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Comparison of Mars rotation angle models

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Abstract. We compare published solutions for the rotation angle $W(t)$ which describes the location of the prime meridian of Mars with respect to the ICRF equator in IAU recommendations. If the model for W includes a very long period term, we transform it into a quadratic polynomial with updated epoch value and rate, resulting in a difference in the mean epoch value up to 200 km. The mean and true epoch rotation angles are about 800 mas (13 m) apart in J2000 and should not be confused with each other in order to accurately locate the prime meridian. We identify two groups of radio-science solutions for W, which can be distinguished by the prime meridian location they used as a priori that differ from each other by about 100 m at J2000.

Keywords. Mars rotation, geodesy, dynamics, Viking, longitude, radio science, prime meridian

1. Introduction

In order to describe the orientation of the spin axis of a planet, the IAU Working Group on Cartographic Coordinates and Rotational Elements (WGCCRE) uses the equatorial coordinates (right ascension α and declination δ) orienting the planet with respect to the equator of the International Celestial Reference Frame (ICRF), see for example Archinal *et al.* (2018). A third angle, denoted W and which has a diurnal periodicity, is used to position the prime meridian with respect to the node of the planet equator over the ICRF equator. For Mars, Yseboodt *et al.* (2023) showed that if a precision of about 5 mas is targeted in the 1970-2030 time interval, α, δ and W should be modeled by the sum of a quadratic polynomial and of a periodic series. The main periodicities correspond to harmonics of the orbital (i.e. annual) period.

To describe the orientation and rotation of Mars, the radio-science community more commonly uses Euler angles, including the obliquity ε and node longitude ψ defined with respect to the planet mean orbit at a chosen epoch. The rotation angle positioning the prime meridian with respect to the node of Mars equator over Mars mean orbit is denoted ϕ. This set of angles $(\varepsilon, \psi, \phi)$ can be precisely converted into equatorial coordinates (α, δ, W), and vice versa, using the analytical method proposed by Yseboodt *et al.* (2023). This method allows us to compare different rotation models whatever the set of angles used.

Thanks to the large number of spacecraft missions sent to the planet Mars in the past 50 years, numerous rotation solutions have been published. The latest IAU-like Mars solutions (in Earth equatorial coordinates and denoted here "IAU angles") were obtained by following a method which first converts the periodic series of Euler angles estimate in the time domain, then numerically transforms them into IAU angles time series, and finally fits the IAU angles periodic series using frequency analysis techniques (Jacobson 2010; Kuchynka *et al.* 2014 and Jacobson *et al.* 2018). In these solutions, the expression for each IAU angle includes a very long period ($\gtrsim 70,000$ years) term that

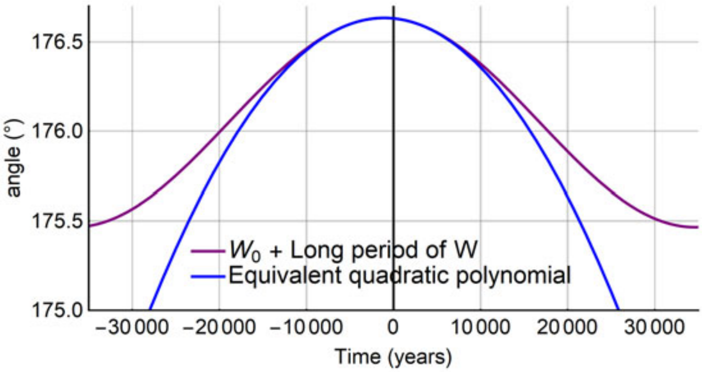

Figure 1. Locally (close to $t = 0$ here), a very long period term (in purple) can mimic the behavior of a quadratic function (in blue).

results from the lack of the quadratic term in the model used for the fit. Indeed, by choosing adequate amplitude, phase and frequency, long period signals can well mimic quadratic signals over an interval of a few decades around J2000. This is not anecdotal for the definition of a rotation model since adding a long period modulation instead of a quadratic term largely and artificially alters the epoch value and rate of each angle.

In Section 2, we transform the very long period term in W of the published IAU-like models into a quadratic function. We discuss the difference between the true and mean epoch value of W in Section 3. Then, we compare different published solutions for the rotation angle in Section 4. Finally we discuss some implications regarding the definition of the prime meridian of Mars.

2. Quadratic polynomial versus long period term

As explained in Yseboodt *et al.* (2023) and mentioned above, when no quadratic term is present in a IAU-like model obtained from a numerical fit, a very long period signal is needed to mimic the expected behavior on an interval of a few tens of years around the reference epoch, as shown in Fig. 1. This is demonstrated below where a long period modulation in the angle W plus a degree 1 polynomial is showed to be equivalent, to the second order in frequency f, to a degree 2 polynomial with no periodic terms:

$$\begin{aligned} W_0 + \dot{W}_0\, t + A \sin(f\, t + \varphi_0) &\approx (W_0 + A \sin\varphi_0) + (\dot{W}_0 + A\, f \cos\varphi_0)\, t - \frac{A\, f^2 \sin\varphi_0}{2}\, t^2 \\ &\approx W_0' + \dot{W}_0'\, t + W_Q\, t^2 \end{aligned} \tag{1}$$

Adding a long period modulation instead of a quadratic term significantly changes the epoch value and the rate at the initial epoch. As shown in Eq. (1), the ”real” epoch value (W_0') and rate ($\dot{W}_0'$) are then both split into a sum of a term coming from the degree 1 polynomial (W_0 and $\dot{W}_0$, respectively) and a term coming from the long period variations ($A \sin\varphi_0$ and $A\, f \cos\varphi_0$, respectively). The same reasoning applies to the α and δ angles.

Published solutions for the spin angle of Mars at t_0 are reported in the third column of Table 1. Corrected values (W_0') are provided for the three solutions that include very long period terms (column 4). Since Jacobson (2010) is the transformation of Konopliv *et al.* (2006), by construction $W'_{0\,(Ja10)} = W_{0\,(Ko06)}$. The same applies for $W'_{0\,(IAU15)} = W_{0\,(Ku14)}$ and for $W'_{0\,(Ja18)} = W_{0\,(Ko16)}$. For Jacobson *et al.* (2018), the correction is as high as 3.3° (or 200 km at the Martian surface) for W_0 and 100 mas/year for the rate of W. For Kuchynka *et al.* (2014), the shift is about 34 km, see first panel of Figure 2.

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Table 1. Comparison of the rotation angle W values at $t_0 = J2000$ for a selection of published rotation models (column 2). We transform the published solutions of Konopliv *et al.* (2006, 2016); Kuchynka *et al.* (2014) expressed in Euler angles into Earth equatorial coordinates by using the method described in Yseboodt *et al.* (2023). The fourth column provides the corrected epoch value (W'_0) for these three solutions. The fifth and sixth columns present the values of the periodic variations at epoch ($\Delta W(t_0)$) and the true epoch values ($W(t_0)$). The last columns present the difference between $W(t_0)$ and the true epoch value of the IAU15 solution, in degree and meter.

	Reference	W_0 Mean value (deg)	W'_0 (deg)	$\Delta W(t_0)$ Periodic (deg)	$W(t_0)$ True value (deg)	$W - W_{IAU15}(t_0)$ (deg)	(m)
IAU82	Davies *et al.* (1983)	176.655	-	0	176.655	0.022940	1357.1
IAU00	Seidelman *et al.* (2002)	176.630	-	0	**176.630**	-0.002060	**-121.9**
Ko06	Konopliv *et al.* (2006)	176.630239	-	0.000217	176.630456	-0.001604	-94.9
Ja10	Jacobson (2010)	176.076538	176.630239	0.553918	176.630456	-0.001604	-94.9
Ku14	Kuchynka *et al.* (2014)	176.631818	-	0.000241	176.632059	0	-0.1
IAU15	Archinal *et al.* (2018)	176.049863	176.631819	0.582197	**176.632060**	0	0
Ko16	Konopliv *et al.* (2016)	176.631896	-	0.000248	176.632145	0.000085	5.0
Ja18	Jacobson *et al.* (2018)	173.308792	176.631896	3.323352	176.632145	0.000085	5.0
LM23	Le Maistre *et al.* (2023)	176.631969	-	0.000216	176.632186	0.000126	7.5

3. Mean and true epoch rotation angles

When periodic terms are included in the definition of W, a distinction between the **mean** (W_0) and **true** ($W(t_0)$) epoch values can be made. Before IAU15 (Archinal *et al.* 2018), the successive IAU reference solutions for the orientation and rotation of Mars did not include periodic terms. This means that for the IAU00 solution for instance, the degree zero term in the time polynomial for the spin angle, W_0, is both the mean and true epoch value of W at the same time. The link between these two quantities is simply written here as:

$$W(t_0) = W_0 + \Delta W(t_0) \tag{2}$$

where the periodic series $\Delta W(t_0)$ can reach 0.00021-0.00025° (i.e. 780-890 mas or 12.4-14.3 m at the surface) at the J2000 epoch (only considering short period terms). A selection of published numerical values of these periodic variations is given in the fifth column of Table 1, while the corresponding true epoch values ($W(t_0)$) are given in the sixth column.

The radio-science community has long been ahead of the IAU WGCCRE regarding the addition of periodic terms in the model of orientation and rotation of Mars. The "Pathfinder model" of Folkner *et al.* (1997) already included nutation and seasonal spin variations (see also Konopliv *et al.* 2006). For the sake of consistency with the IAU solution, Konopliv *et al.* (2006) fixed the value of ϕ_0 so that the longitude of the Body Frame X-axis almost coincides (only 20 cm difference) with the one of the IAU00 solution at the J2000 epoch. This requires coinciding mean epoch values on one side with true epoch values on the other.

4. Comparison of published prime meridian positions

The prime meridian of Mars is defined as that meridian which passes through the center of the 500 m diameter Airy-0 crater (de Vaucouleurs *et al.* 1973). The last two estimations of its location (and therefore of W_0) are those of Duxbury *et al.* (2001) and Kuchynka *et al.* (2014). The first one (Duxbury *et al.* 2001), used for the IAU00 solution, is the average of various estimations based on different methods and mainly based on MOLA observations. Because the range of values covers about 500 m on the Martian surface, the uncertainty on the adopted $W_0 = 176.630°$ is about 250 m ($\sim 0.004°$). The

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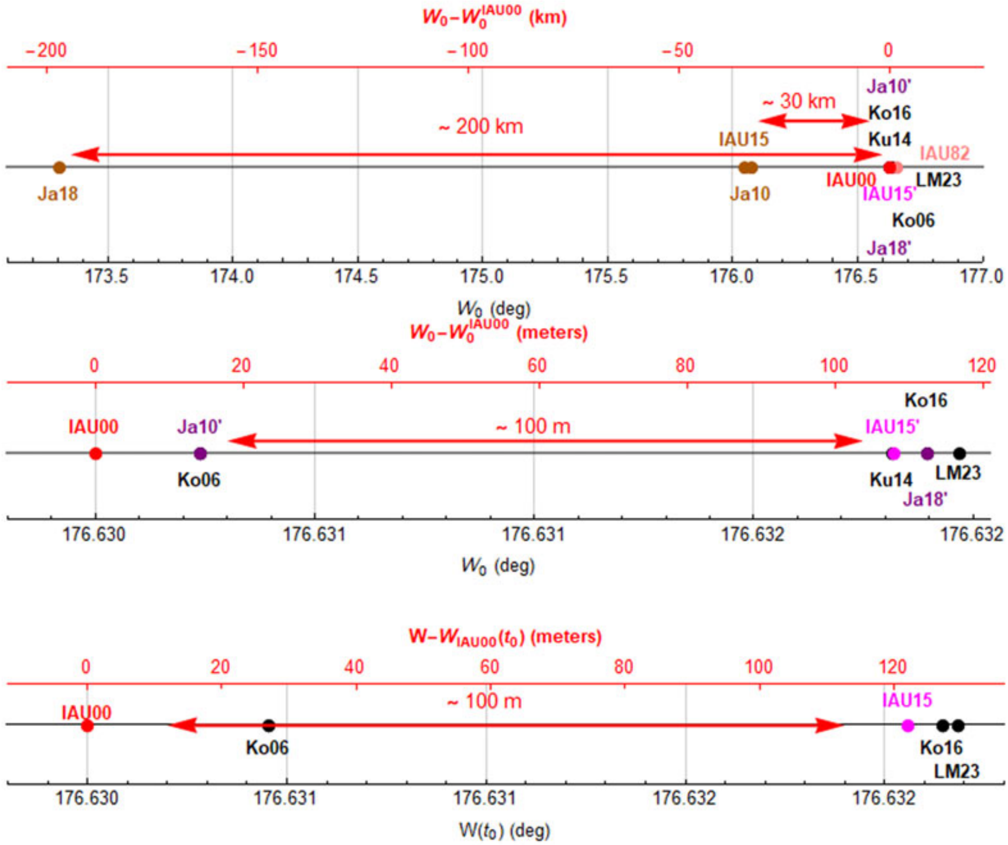

Figure 2. Comparison of the rotation angle W at the J2000 epoch for different published solutions. The names with $'$ are models with the very long period integrated into the epoch value. The first two panels show the mean epoch value W_0 while the last panel shows the true epoch value $W(t_0)$, taking into account the periodic variations at t_0. The red axis and values show the comparison to W_{IAU00} in meters. Since there are large and small differences between all the numerical values, 2 different intervals are shown: about 200 km for the first panel and about 120 meters for the other panels.

second one, estimated by Kuchynka *et al.* (2014) and used as the IAU15 solution, equals $W_0' = 176.631819°$ (after correction from long-period bias as explained in Sec. 2 and shown in Table 1) with an alleged precision of the order of 10 meters. This estimation of W_0 is based on Mars Global Surveyor's Mars Orbiter Camera (MOC) data, used to determine the cartographic longitude of Viking 1 lander relative to Airy-0, using pyramidal georegistration of lower to higher resolution imagery.

Note that the difference in X-axes longitudes between the IAU00 and IAU15 models is of about 100 m at J2000, within the error bar on the epoch value of W of the IAU00 model. The difference between the IAU82 and IAU00 solutions for W_0 was about 1.3 km. The IAU82 solution, issued with the second report of the WGCCRE, was the first providing the epoch value W_0 at the J2000 epoch and with respect to the J2000 ICRF.

The difference between the location of the prime meridian of the IAU00 and IAU15 solutions remains in the pure radio-science estimates of W_0, because radiometric data have no sensitivity to Airy-0 crater location. Thus, two groups of solutions for W epoch value can be identified. In the first group, Konopliv *et al.* (2006, 2011) choose their ϕ_0 value so that their prime meridian coincides with that of the IAU00 solution at the J2000

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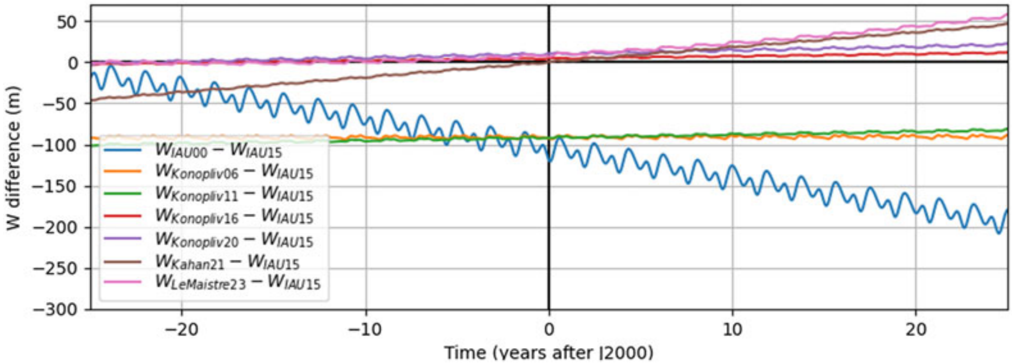

Figure 3. Difference between rotation angle $W(t)$ solutions and $W_{IAU15}(t)$, as a function of time, in meters.

epoch. In the second group, the prime meridian location is close to that of IAU15, either by fixing ϕ_0 value (Le Maistre *et al.* 2023) or by fixing the Viking 1 longitude to the Kuchynka *et al.* (2014) cartographic estimation, as done for example by Konopliv *et al.* (2016, 2020); Kahan *et al.* (2021). These two groups of solutions can be clearly identified on the left and right sides of the second panel of Figure 2 and in the last two columns of Table 1.

We also compare some published solutions at different times, see Figure 3. The difference between the IAU00 and IAU15 solutions clearly shows periodic variations, because the IAU00 solution did not have any periodic variations, unlike IAU15. Close to J2000, we see the two groups of solutions mentioned above: the first one close to IAU00, and the second one close to IAU15. However, the distinction between the groups is less clear away from J2000, mainly due to differences in the rotation rates.

5. Summary and discussion

Future IAU solutions shall replace the artificial terms at very long period by degree 2 polynomials in order to avoid large bias in the definition of the mean epoch values and rates and, consequently in the prime meridian location. Such a bias in the rotation angle $W(t)$ reaches $\sim$200 km in Jacobson *et al.* (2018) solution and 30 km in Jacobson (2010) solution and in the current IAU solution (i.e. IAU15) adopted from Kuchynka *et al.* (2014) (see Figure 2). Even though they accurately represent the temporal evolution of the angles within a few decades around J2000, models with long period terms lead to confusion as to the location of the prime meridian. For example, if one decided to ignore all the periodic terms (short and long) of these models, the prime meridian would be very far from reality.

The exact location of the Airy-0 crater at the J2000 epoch is not defined by the mean epoch rotation angle W_0, but by the true epoch rotation angle $W(t_0)$, the difference being the rotation variations evaluated at J2000. As the periodic variations in W were not taken into account by Duxbury *et al.* (2001), their estimation of W_0 is possibly biased by up to a few tens of meters, which is negligible compared to the 250 m error coming from MOLA observations.

Following IAU recommendations, the radio-science teams generally adjust their solutions so that their prime meridian is close to that of the IAU model at the J2000 epoch. This is why the recent radio-science solutions are divided into two groups, one following IAU00 and one following IAU15, which are based on two different estimates for the mean epoch value W_0. Only a work based on cartographic product and images more precise

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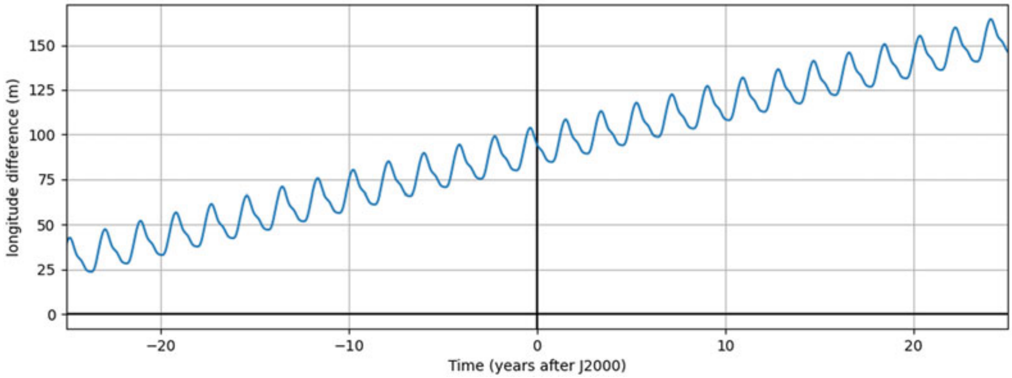

Figure 4. Comparison of the prime meridian position (see equation 3) between IAU00 and IAU15, as a function of time, in meters.

than used before can give a better estimation of W_0, which would then be used to give birth to a third group of Mars rotation models, hopefully close to the IAU15 group.

The intersection between the prime meridian and Mars true equator defines the X-axis of Mars body frame, reference for the cartographic longitude. Since any change in the IAU orientation angles displaces the true equator, and therefore the node from which W is measured, the location of the prime meridian is also affected by any difference in α and δ, besides the difference in W. The difference in longitude between the X-axes of two different orientation and rotation models expressed in IAU angles, $(\alpha_1, \delta_1, W_1)$ and $(\alpha_2, \delta_2, W_2)$ can be expressed, correct up to the first order in angles' differences, by

$$\Delta\lambda_{PrimeMer} = (W_1 - W_2) + (\alpha_1 - \alpha_2)\sin\delta \tag{3}$$

$$= (\phi_1 - \phi_2) + (\psi_1 - \psi_2)\cos\epsilon \tag{4}$$

where δ can be replaced either by the true epoch value of δ_1 or δ_2, or even by one of the mean epoch values, provided that the two models are not expressed with long period terms. From Eq. (3), we see that the prime meridian location changes when W changes, but also when α changes. Close to J2000, this $(\alpha_1 - \alpha_2)\sin\delta$ term equals -27 m for the difference IAU00-IAU15, which add to the 122 m difference in their W, leading to a difference in the prime meridian location of 95 meters at J2000 (see Figure 4).

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