PLASMA DISTRIBUTION NEAR PULSARS

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We argue that the aligned rotator model originally introduced by Goldreich and Julian possesses "static" solutions wherein particles of both sign are trapped within the magnetosphere. Simple models of such distributions are given. For such models to have dynamic properties (constant particle emission, relativistic stellar wind, etc.) one must suppose that the equatorial particles are transported away by (e.g.) flux tube interchange, rather than by flow along field lines as originally proposed.

1. INTRODUCTION

There is an interesting dichotomy in pulsar theory in that most efforts to quantitative analyse the physics of pulsars assume that ions and electrons are freely available from the surface of an aligned rotator, while most phenomenological models assume a pair-production cascade in an oblique rotator, as shown in Figure 1. Nevertheless it should be recognized that despite their obvious differences, the qualitative models borrow heavily from the quantitative ones, even when the latter are not "pulsars" per se. It is often assumed, for example, that the aligned rotator would be a directional radio source and that the obliquity serves mainly to make it a pulsed radio source.

The prototypical aligned rotator model is that of Goldreich and Julian (1969: GJ). There the rotationally induced electric field pulls plasma from the surface, this plasma requires $\underline{E} \cdot \underline{B} = 0$ everywhere in the magnetosphere which in turn forces rigid corotation of the magnetosphere with the star (this is the issue treated in this work) and rigid corotation leads to particle loss beyond the light cylinder which in turn requires emission from the surface to replace the lost particles. Qualitative models are often based on, for example, radio frequency radiation by the particles when they are emitted from the surface.

There are several difficulties with the GJ model that have carried forward to today: the structure of the resultant stellar wind (giant

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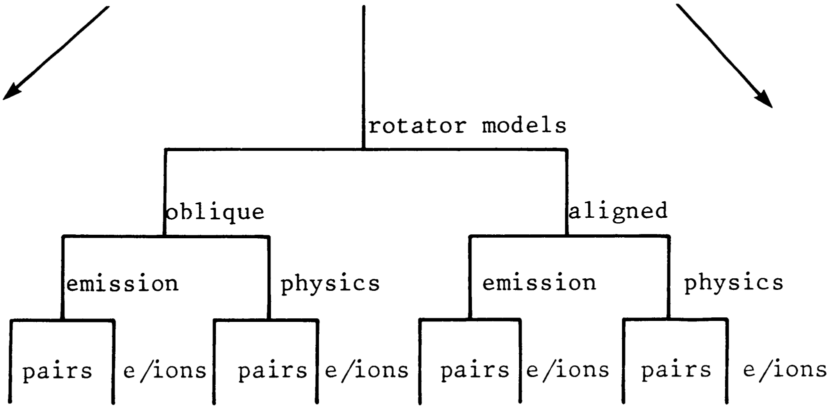

Fig. 1: Pulsar model "family tree". Most efforts to find exact quantitative solutions follow the extreme right hand branch while qualitative or semi-phenomenological models tend to follow the extreme left-hand branch. Other additional branchings (massive magnetospheres, etc.) exist but are not illustrated.

waves or frozen-in flux), the matching of the corotation region into the wind region, and even the fact that the field lines are curved introduces a problem in that the space charge required for $\underline{E}\cdot\underline{B} = 0$ in the magnetosphere does not match (except at a single point) the space charge that would result from the relativistic flow of charged particles along field lines.

There have been several attempts to resolve the latter problem, since it is so well posed, by postulating non-rigid corotation or a two-component plasma - either charges of the same sign but two different velocities or even particles of each charge with distinct velocities. All represent important departures from the original GJ model. We propose here an even more radical departure:

The aligned rotator has static solutions.

Clearly this conclusion resolves the various technical problems. One needs not match onto a wind solution because there is no wind and one does not need a two-component plasma because the meridional velocity is zero, not c. Our conclusion is therefore that the GJ model fails at the point where they assume the magnetosphere to be entirely filled with plasma, at which point one is no longer obliged to accept the remaining chain of arguments leading to a dynamic rotator model (particle emission, etc.). (In their paper, Goldreich and Julian spoke often of the "uniqueness" of their solution, but ended by assuming it, not demonstrating it.)

2. DISCUSSION

There are three elements that lead one to a static aligned rotator: (1) "New" physics - the existence of stable discontinuities in a magnetized charge-separated plasma, (2) the interrelationship among these discontinuities and force-free surfaces, and (3) the electrostatics of an aligned rotator. Only the latter point was something that could have been easily deduced at the time of the GJ paper.

2.1 Discontinuities

In Fig. 2 we show what we mean by a discontinuity, namely an abrupt drop in plasma density to zero (i.e. vacuum) at a sharp interface where the interface is pierced by magnetic field lines. In conventional plasma physics, where one has a quasi-neutral medium of electrons and ions, such interfaces are non-existent. In general some parallel electric field (i.e. $E_{\parallel} \equiv \underline{E}\cdot\hat{\underline{B}}$) exists in the vacuum region and consequently either the electrons or ions will be accelerated away if they drift across the interface. Ambipolar diffusion then carries away the neutralizing charges and the plasma simply invades the vacuum region by flowing along field lines. For a charge-separated plasma one requires only that the weak parallel electric field acts to return the particles to the interface (this field must decline to zero as the discontinuity is approached from the vacuum side).

Such discontinuities have been described in detail (Michel, 1979) but were implicit in the proposal of Holloway (1973) who argued that charges of one sign could not be replaced in the GJ model, leading to a splitting open of the magnetosphere along the zero-charge-density sur-

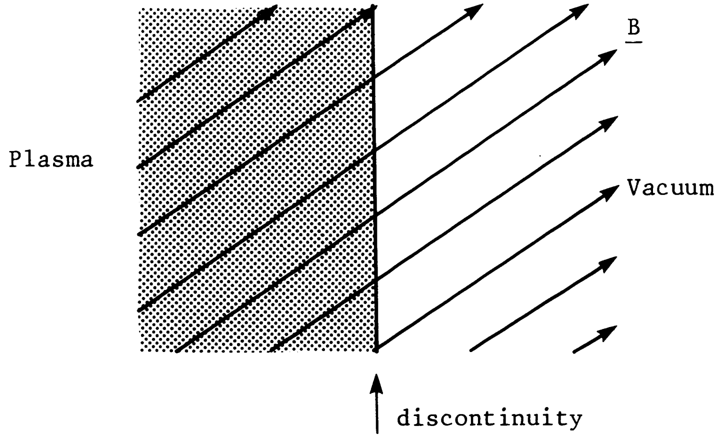

Fig. 2: Discontinuity. The plasma density drops discontinuously to zero to separate the region of finite space charge (but $\underline{E}\cdot\underline{B} = 0$) from regions of non-zero parallel field (but zero density). The latter returns particles to the discontinuity, giving stability.

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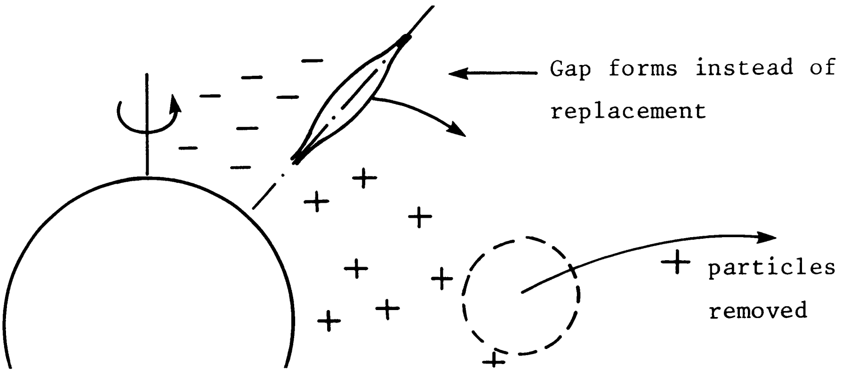

Fig. 3: Holloway's Gap. If positive particles are removed from the equatorial regions of the GJ solution, replacement seems impossible. Holloway proposed that the zero charge surface splits to create a vacuum gap.

face as shown in Figure 3. In the models discussed here, the discontinuities are even "stronger" than envisioned by Holloway, in that the space-charge density does not fall to zero and then remains zero but in fact drops abruptly from finite values to zero.

2.2 Force-Free-Surfaces and Discontinuities

It has been noted by Jackson (1976) that many vacuum field configurations possess force-free-surfaces (FFSs) which are surfaces which would be potential maxima or minima (along magnetic field lines) for charged particles. Such minima do not exist in the absence of a force of constraint such as provided by the magnetic field (Ernshaw's Theorem). If we allow particles of the appropriate sign to "pool" in this potential minimum, the FFS is split into two discontinuities which bound the charge-separated plasma on each side.

Geometrically, the equatorial plane of the aligned rotator is always a FFS and consequently the vacuum solution can always be replaced by a thick disk of charge confined to this plane (Figure 4). A disk as described above does not corotate with the star because the potential is no longer constant along a field line and therefore does not match the surface potential at the star. A "light cylinder" may or may not exist and there is no requirement in any event that the disk extends to such a light cylinder (or be replaced if it did). Nevertheless such a magnetosphere is "force free" in the sense that

$$\rho(\underline{E}\cdot\underline{B}) = 0$$

with the discontinuities separating the $\rho = 0$ (vacuum) regions from the $\underline{E}\cdot\underline{B} = 0$ (space-charge regions).

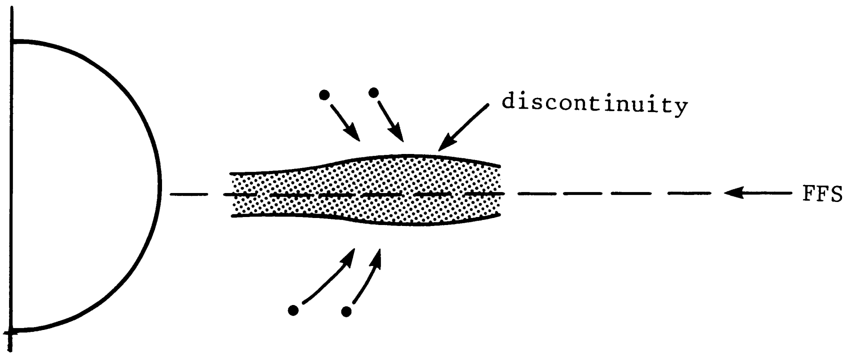

Fig. 4: Discontinuities embrace force-free-surfaces. The discontinuities describe space charge accumulated at a (vacuum) force-free-surface. In a sense, each discontinuity can be regarded as "half" an FFS.

2.3 Pulsar Electrostatics

Whatever might happen in the pulsar magnetosphere, one nevertheless expects a force-free interior, owing to the high conductivity predicted there. If we represent the magnetic field by a point magnetic dipole at the stellar center, the interior space charge is just the GJ charge density plus a point charge at the center. The latter charge may seem a bit surprising, since it has not received a lot of attention in the literature, but follows immediately from $\underline{E}\cdot\underline{B} = 0$ (which for a dipole gives E_r of one sign everywhere) and Gauss's Law. These two charge distributions are not sufficient, however, to give $\underline{E}\cdot\underline{B} = 0$; a third term is required from some external distribution of charges which must be arranged to give the third term in the local potential

$$V = \frac{5}{r} - \frac{3}{r^3} P_2 - 2r^2 P_2 \tag{1}$$

seen at the stellar surface, r = 1. The internal potential is then

$$\Phi = 15 \sin^2 \theta / 2r . \tag{2}$$

If the object were surrounded by vacuum, the third term would be from a surface charge. For pulsars, which have great difficulty retaining surface charges, the surrounding magnetosphere must provide instead this term. [In the GJ model the charge generating this term would reside in the supposed transition region from corotation to wind.] The important point here is that the star itself imposes the perturbation potential.

$$V_* = \frac{5}{r} - \frac{3}{r^3} P_2 \tag{3}$$

and the external magnetosphere must provide the neutralizing $-2r^2P_2$ term. It is easy to see that (3) has a minimum along the polar axis at

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$r_o \approx 1.4$ $(r^2 = 9/5)$ and in fact this FFS is in the form of a sphere centered on the axis with one side passing through $r = r_o$ and the other through the origin, $r = 0$, with an identical sphere for the other pole (Figure 5). Another FFS is in the equatorial plane. As we will see, this system constitutes an "ion trap" which concentrates particles of one sign about the dome-like FFS's, the polar caps and the other sign in the equatorial FFS (actually more in the form of a torus, close the star, than a disk). The nature of the trapping in the dome is different from that at the equator in that the equatorial particles would have to cross magnetic field lines to escape whereas the polar particles could follow magnetic field lines away from the star but are electrostatically trapped.

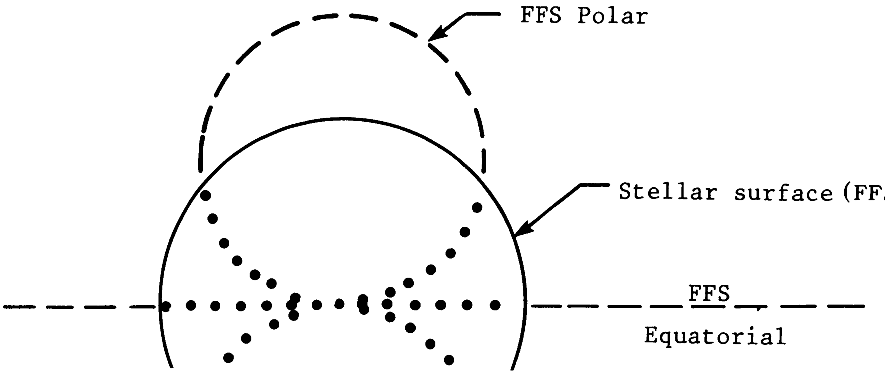

Fig. 5: Force-free-surfaces. Structure for dipole magnetic field with an electric field given by a point charge and a quadrupole field. Dotted lines are extensions of FFS within the star.

3. ZEROTH ORDER SOLUTIONS

Numerical calculations are in progress but are difficult owing to the three surfaces involved. Here we will simply present some simplified models illustrating roughly the expected behavior. The least difficult particle population seems to be the equatorial particles. In any likely charging scenario, these particles are emitted at low latitudes and therefore are bound close to the star ($r \lesssim 1.5$). Then in the outer magnetosphere they are seen mainly as a point charge (plus quadrupole correction, and then successively smaller terms from higher multipoles). A second point is that we need not worry about image charges since in the limit of free particle-availability from the surface it is just the "image charges" (which would be surface charges on a conducting star) that are emitted, hence there are no image charges in that limit. Accordingly, for intermediate approximations one is free to include or ignore these charges as one wishes. We ignore them. Finally we assume the total charge of the star plus magnetosphere is zero for definiteness. Actually a continuum of solutions should exist, parame-

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terized according to total system charge.

3.1 Point Charges Over Polar Caps

Here we simply place two point particles of charge q over each polar cap, concentrating the remaining charge (-2q) at the stellar center. The force on these point charges is

$$\frac{2q}{r^2} - \frac{9}{r^4} - \frac{q}{4r^2} = 0 \tag{4}$$

the first term being the net "central" charge, the second being the induced quadrupole and the third being the mutual repulsion of the charges. This "magnetosphere" wherein all the charges have been lumped together will give the correct external quadrupole moment $(-2r^2P_2)$ if $q/r^2 = 1$, hence $r^4 = 36/7$ or $r \sim 1.5$. (In this example the total central charge is $2r^2 \sim 4.6$ which is <u>less</u> than 5 and corresponds physically to leaving some -5 neutralizing charge behind. This is because concentrating the particles at the pole makes them maximally effective (i.e. too effective) at creating the P_2 moment.) Models with net charge would correspond to simply varying the coefficient of the first term in (4).

3.2 Shells of Charge About the Star

A more realistic charge distribution is obtained if we smear out the point charge. We can accomplish this without introducing multipoles beyond second order by distributing the polar charges according to $\cos^2\theta$ and the equatorial charges as $\sin^2\theta$. The calculation is a bit more elaborate here, but if we locate the two shells at r = b and r = a respectively, one finds that $\underline{E}\cdot\underline{B} = 0$ everywhere on the surface and everywhere[7] on the "polar" shell if

$$b^5(1+2a^3) - 2b^3(3+2a^5) + 4a^3(a^2-3) = 0 . \tag{5}$$

This relation is not very transparent, but if we set a = 1 (i.e. close confinement of the equatorial particles), then b = 1.925... Note that this solution is <u>exact</u> insofar as neglect of self-repulsion within the shell goes, and corresponds to the situation wherein particles of the equatorial sign are trapped at the surface (i.e. ions, cf. Ruderman and Sutherland, 1975). Here the charge involved is somewhat larger ($\sim$ 12 units of central charge). This increase reflects the fact that the further away one puts the major magnetospheric charges, the more charge one needs to get the $-2r^2P_2$ term.

3.3 Conjectured Solution

Figure 5 forms the basis for a hypothetical solution consistent with the simple models above. One simply "fills" the spherical FFS's to form a dome over each polar cap and the equatorial FFS to form a disk. Given axial symmetry, such a configuration would be perfectly stable.

Such solutions could still leave us with the same dilemma found in the GJ model: if part of the disk is crossed by field lines leading to the domes of opposite charge, how could disk particles be replaced if lost or removed? However, since these field lines are now open-circuited, there is nothing to suppress flux-tube interchange as a transport mechanism. Thus additional particles could instead be injected onto field lines near the equator and then be interchanged outward. Moreover, a steady loss of particles could then be supported, disk particles by interchange and dome particles along field lines as usual. The physics of such a disk is a complete departure, of course, from the original GJ model.

4. CONCLUSIONS

If the aligned rotator had historically been recognized to possess static solutions, attention might well not have been diverted from several essential problems with all magnetized rotator models, namely how can these objects emit particles of <u>both</u> sign and drive an outflowing plasma. These are physical requirements that have previously been assumed, in many qualitative models, to "take care of themselves". The prospect of perfectly static solutions seems to be closing the door on that happy thought. But at the same time another door may be opening: these solutions may in fact only be static for axial symmetry with flux-tube interchange as a means of transporting equatorial particles away from the star. Thus we find a quite different picture than that of GJ, and also a host of new problems to be worked out. Whatever the ultimate resolution of this issue, it is evident that the two extreme branches of Figure 1 are in fact closely related.

[1] In principle one can only set $\underline{E}\cdot\underline{B} = 0$ at a single point on the shell, however for this problem there is an accidental calculation everywhere on the shell.

REFERENCES

Goldreich, P. and Julian, W.H.: 1969, Astrophys. J. 160, p. 971.
Holloway, N.J.: 1973, Nature Phys. Sci. 246, p. 6.
Jackson, E.A.: 1976, Astrophys. J. 206, p. 831.
Michel, F.C.: 1979, Astrophys. J. 227, p. 579.
Ruderman, M.A. and Sutherland, P.G.: 1975, Astrophys. J. 196, p. 51.

DISCUSSION

MESTEL: I am interested to know what are the maximum values of γ you expect in your models. If the value becomes large enough for large rotation rates, the radiation damping could enforce a circulation, so perhaps linking your models with the Sussex proposal, with dissipation playing an "active" role.

MICHEL: In this type of model the disk would "subrotate" if the total system charge is zero (then the electric field is dominated by the next multipole, the quadrupole, and falls as r^{-4} whereas B falls as r^{-3} giving a drift velocity falling as r^{-1}). Thus the most rapid velocities would be near the star and similar to the rotation velocity of the neutron star ($v \lesssim 10^{-2}$ c). Consequently, such a magnetosphere would not radiate at an "interesting" (i.e. pulsar-like) rate.

KAHN: What happens to your model if the magnetic axis is slightly inclined to the rotation axis?

MICHEL: In the limit of an angle of 90° between the rotation axis and the magnetic moment the trapping regions appear to persist, becoming two "clouds" of charge (one +, one -) above the rotation plane and two below the rotation plane. However, one can lose particles of both sign along field lines in the latter case, so that these clouds would not "block" particle loss. What happens in intermediate cases (your question) remains to be modeled.

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