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Some problems concerning cluster processes and other point processes

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The several problems studied in the dissertation revolve around the preservation, under sufficiently smooth transformations, of global properties of stochastic systems related to point processes.

The number of busy servers process P_{ns} of a $G/G/\infty$ queue can be viewed as a transformation of its component processes, the arrival and server processes. Stationarity, mixing, strong laws, the central limit theorem and rates of functional convergence for P_{ns} are shown to follow from those of its components under reasonable conditions; ergodicity does not, while rates of one dimensional convergence seem to depend on a factor associated with the transformation operation.

More generally, we consider cluster processes defined as the aggregate of all subsidiary (finite) point processes each of which can be viewed as being triggered by a point of a principal or centre process (thus, the components are the centre process and the sequence of subsidiary processes). It is shown, under suitable conditions which are predictably more severe than those for one dimensional convergence, that the functional central limit theorem for the cluster process holds when its components satisfy a functional central limit theorem; the same is true for the functional strong law, and, for processes with right-hand clusters, for the functional law of the iterated logarithm.

Next, we deal with preservation of various forms of asymptotic independence under the clustering operation. Strong, ϕ- , and complete

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mixing possess an increasing degree of uniformity in their asymptotic independence, and this turns out to be a significant factor in their preservation. In particular, it is indicated that ϕ-mixing may be maintained only under very stringent conditions (bounded clusters), whereas strong mixing is maintained under considerably milder conditions.

The complement of our theme (non-conservation of local properties, that is, those depending on fine structure) is demonstrated via characterizations concerning renewal processes. It is proved that a stationary single server queue with finite capacity, exponential service times and a renewal input process can never have a renewal output process. We also look briefly at the problem of whether n ($\geq$ 2) independent, identically distributed point processes can be superposed to produce a renewal process. It is conjectured that all processes must then be Poisson, and proved in the case of the superposed processes being alternating renewal processes; counter-examples in the non-identical case have come from this area.

The thesis concludes with a list of unsolved and partially solved problems and generalizations, including an initial foray into the problem of identifying the cluster structure of a stationary cluster process from a complete centre process - subsidiary process record.

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