OVERVIEW, FORMULATION AND CURRENT SITUATION FOR PRECESSION-NUTATION

N. CAPITAINE
Observatoire de Paris, URA1125,
61, avenue de l'Observatoire, 75014 Paris, France
E-mail:capitaine@obspm.fr

Abstract. This paper describes the conventional models for precession and nutation, the current procedure for taking them into account in the reduction of highly accurate observations, the recent IAU recommendations on this topic, as well as the current situation for theory and observations. This emphasizes the imperfections in the conventional models and in the current procedure which are not in consistency either with the adoption of the new International Conventional Celestial Reference System (ICRS) or with the high accuracy and resolution of the current observations of Earth rotation. This addresses the question of the adoption of a new formulation and of an improved model combining precession and nutation of the equator with respect to the ICRS as well as the adoption of a revised definition of the Celestial Ephemeris Pole.

1. Current situation for precession and nutation

1.1. CURRENT CONVENTIONAL MODELS

The conventional models for precession/nutation since 1984 are based upon :

(1) the 1976 IAU System of Astronomical Constants adopted by the IAU XVI GA of the IAU, including values, at J2000.0, for the constant of precession and the mean obliquity,
(2) numerical expressions for the precession quantities w.r.t. epoch J2000.0 (Lieske *et al.*, 1977),
(3) the 1980 IAU theory of nutation (Seidelmann, 1982) providing the nutation in longitude and obliquity (Kinoshita, 1977; Wahr, 1981) for the Celestial Ephemeris Pole, CEP.

These conventional models are associated with the use of :

- the FK5 as the Conventional Celestial Reference System (i.e. the Reference Frame at J2000.0 as realized by the FK5 catalogue + the use of (1), (2), (3)),
- of the relationship between Greenwich Mean Sidereal Time (GMST) and UT1 as given by Aoki *et al.* (1982).

These conventional models have been shown, for more than ten years, to present deficiencies w.r.t. VLBI and LLR observations and to be therefore inadequate for highly accurate observations. Consequently, IERS monitors the so-called "celestial pole offsets" as the observed estimates of the corrections to the 1976 IAU precession and 1980 IAU nutation.

1.2. RECENT IAU RECOMMENDATIONS

At the 1994 XXII GA of the IAU, following the recommendations of the Joint Discussion 14 untitled "Towards the establishment of the astronomical standards" and of the Joint Discussion 19 untitled "Nutation", two resolutions (C1 and C6) have been adopted and an IAU/IUGG WG on "Nutation for a non-rigid Earth Nutation theory" has been established.
The IAU resolution C1 :

- urged that observations of the offset of the celestial pole w.r.t. the pole defined by the 1980 theory of nutation be made with the most precise techniques available,

J. Andersen (ed.), Highlights of Astronomy, Volume 11A, 153–157.

https://doi.org/10.1017/S1539299600020311 Published online by Cambridge University Press

- asked the IERS to provide an empirical model for corrections to the 1980 theory of nutation to be used for *a priori* estimates of the CEP offsets.

The IAU resolution C6 recommended that :

- the present 1976 IAU System of Astronomical Constants be retained,
- an IAU File of Current Best Estimates of Astronomical Constants be established.

1.3. CURRENT SITUATION

The 1994 IAU resolutions C1 and C6 have been followed.

The celestial pole offsets are monitored on a regular basis by the IERS with an improving accuracy (0.2 milliarcsecond (*mas*) in 1996 for daily values) and an interim precession-nutation model, the "IERS 1996 Theory of Precession/Nutation", has been provided in the IERS Conventions 1996 (McCarthy, 1996) that match the observations with uncertainties of ± 1 *mas*.

The 1996 IERS Conventions have provided a list of IERS Numerical Standards for Current Best Estimates of a few constants (including the precession constant, p_1, and the mean obliquity, ϵ_0, at J2000.0), the 1976 IAU System of Astronomical Constants being retained for all the other constants. However, consistency has to be ensured between the IERS Numerical Standards for p_1 and ϵ_0 and the numerical values for precession of the IERS 1996 Precession/Nutation theory.

The IAU/IUGG WG on "Non-rigid Earth Nutation theory" has given rise to a large number of studies for improving the theory of nutation for a Rigid Earth Model as well as for the computation of the geophysical nutations and relativistic effects. A large number of estimations of the rate of precession, obliquity rate and amplitudes of nutation have also been performed since 1994, using the most accurate data.

Much progress have therefore been done since the previous IAU General Assembly in the accuracy of the available models and observations for precession and nutation and it is clear that the conventional models have now to be re-considered in order to be in consistency with :

- the new available planetary theories,
- the new theories of Earth Rotation (rigid body, geophysical nutations and relativistic effects),
- the new International Conventional Celestial Reference System (ICRS) adopted by this IAU General Assembly,
- the present observations (accuracy and method of observations).

2. Current procedures for taking precession and nutation into account

2.1. FORMULATION FOR PRECESSION

The current formulation for precession is based on the general precessional motions as defined by Woolard and Clemence (1966) and on the accumulated precessional angles as defined by Lieske *et al.* (1977). The numerical expressions for the precession quantities have been given as functions of two time parameters in Julian centuries, t from the basic epoch (J2000.0) to the date and T, the difference between an arbitrary fixed epoch and the basic epoch. Simplified expressions, when the arbitrary epoch is chosen to be J2000.0 (i.e. $T = 0$) are given in the IERS Conventions.

A few comments can be made on the current formulation for precession :

- it combines the precession of the equator and the precession of the ecliptic (i.e. planetary precession) into the IAU "Constant of precession", p_1, (the speed of general precession in longitude at J2000.0) and into the quantities of precession, the basic quantity, $p_A = \Lambda_A - \Pi_A$, the general precession in longitude, being the difference between two precession angles, one along the ecliptic of date and one along the ecliptic of epoch,
- the motion of the pole of the ecliptic, used in the quantities of precession, has been derived from Newcomb's theory of the Sun (1894) with only improved values for planetary masses, whereas improved planetary theories are available (Simon *et al.*, 1994).

The usual precession quantities depend both upon the precession of the Earth's equator with respect to a fixed celestial frame, due to perturbing torques produced by the Moon, the Sun and the planets on the oblate Earth and upon the motion of the ecliptic of date with respect to a fixed celestial frame, due to planetary perturbations. A better consistency between the models and the real motions would require to clearly separate, in the precession quantities, the part which is due to the motion of the equator with respect to the ecliptic of epoch and the part which is due to the motion of the ecliptic of date, and in the same way to express the angles of nutation, from the ecliptic of epoch.

2.2. FORMULATION FOR NUTATION

The basic variables used in the theory of the rotation of the Earth around its centre of mass are either Euler angles between the Terrestrial Reference System (TRS) and the Celestial Reference System (CRS) (Woolard, 1953), or Andoyer canonical variables plus Oppolzer terms (Kinoshita, 1977). They provide the precession and nutation of the equator in longitude ψ and obliquity ϵ respectively, as well as the angle of Earth's rotation in space, ϕ, as : $\psi_A + \Delta\psi$, $\epsilon_A + \Delta\epsilon$, $\phi_A + \Delta\phi$, the quantities being separated into a secular term (e.g. ψ_A for precession), and a sum of periodic terms represented here by Δ (e.g. $\Delta\psi$ for nutation in longitude).

The current solution for the motion of the equator is referred to the ecliptic of date and represent therefore the motion of the equator with respect to a moving plane and not to the ICRS.

2.3. CURRENT PROCEDURE

The coordinate transformation to be used from the TRS to the CRS at the date t can be written as : $[CRS] = PN(t)R(t)W(t)[TRS]$, where $PN(t)$, $R(t)$ and $W(t)$ are the transformation matrices arising from the motion of the CEP in the CRS, from the rotation of the Earth and from the motion of the CEP in the TRS respectively.

The current procedure, which appears as option 1 in the IERS Conventions (McCarthy 1996), separates the celestial motion of the CEP into two transformation matrices P and N for precession and nutation respectively and makes use of the equinox for realizing the intermediate reference frame of date. It uses, the precession quantities z_A, ζ_A, θ_A given by Lieske *et al.* (1977) in the transformation matrix P, the nutation quantities in longitude, $\Delta\psi$, and obliquity, $\Delta\epsilon$, referred to the mean ecliptic and equinox of date in the transformation matrix N, and apparent Greenwich Sidereal Time, GST, as a function of UT1 in the transformation matrix R.

The numerical values for the precession quantities at date t are those of Lieske *et al.* (1977) with improved values for the precession rate of the equator in longitude and obliquity associated with the IERS 1996 series of nutation. The numerical values for the nutation angles at date t are those of the IERS 1996 series of nutation, but the IERS "celestial pole offsets" are the observed differences with respect to the conventional celestial pole position defined by the IAU model for precession and nutation. The numerical expression of GST is obtained from the relationship between GMST and UT1 (Aoki *et al.*, 1982) and the expression of the "equation of equinoxes", with two complementary terms since 1 January 1997.

The following comments can be made on the current procedure :

(1) it artificially separates the motion of the equator into precession and nutation; such a separation, due to technical constraints belonging to past, is no more necessary with the modern observations and computations,
(2) it refers to the "true equinox of date" and therefore GST includes both Earth's rotation and accumulated precession-nutation of the equinox along the moving equator; this is for historical reasons and is related to the use of the FK5, which has to provide the pole and the equinox at each date, which is no more the case of the ICRS,
(3) classical solutions and quantities for precession and nutation refer to the ecliptic of date instead of a fixed plane to be related to the ICRS,
(4) the conventional precession and nutation model is clearly inadequate and an improved model is necessary.

The "1996 Theory of precession/nutation" is an improved model as compared to the IAU conventional models; however, it is not meant to replace the IAU model, but rather to be used for prediction purposes, as well as when accurate *a priori* estimates of nutation are necessary. Improved theoretical series could be used, but the advantages of the use of a conventional model which may include few thousands of terms has to be compared with that of the IERS 1996 series.

https://doi.org/10.1017/S1539299600020311 Published online by Cambridge University Press

An other point to be stressed is that the arguments of nutation have to be in consistency with the planetary theories used in the solution for nutations in order not to introduce spurious differences in the nutation quantities (Chapront-Touzé and Chapront, 1994).

2.4. ALTERNATIVE PROCEDURE

An other procedure can be used for taking into account precession and nutation of the equator. The celestial pole coordinates (X, Y) (Capitaine, 1990), which include both precession and nutation of the equator and moreover refer to the ICRS, would be variables more consistent to the real motions.

The relationship between these quantities and the classical quantities for precession and nutation is such that (Capitaine, 1990) :

$$\begin{aligned} X &= \chi_o + \sin(\omega_A + \Delta\epsilon_1)\sin(\psi_A + \Delta\psi_1) \\ Y &= \eta_o + \sin\epsilon_o\cos(\omega_A + \Delta\epsilon_1) + \cos\epsilon_o\sin(\omega_A + \Delta\epsilon_1)\cos(\psi_A + \Delta\psi_1) \end{aligned}$$

χ_o, η_o being the celestial offsets between the computed pole for J2000.0 and the pole of the ICRS, ω_A the inclination of the mean equator on the fixed ecliptic and index 1 being for nutation quantities referred to the fixed ecliptic. The development as function of time of these quantities can also be directly provided by the theory if it uses such kind of variables.

Concerning Earth rotation, the stellar angle $\theta = \theta_o + \int_{t_o}^{t} \omega_3\, dt$, kinematically defined by Guinot (1979) such that its time derivative is equal to the Earth's angular velocity, ω_3, around the axis of rotation can advantageously replace GST; it makes an explicit use of the "non-rotating origin" (NRO), which has to be implicitly used to express any accumulated angle along the moving equator (such as the accumulated precession and nutation in the complete expression of GST).

This alternative procedure is described in option 2 of the IERS Conventions.

3. The Celestial Ephemeris Pole

The intermediate pole defines an intermediate reference direction in the transformation between the CRS and the TRS. In the IAU-1964 series of nutation, the considered pole was the pole of instantaneous rotation which has been shown to be not observable by the classical astrometric observations. In the IAU-1980 series of nutation, the CEP has been adopted as being a pole closer to the actually "observed pole" at that time. A conceptual definition has been given for that point so that it has no diurnal motion w.r.t. TRS or CRS. The corresponding conventional definition is the realization of this pole by the precession-nutation model. This leads to several practical realizations of the CEP :

(1) model IAU-1976 precession + IAU-1980 nutation,
(2) model (1) + estimated celestial offsets.

The realization (1) shows the imperfections due to the models IAU-1976 and IAU-1980 as diurnal terms in the TRS, with amplitudes of the order of 10 *mas* (total). The realization (2) has the imperfections due to the adjustment procedure and for example, every diurnal prograde motion is absorbed in the residuals if they are not *a priori* estimated.

The conceptual definition given for the CEP is no more consistent with the theory of nutation which now includes diurnal terms of the CEP in the CRS (Bretagnon *et al.*, 1997), as well as with the model for pole motion which includes diurnal motion of the CEP in the TRS (Gross, 1993; Herring and Dong, 1994).

It is no more consistent with the sub-daily determinations of the Earth Orientation Parameters (EOP), with a level of accuracy of the order of less than 1 *mas*, as given through intensive campaigns, which make less clear the distinction between high-frequency polar motion, dominated by diurnal and semi-diurnal periodic signals and nutation dominated by diurnal retrograde polar motion.

The use of the CEP has therefore to be re-discussed and a new definition of the "actually observed pole" is necessary to be consistent with the new theory including diurnal terms both in space and within the Earth and according to new observing procedures.

4. Conclusion : Questions to the Joint Discussion

A new ICRS has been adopted by the IAU which is no more linked to the pole and the equinox. Present techniques of observations, which are sensitive to the orientation of the equator with respect to the ICRS, are improving in accuracy and resolution. New theoretical developments are available for precession and nutation.

The questions which have to be addressed are therefore the following :

(1) Is it necessary to keep a formulation combining precession of the equator and of the ecliptic ? (i.e. keep the constant of precession as a primary constant of the IAU System of Astronomical Constants ?)
(2) Is it necessary to adopt an improved model for the ecliptic pole motion ?
(3) Is it time to adopt an improved, consistent and combined formulation and model for the precession and nutation of the equator referred to the ICRS ?
(4) Is it time to abandon the equinox as the point of reference on the equator ?
(5) How can the definition of the CEP be revised to be consistent with the present theory of nutation and pole motion as well as to the present observing procedures ?

References

Aoki, S., Guinot, B., Kaplan, G.H., Kinoshita, H., McCarthy, D.D. and Seidelmann, P.K. (1982) *Astron. Astrophys.*, **105**, pp. 359.
Bretagnon, P., Rocher, P. and Simon, J.-L. (1997) *Astron. Astrophys.*, **319**, pp. 305.
Capitaine, N. (1990) *Celest. Mech. Dyn. Astr.*, **48**, pp. 127.
Chapront-Touzé, M. and Chapront, J. (1994) in *Journées Systèmes de Référence spatio-temporels*, ed. N. Capitaine, Observatoire de Paris.
Guinot, B. (1979) in *Time and the Earth's Rotation*, eds. D.D. McCarthy and J.D. Pilkington, D. Reidel Publishing Company, pp. 7.
Herring, T.A. and Dong, D. (1994) *J. Geophys. Res.*, **99**, B9, pp. 18 051.
Kinoshita, H. (1977) *Celest. Mech.*, **15**, pp. 277.
Lieske, J.H., Lederle, T., Fricke, W. and Morando, B. (1977) *Astron. Astrophys.*, **58**, pp. 1–16.
McCarthy, D.D. (ed.) (1996) *IERS Conventions*, IERS Technical Note, **21**, Observatoire de Paris.
Seidelmann, P.K. (1982) *Celest. Mech.*, **27**, pp. 79.
Simon, J.L., Bretagnon, P., Chapront, J., Chapront-Touzé, M., Francou, G. and Laskar, J. (1994) *Astron. Astrophys.*, **282**, pp. 663.
Wahr, J.M. (1981) *Geophys. J. R. Astron. Soc.*, **64**, pp. 651.
Woolard, E.W. (1953) *Astr.Pap.Amer. Eph. Naut. Almanach*, **XV**, I, pp. 1–165.
Woolard, E.W. and Clemence, G.M. (1966) *Spherical Astronomy*, Academic Press, New-York.