COMPARISON OF ELP-2000 TO A JPL NUMERICAL INTEGRATION

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This contribution is an outline of the main results obtained by the authors in comparing their solution ELP-2000, to a JPL numerical integration, LE-51. A full paper containing discussions and comments on the results will be proposed to *Astronomy & Astrophysics*.

INTRODUCTION.

A solution for the orbital motion of the Moon has been built by the authors. It is named ELP-2000, the epoch of reference being J2000. It is a semi-analytical solution, its structure being quite similar to Brown-Eckert's one, as it appears in the Improved Lunar Ephemeris, ILE, j=2, (Eckert et al., 1954). The main purpose of this work is to present the results of a comparison of a provisional but complete solution, to an external numerical integration, LE-51, built at JPL (Williams, 1980), and fitted to lunar laser rangings. The JPL numerical integration is regarded as an "observational model". It is a first attempt to compare as a whole, a new lunar ephemeris, derived from a semi-analytical theory, to observations, via a numerical integration.
The results should not be considered as definite. In its present state, this study allows us to discuss the following points :

- To evaluate the global precision of our model, and put in evidence the important factor in accuracy which has been gained as compared to the ILE.
- To study the problems connected with the adjustment of constants (lunar and solar parameters).
- To determine the reference frame.
- To compute explicitly the mean motions for the node and perigee, within an accuracy better than 1"/cy.

If several improvements have to be brought to our solution in its present state, it is nevertheless possible to anticipate the construction of a lunar ephemeris from a semi-analytical theory, at the level of accuracy of a few meters, and to introduce it in the French ephemerides, the *Connaissance des Temps*. Presently, the maximum errors over the range of 20 years, are less than 20 meters in longitude, and less than 12 meters in radius vector.

O. Calame (ed.), High-Precision Earth Rotation and Earth-Moon Dynamics, 257–264.

1. THE SOLUTION ELP-2000 AND ITS COMPLEMENTS.

The position of the Moon is represented by its three coordinates : the longitude V, the latitude U and the radius vector r. Each of the coordinates, designated by σ, is expressed in form of Fourier series, with mixed terms proportional to the time t, such as :

$$\sigma = \sum_i (\alpha_i + \beta_i t) \sin (D_i + k_i T + P_i + k'_i \zeta + \phi_i) \quad (1)$$

α_i and β_i are numerical coefficients; k_i and k'_i are integers; ϕ_i is a phase.
D_i is a linear combination of the four Delaunay's arguments : D, F, l and l'. T is the heliocentric mean longitude of the Earth-Moon barycenter. Instead of Delaunay's arguments, we also introduce, with Brown's notations :

$w_1 = D + T + 180°$	Mean longitude of the Moon.
$w_2 = D - l + T + 180°$	Longitude of perigee.
$w_3 = D - F + T + 180°$	Longitude of the node.

and again,

$l' = T - \tilde{\omega}'$; $\tilde{\omega}'$ being the heliocentric longitude of the Earth-Moon perihelium.
In (1), P_i is a linear combination of the heliocentric mean longitudes of the planets, from Mercury to Neptune. They are introduced by the planetary perturbations of the Moon, as well as the β_i terms in (1), because of the secular variations of the planetary elements. Finally,

$\zeta = w_1 + \psi_1 t$; ψ_1 being the precession in longitude. This argument appears when computing the perturbations due to the Earth figure.
We shall now describe briefly the various components of our solution.

The Main Problem (M. Chapront-Touzé, 1980).
The nominal solution has been computed with the following parameters, as they are introduced in the ILE, and which are conventional :

Lunar elements.

ν : Sidereal mean motion of the Moon.	1 732 559 343″.18/cy	
$2E$: Coefficient of the $\sin l$ term in longitude.	22 639″.55	(2)
2Γ : Coefficient of the $\sin F$ term in latitude.	18 461″.40	

Solar elements.

e' : solar eccentricity (Newcomb J2000)	0.016 709 24
n' : sidereal mean motion (Newcomb J2000)	129 597 742″.34/cy

Physical parameters.

σ_1 : mass ratio $L/(T + L)$	0.012 150 568
σ_2 : mass ratio $(T + L)/(S + T + L)$	$3.040\,423\,956\ 10^{-6}$

L, T and S are respectively the Moon, Earth and Sun masses.
The Main Problem includes the partials with respect to the quantities : n'/ν, E, Γ, e', a/a' (ratio of the semi major axis) and σ_1. They will be useful when fitting the solution to an external model.

Perturbations due to the Earth figure (M. Chapront-Touzé, 1982).
Contributions of the harmonics J_2 and J_3 are included, as well as precession of the Earth equator and the largest perturbations due to nutation.

Planetary perturbations (M. Chapront-Touzé, J. Chapront, 1980).
A solution for the planetary motions (Earth-Moon barycenter, inner and outer planets) has been derived up to the third order with respect to planetary masses by Bretagnon (1980, 1981). It has been used to compute the planetary perturbations of the Moon, either in the direct and indirect case. It is worth noticing that Bretagnon's theory has been fitted to observations via a numerical integration due to Oesterwinter and Cohen (1972). When comparing the lunar solution to LE-51, the planetary and solar elements will have to be modified.

Motion of the reference frame.
The natural plane of reference for the Moon is the ecliptic of date. The perturbations induced by the motion of this plane have been computed using the secular variations of the ecliptic pole, as proposed by Bretagnon.

Secular variations of e' and $\tilde{\omega}'$.
We have used the following values :

$$\dot{e}' = -8''.67095/\text{cy}$$
$$\dot{\tilde{\omega}}' = 1161''.2141/\text{cy}$$

They give rise to mixed secular terms as β_i in (1), and also to secular accelerations in node and perigee motions.

Perturbations due to the short periodic motion of the Earth-Moon barycenter.
Bretagnon has computed the perturbations due to the couple Earth and Moon, on the Earth-Moon barycenter. When the series are substituted in the differential equations for the lunar motion, they produce by a coupling effect, very important secular variations in the node (-8''.2347/cy) and the perigee (3''.4821/cy)

Tidal forces.
With a modelling analogous to the one of Williams et al. (1978), we have built the perturbations due to tidal forces. Our value for the secular acceleration is :

$$g_{ELP} = -11''.9374/\text{cy}^2$$

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COMPLEMENTS ADDED TO ELP-2000.

Perturbations due to the lunar figure.
They come from Henrard's work (1980) which gives a development in terms of $C_{2,0}$ and $C_{2,2}$, the two fundamental harmonics of the lunar gravity field.

The relativistic effects.
Among them, two groups have been retained, the terminology being derived from the newtonian formulation.

The main effect : It acts on the lunar motion because of the metrics generated by the two bodies, Earth and Sun, in the case of General Relativity, formulated in isotropic coordinates. We have used the developments of relativistic perturbations obtained by Brumberg (1972). The main contributions appear in node and perigee motions :

$$\delta_R w_2 = 1''.83/\text{cy (perigee)}$$
$$\delta_R w_3 = 1''.90/\text{cy (node)}$$

as well as in the constant part of the radius vector :

$$\delta_R r = -8.264 \text{ meters}$$

The indirect effect : In the frame of the Sun's theory, perturbations on the Earth-Moon barycenter, in isotropic coordinates, in the case of the Schwarzschild metrics, induce an indirect effect on the Moon described by Lestrade et al. (1982), mostly in node and perigee motions :

$$\delta_I w_2 = 0''.690/\text{cy}$$
$$\delta_I w_3 = -0''.199/\text{cy}$$

2. COMPARISON TO LE-51

Using the series described above, an ephemeris has been built covering the range of time 1980-2002, by step of two days. The time interval covers the nodal period and is convenient for studying trends and accelerations between the two models, ELP-2000 and LE-51.
We first modified the eccentricity and the mean motion of the Sun, starting with Bretagnon's values instead of Newcomb's :

$$e' = 0.016\,708\,772 \qquad (3)$$
$$n' = 129\,597\,741''.42/\text{cy}$$

The values of the angles at the epoch J2000 (Julian date 2 451 545) are taken from Bretagnon's planetary elements. For the lunar angles, our starting values are computed from the ILE, for J2000, such as :

$$D_0 = w_1^{(0)} - T_0 + 180° = 297°51'00''.967$$
$$F_0 = w_1^{(0)} - w_3^{(0)} = 93°16'22''.179$$
$$l_0 = w_1^{(0)} - w_2^{(0)} = 134°57'46''.522$$

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as well as for $l'_0 = T_0 - \tilde{\omega}'_0$:

$$l'_0 = 357°31'31''.548 \quad \text{(ILE)}$$

$$T_0 = 100°27'58''.427 \quad \text{(Bretagnon).}$$

The secular variations of the three angles w_i (i = 1,2,3) computed with the above constants, given in (2) and (3), have the general form :

$$w_i = w_i^{(0)} + w_i^{(1)} t + w_i^{(2)} t^2$$

t is expressed in julian century. They have the following numerical values :

$$w_1 = w_1^{(0)} + 1\,732\,559\,343''.1800t - 2''.5295t^2$$

$$w_2 = w_2^{(0)} + 14\,643\,418''.3951t - 38''.3249t^2$$

$$w_3 = w_3^{(0)} - 6\,967\,918''.9653t + 6''.3785t^2$$

Before adjusting the two models, we have to modify the radius vector in ELP of the constant deviation:

$$\delta_G r = -7.0795 \text{ meters}$$

because of the determination of the ratio AU/km made in the numerical integration DE-102 (Standish, 1980). Indeed, in ELP, we use the quantity $G(L + T)$ recommanded by IAU, where G is the gravitational constant. To bring the two models in the same reference frame, we have also to rotate the equator of DE-102 with the two angles (P. Bretagnon, J. Chapront, 1981) :

$$\Delta\varepsilon = \varepsilon_1 - \varepsilon_0 = -0''.039$$

$$\Delta\phi = \gamma_0\gamma_1 = -0''.230$$

where $(\varepsilon_0, \gamma_0)$ and $(\varepsilon_1, \gamma_1)$ are respectively the value of the obliquity and the vernal point in DE-102 and in ELP. The obliquity is used when going from equatorial coordinates to ecliptic coordinates, since the ecliptic is our reference frame. We also have to use the following formula to compute the precession in longitude, which is consistant with the theory of the Sun (Bretagnon) that was introduced :

$$\Psi = 5029''.0966t + 1''.11370t^2 + 0''.000076t^3$$

Adjustment of the two models :

Using the partials of the Main Problem as mentioned above, we have adjusted the two models in varying the following quantities :

a) Lunar parameters.

$$\delta\nu,\ \delta E,\ \delta\Gamma,\ \delta w_1^{(0)},\ \delta w_2^{(0)},\ \delta w_3^{(0)}$$

b) Solar parameters.

$$\delta n',\ \delta e',\ \delta\tilde{\omega}',\ \delta T_0$$

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c) Rotation of the reference frame.

$\delta\Delta\varepsilon$, $\delta\Delta\phi$

d) Trends and accelerations.

$\delta w_2^{(1)}$, $\delta w_3^{(1)}$, $\delta w_1^{(2)}$

If we consider LE-51 as an observational model, to vary the parameters *a)* to *c)* is legitimate since they are either a choice of constants of integration, of physical parameters or of a reference frame. In the *d)* case they represent either a lack of accuracy of our model and/or a difference in the two modellings. The results are summarized in the table below :

$\delta w_1^{(0)} = 0''8040 \pm 0''0001$	Lunar elements
$\delta w_2^{(0)} = -0''860 \pm 0''001$	
$\delta w_3^{(0)} = 3''489 \pm 0''001$	
$\delta \nu = 0''023 \pm 0''002/\text{cy}$	
$\delta E = 0''01752 \pm 0''00002$	
$\delta \Gamma = -0''08312 \pm 0''00003$	
$\delta T_0 = 1''104 \pm 0''002$	Solar elements
$\delta \tilde{\omega}'_0 = -12''094 \pm 0''014$	
$\delta n' = 0''900 \pm 0''015/\text{cy}$	
$\delta e' = -0''0456 \pm 0''0002$	
$\delta\Delta\phi = 0''0003 \pm 0''0005$	Reference frame
$\delta\Delta\varepsilon = -0''0016 \pm 0''0002$	
$\delta w_2^{(1)} = -0''7731 \pm 0''0076/\text{cy}$	Trends and accelerations
$\delta w_3^{(1)} = 0''1252 \pm 0''0096/\text{cy}$	
$\delta w_1^{(2)} = -4''527 \pm 0''010/\text{cy}^2$	

After this adjustment, the differences on the three coordinates, between the two solutions are :

Quadratic errors	*Maximum errors*	
$\sigma_V = 0''003$	$E_V = 0''010$	
$\sigma_U = 0''0015$	$E_U = 0''005$	(4)
$\sigma_r = 4$ meters	$E_r = 12$ meters	

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In fact, in LE-51, the tidal acceleration which has been used is :

$$g_{LE51} = -13''.5/cy^2$$

Correcting $\delta w_1^{(2)}$ of the difference $g_{LE51} - g_{ELP}$, it remains :

$$\delta w_1^{(2)} = -2''.9/cy^2.$$

3. PERIGEE AND NODE MOTIONS.

The differences between the two solutions are less than 1"/cy for the perigee and node motions. In the following table, we have gathered the various contributions to the motions computed in ELP-2000.

	PERIGEE ("/cy) w_2	NODE ("/cy) w_3
Main Problem	14 642 537.9368	- 6 967 167.2643
Corrections (3) to e' and n'	- 1.0385	0.1904
Planetary perturbations	243.7535	- 135.8394
Earth-Moon barycenter	3.4821	- 8.2347
Tidal forces	0.0663	0.0002
Earth figure	633.4034	- 592.5357
Lunar figure (Henrard)	- 1.7280	- 16.9830
Relativity :		
Main effect (Brumberg)	1.8300	1.9000
Indirect effect	0.6896	- 0.1989
Corrections after fitting constants to LE-51	2.7407	- 0.6344
TOTAL	14 643 421.1359	- 6 967 919.5998
LE51 - ELP2000	- 0.7731	0.1252

CONCLUSION.

The final results as illustrated by (4) show that it does not seem unrealistic to built a semi-analytical solution with an internal precision of a few meters, the length of validity covering several centuries. In its present state, we are looking to introduce ELP-2000 to replace the ILE in the *Connaissance des Temps*, since a factor better than 50 at least has been gained with respect to the conventional ephemeris. The new values that we propose for perigee and node motions, with an accuracy better than 1"/cy, bring some light in the discussion that was started by Eckert (1965) and continued by Martin and Van Flandern (1970) on the differences between the observed and computed values.

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DISCUSSION

Abalakin : If we make use of your ephemeris developments, do we obtain true geometric or apparent values of the Moon's coordinates ?

Chapront : We get true geometric coordinates.