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A REFLEXIVE BANACH SPACE THAT IS LUR AND NOT 2R

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ABSTRACT. An example of the type described in the title is given.

A Banach space B is *locally uniformly rotund* (LUR) [2] if the conditions $\|x\|=\|x_n\|=1$ and $\lim_{n\to\infty}\|x+x_n\|=2$ imply $\lim_{n\to\infty}\|x-x_n\|=0$.

A Banach space B is *fully 2-rotund* (2R) [1] if the conditions $\lim_{n\to\infty}\|x_n\|=1$ and $\lim_{m,n\to\infty}\|x_m+x_n\|=2$ imply the sequence $\{x_n\}$ is convergent.

The purpose of this note is to answer negatively the following question posed by V. D. Mil'man [3, p. 97]: Is every reflexive, locally uniformly rotund Banach space fully 2-rotund?

For $x=(x^j)_{j=1}^{\infty}$ a member of ℓ^2, define

$$\|x\|=\max\{\sup_{\substack{i,j\\ i\neq j}}(|x^i|+|x^j|),\|x\|_2\}$$

where $\|\cdot\|_2$ denotes the usual ℓ^2 norm, and for each positive integer k let $R_kx=\sum_k^\infty x^je_j$ where $\{e_j\}$ denotes the usual unit vector basis for ℓ^2. Now, define

$$\|x\|_1=\sum_1^\infty 2^{-k}\|R_kx\|.$$

It is easy to verify that $\|\cdot\|$, and consequently $\|\cdot\|_1$, is a norm on ℓ^2 that is equivalent to $\|\cdot\|_2$. Finally, for $x=(x^j)_{j=1}^\infty$ in ℓ^2 define the equivalent norm:

$$\|x\|_M=(\|x\|_1^2+J^2(x))^{1/2}$$

where $J^2(x)=\sum_1^\infty 2^{-j}|x^j|^2$.

It follows from the proofs of Theorem 1.7 and Theorem 1.10 of [3] that $(\ell^2;\|\cdot\|_M)$ is locally uniformly rotund.

To see that $(\ell^2;\|\cdot\|_M)$ is not fully 2-rotund, let $x_n=e_n$. Then $\lim_{n\to\infty}\|x_n\|_M=1$ and $\lim_{m,n\to\infty}\|x_m+x_n\|_M=2$, but $\{x_n\}$ is not a convergent sequence.

REFERENCES

1. K. Fan and I. Glicksberg, *Fully convex normed linear speces*, Proc. Nat. Acad. Sci. U.S.A. **41** (1955), 947–953.

2. A. R. Lovaglia, *Locally uniformly convex Banach spaces*, Trans. Amer. Math. Soc. **78** (1955), 225–238.

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3. V. D. Mil'man, *Geometric theory of Banach spaces* II: *Geometry of the unit sphere*, Uspehi Mat. Nauk **26** (1971), 73–149; Russian Math. Surveys **26** (1971), 79–163.

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