PREDICTIONS OF OCCULTATIONS BY MINOR PLANETS AS A TEST OF ACCURACY

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ABSTRACT. Predictions of tracks of occultations of stars by minor planets give a strong test of the accuracy of observations, star places, and dynamical theories. A critical discussion of the 1983 September 11 occultation of 14 Piscium by (51) Nemausa shows that the systematic regional errors of fundamental stars introduce errors in the long-term predictions of occultations that are larger than relativistic effects.

1. INTRODUCTION AND SURVEY OF OBSERVATIONS

Relativistic corrections to the motions of bodies and light-rays are less than the gravitational radius of the Sun:

$$2k^2M_0/c^2 \backsim 3\ \text{km} \tag{1}$$

where k^2 is the gravitational constant, M_0 the mass of the Sun, and c the velocity of light. For a main belt asteroid ±3 km corresponds to $\pm0\overset{''}{.}003$ in geocentric position, which quantity is far below the precision of present astrometry. Precision of this order of magnitude can, however, be obtained by simple timings of occultations and the precision in the relative distance between star and centre of mass of the planet is mainly limited by the surface irregularities. Comparisons between long-term predictions of ground tracks of occultations with observations give strong tests of the accidental and systematic precision which can be attained for positions and dynamical theories.

For (51) Nemausa, long series of precise and well-documented observations are now available because this particular object was selected (Strömgren, 1950) for an improvement of the declinations of fundamental stars in the equator zone. Initially, my interest in these observations was to ascertain the order of magnitudes of observational errors with special reference to an investigation of relativistic effects and the work started with a numerical development of the general perturbations in elements due to relativistic effects of the Sun and Jupiter. Except for the well-known perihelion motion no secular terms or resonances were found which were not much smaller than 3 km. An accuracy of ±100 km can hardly be obtained so the non-trivial relativistic effects cannot be seen with the present technique. However, the

J. Kovalevsky and V. A. Brumberg (eds.), Relativity in Celestial Mechanics and Astrometry, 113–116.

original aim had a decisive influence on the arrangement of the reductions and computations in the work on (51) Nemausa.

1) From the period 1943 to 1977 published dependencies and the star places for 2240 photographic positions have been compiled by Ole Møller in machine-readable form. The positions were completely rereduced with due account taken of differential refraction. In order to separate different sources of errors and to obtain realistic estimates of the observational mean errors, the O-C residuals of the individual series were analyzed in every detail for effects as atmospheric dispersion, reversal of telescope between exposures, and so forth. In this way some errors were found in the published data, typically interchanges of star numbers or dependencies. These ambiguities were resolved by communication with the observers and practically all observations were saved. The southern ($\delta<-2^0$) reference stars were taken from the Yale Observatory Catalogues and reduced to FK_4 by applying systematic differences from comparisons with the Perth 70 and Bucharest meridian circle catalogues. Except that no systematic differences were applied on the AGK_3 reference stars, most northern ($\delta>-2^0$) observations were worked up by similar lines of direction by Ole Møller.

2) 152 refractor and 63 meridian circle observations from 1858 to 1902 were used to check the mean motion. Only the right ascensions were used and they were completely re-reduced with new clock-corrections and modern star places in the FK_4 system.

3) The above observations formed the basis for a determination of the equator and equinox of FK_4 (Kristensen, 1980) but for the prediction of the 1983 occultation (Kristensen, 1983) about 116 additional observations were available from the period 1977-1982, among which were some very precise Bordeaux transit circle observations.

All these observations form a homogeneous material reduced to FK_4 over the extended period 1858-1982. Differential corrections to the orbit of Nemausa, the Earth, and the equator point of FK_4 were performed in a rigorous manner, which took into account that, for instance, the observations for which the same systematic differences were applied are not statistically independent. Great care was taken to obtain realistic estimates of mean errors, and except for the errors of the system of FK_4, all conceivable errors, including of course those of the orbital elements of the Earth, were taken into account. The corrections to the equinox and equator should be <u>in</u> the system of FK_4, which thus had to be considered free of errors. The mean epoch of FK_4 is 1935 and the errors in the proper motion system (μ) in α are important; in the zone $-20^0<\delta<+20^0$ the internal errors were estimated (Fricke, 1963) to $\overset{s}{.}006$-$\overset{s}{.}012$/century.

2. THE OBSERVED OCCULTATIONS

The occultation of SAO 144417 (=SRS 16138) on 1979 August 17 was observed photoelectrically at Gissar and Alma-Ata with the result (Kristensen, 1981):

$$\cos\delta(\Delta\alpha_{pl}-\Delta\alpha_*) = -\overset{''}{.}077 \pm \overset{''}{.}002 \quad (\Delta\delta_{pl}-\Delta\delta_*) = -\overset{''}{.}147 \pm \overset{''}{.}002 \quad (2)$$

where $\Delta\alpha_{pl}$ and $\Delta\alpha_*$ are corrections to the ephemeris and star place, respectively. The star position was provided by J.A. Hughes (1979) with errors $\pm^{s}.005$ and $\pm''.11$, which alone could account for the observed difference (2).

It was, however, possible to separate ephemeris and star errors by the 1983 occultation of 14 Psc because of the several observations secured before and after the event.

About 50 observations of this occultation were obtained in the United States (Dunham, 1983) and a preliminary reduction has given:

$$\cos\delta(\Delta\alpha_{pl}-\Delta\alpha_*) = +''.136 \qquad (\Delta\delta_{pl}-\Delta\delta_*) = +''.057. \tag{3}$$

A careful investigation was made of the star place (Bien and Schwan, 1983) and errors were estimated to $\pm^{s}.003$ and $\pm''.09$.

Continued observations, from August 5 to December 6, gave by the Bordaux transit circle (Requième and Rapaport, 1985) 37 observations of (51) Nemausa:

$$\cos\delta\ \Delta\alpha_{pl} = +''.25\ (\pm''.16) \qquad \Delta\delta_{pl} = -''.02\ (\pm''.17) \tag{4}$$

where the stated mean errors refer to a <u>single</u> observation. Fourty observations of 14 Psc gave similarly

$$\cos\delta\ \Delta\alpha_* = +''.06\ (\pm''.11) \qquad \Delta\delta_* = -''.16\ (\pm''.20) \tag{5}$$

Eleven observations of Nemausa by the Lick 51 cm astrograph (Klemola and Harlan, 1984) with AGK_3R reference stars gave

$$\cos\delta\ \Delta\alpha_{pl} = +''.16\ (\pm''.12) \qquad \Delta\delta_{pl} = -''.01\ (\pm''.07) \tag{6}$$

If the corrections to the star are converted by eq. (3) to ephemeris corrections, we get

Source	$\cos\delta\ \Delta\alpha_{pl}$	$\Delta\delta_{pl}$
Bordeaux T.C.	$+''.25\pm''.03$	$-''.02\pm''.03$
Lick, 51 cm astrograph	$+.16\pm.04$	$-.01\pm.03$
Bordeaux, star + occ.	$+.20\pm.03$	$-.10\pm.03$
Bien and Schwan + occ.	$+.14\pm.05$	$+.06\pm.09$
Weighted mean:	$+.20\pm.03$	$-.04\pm.03$

(7)

We note especially the consistency of the Bordeaux results indicating that there is no magnitude error between the planet ($V=10^{m}.5$) and the star ($5^{m}.9$). The predicted error of the ephemeris from all sources except the errors of the FK_4 system is $\pm''.03$ in both α and δ.

3. INFLUENCE OF THE MEAN ERRORS OF THE FUNDAMENTAL SYSTEM

A set of preliminary corrections ($\Delta\mu$) to the proper motion system of FK_4 was provided by Bien (1984) and hoping that these corrections could reduce the significant ephemeris correction $+\overset{''}{.}20\pm\overset{''}{.}03$ an approximate adjustment was performed. At the 27 dates of opposition in the period 1943-1982 the corrections $\Delta\alpha=\Delta\mu\ (T-1935)$ were adjusted in the form

$$\Delta\alpha = +\overset{''}{.}020 +\overset{''}{.}144\ (T-19.56)\ \ +\overset{''}{.}004\cos\alpha+\overset{''}{.}012\sin\alpha \qquad (8)$$

This approximation ignores the excentricity and δ, which is always small for (51) Nemausa. Weights were given to the resulting normal places in accordance with the weights in the original adjustment which was not optimal for minimizing the errors of $\Delta\mu$.

At the occultation we obtain

$$\Delta\alpha = +\overset{''}{.}062 \pm\overset{''}{.}24\ \sigma_\mu \qquad (9)$$

where σ_μ ("/century) is the mean error of the regional values of μ.

At the position of the star, $\Delta\mu= +\overset{s}{.}004$/cent., and we obtain

$$\Delta\alpha_* = \overset{s}{.}004\ \ (19.837 - 19.35) = +\ \overset{''}{.}029 \pm .487\sigma_\mu \qquad (10)$$

The improved system does not eliminate the ephemeris correction but gives

$$\cos\delta\ \Delta\alpha_{pl} = +\overset{''}{.}20 - \overset{''}{.}062 + \overset{''}{.}029 = +\ \overset{''}{.}167 \pm \overset{''}{.}09 \qquad (11)$$

the mean errors of which are:

ephemeris	$\pm\overset{''}{.}03$
observations in 1985	$\pm.03$
μ system, Nemausa	$\pm.24\sigma_\mu$
μ system, star	$\pm.487\sigma_\mu$

It is difficult to estimate σ_μ, but if we assume $\sigma_\mu = \pm\ \overset{s}{.}010$/cent., then we obtain the value $\pm\overset{''}{.}09$ stated in (11). The conclusion is that typical errors in long-term predictions of occultations by main belt asteroids will be of order ±100 km due to the errors in the fundamental proper motions and the much smaller relativistic effects cannot be detected.

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