Coulomb cross section is sufficiently large that the ions must shock. Initially, the ions contain all the energy and they must transfer their energy to the electrons before it can go anywhere.

KING: The ions shock there. The difference is that your columns are very long, this column is very short, because the losses are not radiative.

LANGER: In your models here, you cannot possibly have a shock height which is less than the electron-ion equipartition distance. Essentially you have picked that point in the electron velocity distribution where the mean free path of an electron is equal to somewhat more than the distance to the photosphere, as the point where your tail cuts off and at that point the velocity will drop below a Maxwellian. You then compute the net flux of energy into that portion of the distribution driven by the electron relaxation processes, as the amount of energy density that would originally be beyond that cut off, divided by the relaxation time. How was that relaxation time computed and what is its relationship to some attempt to calculate, using the Fokker-Planck equation the energy flux across that cut-off point?

KING: To take the last point first, to do a Fokker-Planck treatment of this, is going to be difficult because you got spatial gradients, so it is the full Boltzmann. The answer to the first part, if I understand you correctly, is that it is the standard approximation used in stellar dynamics, for example.

CRAMPTON: Tapia showed us that, at least he thought, that in all of the AM Her stars the poles were inclined at a small angle to the line of sight. Can you tell us from the models whether you expect the X-rays only to be seen in a small cone angle? Because nearly all of the AM Her stars are found by X-rays.

KING: Well, the point is that the base of these flows is where the pressure in the white dwarf envelope equals the ram pressure and that is not necessarily near where the surface is. So you have a shock that probably lies very close to the surface, most of the X-rays will have to come out through the sides, they certainly cannot go up the column. So you would probably expect to see low energy absorption.

LAMB: There do exist Einstein OGS observations of AM Her which were nearly simultaneous with IUE observations and which require a blackbody temperature of 40-45 eV. The OGS observations do not tolerate a blackbody temperature of less than 35 eV. In order to join the soft X-rays to the UV using the Rayleigh-Jeans tail of a blackbody spectrum, one has to have, instead, a temperature of 27 eV. Taking a blackbody temperature of 40-45 eV does not give a large ratio for L_X/L_{total}, so in the case of AM Her it may be that the ratio is only 2-4 rather than 10 or 100. I just wanted to point this out, because it has been a long standing statement that you can join the soft X-rays and the UV, but it actually does not seem to be true.