3 GEORGE STREET,
EDINBURGH. 15*th June* 1949.

The Editor,

Transactions of the Faculty of Actuaries.

DEAR SIR,

With reference to the Actuarial Note " Widow's Annuity Option in connection with Staff Pension Funds," *T.F.A.*, vol. xviii. p. 344, we are indebted to Mr. J. A. Carson, F.F.A., for an alternative method of approach.

The equation of value connecting $r^{(1)}_{60:y}$ and $r^{(2)}_{60:y}$ given in the middle of p. 349 may be written as

$$r^{(1)}_{60:y}(a_{60|y}-vp_{60:y}a_{61|y+1})+r^{(2)}_{60:y}vp_{60:y}a_{61|y+1}=v^2{}_2p_{60}\bar{a}_{62}$$

and, bearing in mind that

$$r^{(1)}_{60:y}a_{60|y}=vp_{60}\bar{a}_{61},$$

this equation may be simplified to

$$\left(r^{(1)}_{60:y}-r^{(2)}_{60:y}\right)p_y a_{61|y+1}=\bar{a}_{61:\overline{1}|}.$$

By a similar process it may be shown analytically that

$$\left(r^{(n)}_{60:y}-r^{(n+1)}_{60:y}\right)_n p_y a_{60+n|y+n}=\bar{a}_{60+n:\overline{1}|}.$$

This last equation is capable of being explained verbally as follows :—

If an employee has attained age $60+n$ in the service he has the alternatives—

(*a*) of retiring at once, the widow's pension being $r^{(n)}_{60:y}$; or

(*b*) of deferring retirement until age $60+n+1$, the widow's pension in that case being $r^{(n+1)}_{60:y}$.

The difference between the present values of these benefits must equal the difference between the present values of the deductions from the employee's pension, and these differences are represented respectively by the left- and right-hand sides of the above equation.

The equation may be written—

$$r^{(n+1)}_{60:y}=r^{(n)}_{60:y}-\frac{\bar{a}_{60+n:\overline{1}|}}{_n p_y a_{60+n|y+n}}$$

and this last expression may be used to calculate the successive values of the function " *r*." In form it is much more elegant than the

corresponding expression given in the Note, which involves the *ad hoc* function " B," and we have therefore thought it well to put it on record. From the point of view of the amount of numerical work necessary in actual calculation, however, there is little to choose between the two formulae.

Yours faithfully,

A. R. REID.

D. W. A. DONALD.