I. EVOLUTION OF LOW AND INTERMEDIATE MASS STARS
OBSERVATIONS AND MODELS

FUNDAMENTAL PROBLEMS AND BASIC TESTS OF STELLAR EVOLUTION THEORY - THE CASE OF CARBON STARS[†]

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Abstract

Carbon stars are thought to be in the asymptotic giant branch (AGB) phase of evolution, alternately burning hydrogen and helium in shells above an electron-degenerate carbon-oxygen (CO) core. The excess of carbon relative to oxygen at the surfaces of these stars is thought to be due to convective dredge-up which occurs following a thermal pulse. During a thermal pulse, carbon and neutron-rich isotopes are made in a convective helium-burning zone. In model stars of large CO core mass, the source of neutrons for producing the neutron-rich isotopes is the $^{22}Ne(\alpha,n)^{25}Mg$ reaction and the isotopes are produced in the solar system s-process distribution. In models of small core mass, the $^{13}C(\alpha,n)^{16}O$ reaction is thought to be responsible for the release of neutrons, and the resultant distribution of neutron-rich isotopes is expected to vary considerably from one star to the next, with the distribution in isolated instances possibly resembling the solar system distribution of r-process isotopes. After the dredge-up phase following each pulse, the ^{13}C is made by the reactions $^{12}C(p,\gamma)^{13}N(\beta^+\nu)^{13}C$ in a zone of large ^{12}C abundance and small ^{1}H abundance that has been established by semiconvective mixing during the dredge-up phase. There is qualitative accord between the properties of carbon stars in the Magellanic Clouds and properties of model stars, but considerably more theoretical work is required before a quantitative match is achieved.

The observed paucity of AGB stars more luminous than $M_{BOL} \sim -6$ is interpreted to mean that the AGB lifetime of a star more luminous than this is at least a factor of ten smaller than the AGB lifetime of stars less luminous than this, or, at most 10^5 yr. Since, with current estimates of the $^{22}Ne(\alpha,n)^{25}Mg$ reaction rate R_{22}, only AGB model stars more luminous than $M_{BOL} \sim -6$ can produce s-process

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isotopes in the solar system distribution, it is inferred that either (1) the current estimates of R_{22} are too small by one to two orders of magnitude, allowing less luminous AGB stars to contribute, (2) the solar system distribution is not equivalent to the average Galactic distribution, being rather the consequence of a unique injection into the protosolar nebula of matter from a massive intermediate-mass AGB star, or (3) the estimates of the temperatures in the convective shell that are given by extant models are too low by, say, 10 or 15 percent.

The absence of carbon stars more luminous than $M_{BOL} \sim -6$ is suggested to be due primarily to the fact that $\sim 10^6$ yr of AGB evolution is necessary to produce surface $C/O > 1$, rather than to be due to the burning of dredged-up carbon into nitrogen at the base of the convective envelope during the interpulse quiescent hydrogen-burning phase. Thus, the positive correlation between the nitrogen and helium abundances in planetary nebulae is perhaps primarily a consequence of the second dredge-up episode rather than a consequence of processes occurring during the thermally pulsing phase.

I. THE OBSERVATIONAL FACTS

A. Galactic Studies

One of the drawbacks to attempts at understanding carbon stars from observations of stars in the Galaxy is the fact that very few of them are in clusters for which distances can be estimated. Nevertheless, much can be learned by piecing together all of the available evidence.

Abundances of Carbon, Nitrogen, and Oxygen. From a study of the Sun and of stars, HII regions, and molecular clouds in the vicinity of the Sun, one infers that the CNO elements are produced in abundances such that $O > C > N$. This is not to say that every star which contributes to the synthesis of these elements produces them in the stated order of abundances, but that the net result of the synthesis of these elements by all stars is responsible for the observed order in the interstellar medium and in unevolved stars formed from this medium.

That the observed order of abundances must be the result of an averaging of the contributions from many different types of stars is obvious from the very existence of carbon stars which are, by definition, stars in which $C > O$ at the surface and which are known to be contributing matter (with $C > O$) to the interstellar medium by way of a strong wind. As we shall see, the evidence is overwhelming that the relative abundance of C relative to O in carbon stars is due to synthesis of C in the stars themselves.

Neutron-Rich Isotopes. Two other sets of elements (actually, sets of isotopes) which prove to be of interest in connection with carbon stars are found from studies of neutron-rich isotopes in meteorites. These are the so called s-process and r-process isotopes which are thought to be made by successive neutron captures and β decays beginning with the seed nucleus ^{56}Fe. In one group, adjacent stable isotopes are related to one another by $n_i\sigma_i = n_{i-1}\sigma_{i-1}$ $(1 + \Lambda/\sigma_i)^{-1}$, where n_i is the number abundance of the i^{th} isotope, σ_i is its neutron capture cross section (at an energy of ~ 30 keV), and Λ is a universal constant whose value is of the order of (3-5) millibarns. In theoretical nucleosynthesis experiments, such a distribution can most easily be formed if the flux (or density) of neutrons is sufficiently low that a beta-unstable nucleus will decay before capturing a neutron -- hence the name s(low neutron capture)-process. In contrast, the isotopes in the other group display abundance patterns that can be reproduced theoretically most easily if neutron capture takes precedence over beta decay until neutron number reaches the magic values of 82 and 126. These isotopes are therefore called r(apid neutron capture)-process isotopes.

Over the past three decades, an examination of abundances of neutron-rich isotopes in Galactic S-stars (in which C ~ 0) has shown that neutron-rich isotopes are formed in such stars and brought to their surfaces. Of particular interest is the occurrence in many S-stars of Tc. The most stable isotope ^{99}Tc has a half life of only 2×10^5 yr, whereas the progenitor of an S-star must have a main sequence lifetime in excess of a few times 10^7 yr. In the spectrum of Galactic Ba and CH stars, which are not as bright as AGB stars, one finds evidence for neutron capture activity, but recent radial velocity studies have shown that many of these stars are in binaries with a degenerate dwarf companion (McClure, Fletcher and Nemec 1980, McClure 1983a,b). The thought is that the neutron-rich isotopes may have been formed in the AGB precursor of the degenerate dwarf and transmitted by Roche-lobe overflow or by a wind to the star which now shows Ba or CH features. Certainly, the orbital separations of many of the binaries are large enough to have accommodated a star of AGB dimensions. The high degree of variability in the relative abundances of the neutron-rich isotopes from one star to the next suggests that the neutron source is highly variable in strength and duration from one erstwhile AGB companion to the next.

Tc in Miras. Most Mira variables are not at the same time carbon stars. However, approximately half of the brightest fifteen percent of them are (Cahn 1980). Furthermore, except for those of the smallest periods (and hence presumably of the lowest luminosities), many of them show Tc lines and the probability of finding such lines increases with the period (and therefore presumably with brightness) of the Mira (Little-Marenin and Little 1979). We infer, first, that perhaps the production of neutron-rich isotopes and certainly the probability of dredging these isotopes to the surface increases with increasing AGB luminosity and, second, that the conversion of the surface C/O ratio from less than 1 to larger than 1 may require several dredge-up episodes.

Frequency of C-Stars as a Function of Metallicity. As shown by Blanco, McCarthy, and Blanco (1978) there is a pronounced gradient in the space frequency of Galactic carbon stars, with none occurring in the Galactic bulge. This is despite the existence of metal rich AGB stars in the bulge (Whitford and Rich 1983, Frogel and Whitford 1983). The simplest interpretation of these facts is that the larger the abundance of Fe/H, the more difficult it is for an AGB star to develop a surface ratio C/O > 1. Whether this means that, with increasing Fe/H, it is more difficult for dredge-up to occur or whether, because of a larger initial abundance of oxygen, that more carbon must be dredged-up in order for C > O at the surface, or both, is not settled by these observations.

In summary, from an examination of stars in our own Galaxy, we have learned that (1) carbon and neutron-rich isotopes are made in cool, bright AGB giants and brought to the surfaces of these giants; (2) the brighter the giant, the greater is the likelihood of producing and bringing to the surface freshly manufactured elements and isotopes; and (3) the larger C and O are to begin with, the harder it is to achieve C > O at the surface.

B. Magellanic Cloud Studies

The development of infrared technology over the past decade has made it possible to take advantage of the fact that all of the stars in each of the Clouds are effectively at the same distance from the earth. This has permitted us (since we can now compare absolute luminosities of real carbon stars with those of model AGB stars) to establish with a certainty that AGB stars bring freshly synthesized carbon and neutron-rich isotopes to their surfaces and that the conversion from O-star, through S-star, to C-star occurs at a definite point along the AGB branch.

Field Stars. Surveys of selected fields in both clouds show that carbon stars are, in general, confined to a very narrow interval in bolometric magnitude: $-6 \lesssim M_{BOL} \lesssim -4$ (survey by Blanco, McCarthy, and Blanco 1980; bolometric magnitudes by Richer 1981, Cohen et al 1981, Frogel et al 1981, and Frogel and Cohen 1982). Extrapolations from the surveyed regions suggest that there are altogether $\sim 1.4 \times 10^4$ carbon stars in both clouds (Blanco and McCarthy 1983) and that there are only a handful of S-stars (Blanco, Frogel, and McCarthy 1981, Lloyd-Evans 1983). This permits us to say that the S-star phase, when $C \sim O$, is a very transitory one, with the transition from $C \sim O$ x (1-.05) to $C \sim O$ x (1 + 0.05) occurring in perhaps only one dredge-up episode.

The long period variables (LPV's) in the Magellanic Clouds may be assigned to one of two distinct sequences (Wood, Bessel, and Fox 1983) according to whether or not they show evidence for overabundances of the neutron-rich element Zr. Those in which the tracer ZrO appears to be "normal" are in general brighter than $M_{BOL} \sim -7.3$, which corresponds to the magnitude of a theoretical AGB model whose core has reached the Chandrasekhar mass of 1.4 $M_{\odot}$. Those in which strong ZrO bands appear define a sequence in the M_{BOL}-pulsation period plane that is consistent with theoretical AGB models. The observed sequence that is characterized by strong ZrO bands extends, within the uncertainties, up to the maximum brightness reached by theoretical AGB models. The lowest period representatives of this sequence tend to be carbon stars, but the brightest ones are not carbon stars. The number (~ 100) of the non-carbon star LPV's with $M_{BOL} \lesssim -6$ is about a factor of twenty or so less than the number of Cepheids in the Clouds and, since the progenitors of the LPV's are thought to be Cepheids with lifetimes of ~ 10^6 yr, we infer that stars which reach the AGB with $M_{BOL} \lesssim -6$ remain on the AGB for only ~ 10^5 yr and this, as it turns out, is only ten percent or so of the AGB lifetime expected if mass loss via a stellar wind were to occur at roughly the Reimers rate (1975).

The frequency of carbon stars in the SMC is about 3 times larger than it is in the LMC (Blanco, McCarthy and Blanco 1978, 1980; Blanco and McCarthy 1983) and, since the metallicity of SMC stars is on average much less (by perhaps a factor of 3-6) than the metallicity of LMC stars, we recover the result of studies of Galactic C-stars that, for whatever reason, AGB stars of low metallicity find it easier to become carbon stars than do AGB stars of higher metallicity.

Stars in Globular Clusters. Searle, Wilkinson, and Bagnuolo (1980) have suggested an ordering of Magellanic Cloud clusters in an approximate age sequence and Cohen (1982) has shown that this sequence is equivalent to an ordering according to metallicity with, naturally enough, the oldest clusters having the lowest metallicity. Frogel and Blanco (1984, this volume) find that, in the oldest (~ 10^{10} yr) clusters, there are essentially no carbon stars. From this one may infer that, if the initial mass of an AGB star is quite low ($\lesssim 0.8\ M_{\odot}$), either dredge-up cannot occur, or the envelope mass of the star is lost by a wind before dredge-up can lead to C > O, *or both.*

Among stars on the AGB of an intermediate age cluster (0.3×10^9 yr $\lesssim$ age $\lesssim 3 \times 10^9$ yr), Frogel and Blanco find that there is a clear separation in absolute luminosity between the dimmer M-stars (O > C) and the brighter C-stars (C > O), with the luminosity at the transition point between M-stars and C-stars increasing with decreasing cluster age. This result has also been obtained independently by Lloyd-Evans (1983) and it is consistent both with the idea that dredge-up occurs on the AGB and with the theoretical result that the luminosity with which a model star reaches the thermally pulsing stage on the AGB increases with increasing mass (decreasing nuclear burning lifetime) of its main sequence progenitor (e.g., Becker and Iben 1979, 1980).

A remarkable result of the Frogel-Blanco survey is that in young clusters which contain Cepheids (age $\lesssim 10^8$ yr) *there are no C-stars.* And yet, theoretically, AGB stars which have Cepheid progenitors, and hence have masses on the order of (4 - 6) $M_\odot$, are expected to dredge-up carbon with great facility (Iben 1975a). Even more astonishing than the absence of C-stars is the total absence of even M-stars brighter than about $M_{BOL} \sim -6$. Since a model AGB star reaches a luminosity given by $M_{BOL} \sim -7.3$ if its core mass reaches ~ 1.4 $M_\odot$, and since the rate at which a theoretical model brightens (in magnitudes per year) is independent of core mass, the inescapable conclusion is that a real AGB star leaves the AGB considerably before its core mass reaches 1.4 $M_\odot$. The absence of carbon stars in the field brighter than $M_{BOL} \sim -6$ can now also be attributed to the paucity of AGB stars brighter than this rather than to two other possibilities that have been cited in the past: (1) AGB stars brighter than $M_{BOL} \sim -6$ do not dredge-up carbon (contrary to theoretical indications) or (2) such stars convert dredged-up carbon into nitrogen as a consequence of proton captures at the base of the convective envelope during the quiescent, interpulse hydrogen-burning phase.

THE THEORY - OVERVIEW

A. Surface Composition Changes Prior to the AGB Phase

It is important to distinguish the nature of those surface composition changes which may occur prior to the AGB phase from those changes which are due solely to processes occurring during the AGB phase proper. This is not a completely straightforward task, as we cannot deduce from first principles the rates at which rotationally induced forms of mixing actually operate in real stars. Even if we were able to calculate the effectiveness of such forms of mixing, the fact that there is a large spread in rotational velocities among real stars will introduce a large spread in surface composition changes if these forms of mixing are of importance.

Meridional Circulation and Turbulent Diffusion in Main Sequence Stars. In principle, these processes should both increase in effectiveness with increasing rotation rate and might be expected to bring to the surface products of nuclear processing in the interior. In particular, one might anticipate some outward diffusion of products of CN-cycle burning and a reduction in C and an enhancement of N at the stellar surface. However, no evidence has yet been presented by spectroscopists to show that there is a tendency for C/N to decrease in any systematic way from high T_e to low T_e across the main sequence band and there is therefore no compelling evidence that rotationally induced mixing is of importance.

Convective Dredge-Up on the Giant Branch. In contrast, there is good observational evidence to support the theoretical indications that CN-cycle processed material is first brought to the surface as a star begins its initial climb upwards along the (first) red giant branch and the base of the convective envelope extends inward in mass to where C has been converted almost entirely into N (see Iben and Renzini 1984 for a review). The theoretical calculations provide precise quantitative predictions [surface C down by ~ 1/3 x and N up by ~ 2 x] and these predictions are consistent with the observations.

Meridional Mixing on the Giant Branch in Globular Cluster Stars. Kraft et al (1982) and Carbon et al (1982) find that, among red giant stars in the Galactic globular cluster M92, the abundance of carbon decreases with stellar luminosity. This decrease continues to luminosities much larger than the luminosity at which the first convective dredge-up is expected to have run its course. The best explanation for the observed effect is the one given by Sweigart and Mengel (1979) who point out that, in giants, the distance between (a) the region above the main hydrogen-burning shell where C has been converted into N, and (b) the base of the convective envelope, is quite small and argue that meridional circulation may carry C-depleted, N-enhanced material into the base of the convective envelope. Convective mixing will then carry some of this processed material to the surface. The phenomenon appears to be confined to stars in only a subset of clusters and does not appear to occur in stars in the field (Kraft 1979). One infers that close stellar encounters among stars in clusters of the largest spatial concentration may lead to (probably temporary or reversible) spin-ups that initiate the requisite meridional currents.

Convective Dredge-up After Central Helium Exhaustion. In model stars which, as a consequence of hydrogen-burning either before or during the core helium-burning phase, develop a hydrogen-exhausted core larger than about 1 $M_\odot$ by the time that the central helium abundance goes to zero, the base of the convective envelope extends briefly inward in mass into the region through which the hydrogen-burning shell has passed during the preceding core helium-burning phase. Therefore, as the model star once again climbs along the (second) red giant branch and the base of the convective envelope moves inward in mass, fresh ^{4}He and ^{14}N (produced at the expense of both ^{12}C and ^{16}O) are brought to the surface. Thus, the surface abundances of both ^{4}He and ^{14}N increase, whereas the surface abundances of ^{12}C and ^{16}O decrease. This second dredge-up phenomenon occurs only in models with initial main sequence masses larger than ~ (4-5) $M_\odot$ and less than ~ (8-9) $M_\odot$. Within this narrow range of masses, the calculations show that ^{14}N and ^{4}He enhancements as large as a factor of two occur (for N, over and above the factor of ~ 2 enhancement that occurs during the first dredge up episode), with the degree of enhancement increasing with increasing mass within this range (Becker and Iben 1979).

B. Surface Composition Changes During the Thermally Pulsing (TP)-AGB Phase

We have seen that, prior to the AGB phase, enhancements in the surface abundances of N and He and depletions in the surface abundances of C and O (the depletion of C being much more substantial than that of O) occur as a consequence of the convective dredge-up of matter that has experienced hydrogen-burning in varying degrees. The fact that many AGB stars exhibit a surface abundance of oxygen larger than the surface abundance of carbon, in spite of the fact that surface C is depleted when hydrogen-burning products are dredged to the surface, provides compelling evidence that such stars must somehow bring to their surfaces matter that has experienced helium-burning in the interior.

Abundance Changes in Nuclear Burning Regions. During about ninety percent of the AGB lifetime of a model star, helium-burning proceeds at a very low level, with effectively all of the energy which flows to the surface coming from hydrogen-burning in a thin shell (whose mass varies from ~ 10^{-4} M_0 to 10^{-7} M_0 as the mass M_{CO} of the electron-degenerate CO core is varied from ~ 0.5 M_0 to ~ 1.4 M_0). If the mass of the stellar envelope is large enough and if convective mixing is efficient enough, some burning may take place at the base of the convective envelope and, if $M > (4-5)$ M_0, it is even possible that the abundance of ^{12}C decreases and the abundance of ^{14}N increases in the envelope between pulses (Iben 1975a, Renzini and Voli 1981). When the mass of the He-N zone laid down by the hydrogen-burning shell reaches about 100 times the mass of the burning shell, densities and temperatures in this zone become large enough to excite a helium-burning thermonuclear runaway. In the course of this runaway, the rate of energy generation by helium-burning can reach as high as (10^7-10^8) L_0 and temperatures at the base of the runaway zone can reach as high as (300-400) x 10^6K. During the thermal flash, nuclear energy is injected so rapidly that it is converted locally into thermal energy and thence into local expansion energy before it has a chance to leak out from the burning zone by radiative or convective "diffusion." The runaway is quenched as a consequence of the expansion and subsequent cooling.

At the peak of a thermal pulse (or helium shell flash, as it is often called), the entire region between the base of the helium-burning zone and the hydrogen-helium discontinuity is unstable to convection. After the pulse subsides and helium begins to burn quiescently, the abundance of ^{12}C left behind just below the hydrogen-helium discontinuity is about 20 percent by mass. Because burning in the convective zone is far from complete, the amount of ^{16}O left behind is much less than the amount of ^{12}C. In the course of the quiescent helium-burning phase, which lasts for roughly ten percent of the interpulse lifetime, the total amount of helium converted into carbon and oxygen equals the amount of helium-produced during the quiescent hydrogen-burning phase.

Of particular interest is the neutron-capture nucleosynthesis that occurs in the convective helium-burning shell during pulse peak. The nature of this nucleosynthesis is a function of, among other things, the source of neutrons. If the mass of the CO core is larger than a critical value M_{CO}^{crit}, temperatures in the convective zone become large enough for a long enough time that ^{14}N is converted completely into ^{22}Ne and a substantial fraction of this ^{22}Ne is converted into ^{25}Mg and a neutron. Most of the neutrons released are captured by ^{22}Ne, ^{25}Mg and the neutron capture progeny of these isotopes, but the number of neutrons left to be captured by ^{56}Fe and its progeny is precisely what is needed to produce heavy s-process isotopes in the solar-system distribution (Iben 1975b, Truran and Iben 1977). An essential aspect of the environment that produces this result is the overlap in mass between successive convective shells; this overlap ensures that, in any given shell, the fraction of matter which has experienced N neutron exposures is an exponentially declining function of N. The universal parameter Λ characterising the final distribution of s-process isotopes is essentially the average neutron capture cross section of the light elements from ^{22}Ne to, say, ^{27}Al.

Analysis of early studies (Iben and Truran 1978) suggested that $M_{CO}^{crit} \sim 0.95\ M_{\odot}$, but more recent calculations (Becker 1983) give hope that M_{CO}^{crit} may be as small as (0.75-0.80) $M_{\odot}$. It is worthwhile remarking that a full understanding of the theoretical properties of thermally pulsing AGB models will require substantial amounts of computer time on what are, even today, considered to be "supercomputers."

For smaller CO core masses, temperatures in the convective shell do not become large enough for more than a percent or so of ^{22}Ne to be converted into ^{25}Mg and a neutron (Becker 1980, Iben 1982). However, if the metallicity is low enough, ^{13}C is made available as a neutron source as a consequence of a process (Iben and Renzini 1983) to be described in a later section. The neutron densities which are created by the ^{13}C(α, n)^{16}O source are too large to produce neutron-rich isotopes in a distribution resembling the solar system distribution of s-process isotopes. They are also too small to produce, in one pulse, the classical r-process distribution. It remains to be seen if the combination of an intermediate strength neutron flux and an exponential distribution of exposures may perhaps produce the classical r-process distribution.

Dredge-Up Following Thermal Pulses. The rapid expansion and cooling that is initiated in the helium-burning zone extends beyond this zone into the hydrogen-rich layers, with the result that hydrogen-burning is extinguished. There is a sharp but temporary drop in the surface luminosity as a consequence of this extinction. However, even as the total rate of energy generation by helium-burning drops dramatically from its peak values, more and more of the energy produced by helium-burning makes its way outward by radiative diffusion toward the base of the convective envelope and then through the convective envelope, until the major source of surface luminosity becomes helium-burning. Expansion and cooling in the helium-burning layers is eventually reversed and not only does the rate of energy generation by helium-burning increase for a time, but the flux of energy passing through the base of the convective envelope also increases. The net result is that the base of the convective envelope moves inward in mass and, in AGB models of sufficiently large core mass, this base extends into the region where ^{12}C and neutron-rich isotopes have been produced (Iben 1975a, 1976). The freshly made nucleosynthetic products are then convected to the surface. The minimum core mass at which this form of dredge-up occurs depends sensitively on the core mass, the total mass, and the metallicity of the model star as well as on the choice of mixing length/scale height used in the standard algorithm for modeling convective flow (Wood 1981).

For model stars of metallicity appropriate to the SMC, the minimum core mass is on the order of 0.6 $M_{\odot}$ (Iben and Renzini 1982a,b, Iben 1983) and a surface ratio $C/O > 1$ is achieved after the first or second dredge-up episode. At the end of this dredge-up episode, the stellar luminosity corresponds to $M_{BOL} \sim -5$, but the model then settles down into an extended phase of quiescent helium-burning during which the stellar luminosity drops slowly (over about 10 percent of the interpulse lifetime) to about $M_{BOL} \sim -4$ and then rises slowly over the remainder of the interpulse lifetime to reach $M_{BOL} \sim -4.7$ just before the onset of another pulse. Thus, the models suggest that carbon star characteristics will first appear at $M_{BOL} \sim -4$ and that there should be an overlap in M_{BOL} between carbon stars (with $M_{CO} \gtrsim 0.6\ M_{\odot}$) that are decreasing in luminosity following a helium shell flash and M-stars (with $M_{CO} \lesssim 0.6\ M_{\odot}$) that are increasing in luminosity after reaching the luminosity minimum following a helium shell flash. These results are consistent with the distributions of field M-stars and field C-stars in both the SMC and the LMC.

For stars of core mass larger than, say, 0.7 $M_{\odot}$, dredge-up occurs readily, regardless of metallicity, ℓ/H, and total stellar mass (Becker 1983). However, the larger M_{CO} is at the start of the thermally pulsing AGB phase, the larger is the main sequence mass of the progenitor model star. Thus, the larger M_{CO}^{start}, the more ^{12}C-rich material must be dredged-up to produce a surface $C/O > 1$. In models with $M_{CO}^{start} \gtrsim 0.8\ M_{\odot}$ (corresponding to a maximum luminosity prior to a thermal pulse of $M_{BOL} \sim -6$), roughly 10^6 yr of AGB evolution is required to achieve $C/O > 1$ (Iben and Truran 1978, Iben 1981, Renzini and Voli 1981).

Since M_{CO}^{start} decreases with decreasing main sequence mass (and increasing main sequence lifetime) of a progenitor star, the theory suggests that the minimum luminosity at which C-star characteristics can occur in a population of a given age will decrease as the age of the population increases. This, of course, is what is known to be the case among globular clusters in the Magellanic Clouds (Frogel and Blanco 1984, Lloyd-Evans 1983). Not only is the agreement between theory and observation qualitatively satisfactory, it is also consistent quantitatively.

The absence of luminous single carbon stars in the oldest clusters in the Clouds can be understood in terms of the theoretical results that: (1) $M_{CO}^{start} \sim 0.53\ M_{\odot}$ for stars of initial mass less than $\sim 2\ M_{\odot}$; (2) the oldest stars (age $\sim 10^{10}$ yr) have initial main sequences masses $\lesssim 0.8\ M_{\odot}$; (3) dredge-up does not occur until $M_{CO} \gtrsim$

0.6 M_O; and (4) wind mass loss causes the hydrogen-rich envelope of an AGB star of such a low mass to evaporate before its luminosity exceeds the luminosity at the top of the first red giant branch in Galactic globular clusters ($M_{BOL} \gtrsim -3.5$, $M_{CO} \lesssim 0.53$). Such stars have therefore at best just reached the thermally pulsing AGB phase just before ceasing to exist as AGB stars.

The observed fact that the frequency of carbon stars among AGB stars with intermediate age progenitors decreases with increasing Z can be understood as the consequence of two effects. The first effect is the obvious one that, the larger Z (and hence presumably the larger O is to begin with), the more C-rich material must be dredged-up before surface C > O. The second effect is that, the larger Z is, the more extended is the envelope and the smaller is the gas pressure relative to the radiation pressure. Since the radiative gradient at any point in the envelope is proportional to κ (L/M) (1 + P_{gas}/P_{rad}), the smaller P_{gas}/P_{rad}, the less likely is the adiabatic gradient to be smaller than the radiative gradient (Iben 1983a).

Implication of the Paucity of Bright AGB Stars for s-Process Nucleosynthesis. The absence in the Magellanic Clouds of carbon stars brighter than $M_{BOL} \sim -6$, coupled with the paucity of M-stars brighter than this, may have dramatic implications not only for our understanding of the origin of the s-process isotopes in the solar system but also for our understanding of Galactic nucleosynthesis in general. If one adopts the standard choice for the $^{22}Ne(\alpha,n)^{25}Mg$ reaction (Fowler, Caughlan, and Zimmerman 1975), then early studies suggest that ^{22}Ne is not converted substantially into ^{25}Mg and a neutron unless $M_{CO} > M_{CO}^{crit} \gtrsim 0.95$ M_O (Iben 1976, Truran and Iben 1977, Iben and Truran 1978). Do enough stars achieve core masses this large and maintain themselves as thermally pulsing AGB stars long enough to produce s-process isotopes in sufficient quantity to account for the solar-system s-process distribution on the assumption that this distribution is equivalent to the average Galaxy-wide distribution of s-process isotopes?

From Becker and Iben (1979) one has that, when Z ~ 0.01 and Y ~ 0.28, M_{CO}^{start} and initial main sequence mass M_{MS} are related by M_{CO}^{start} 0.85 + 0.053 (M_{MS} - 4) when $4 \lesssim M_{MS} \lesssim 8$. Masses and luminosities are here and hereinafter in solar units unless otherwise specified. For all $M_{MS} \lesssim 2$, $M_{CO}^{start} \sim 0.53$ and $M_{CO}^{start} \sim 0.53 + 0.16$ (M_{MS} - 2) for $2 \lesssim M_{MS} \lesssim 4$.

In the absence of convective dredge-up following thermal pulses, core mass grows according to M_{CO} (M_O yr^{-1}) ~ 10^{-6} (M_{CO} - 0.5) so that, after t_{AGB} years of evolution, $\Delta M_{CO} \sim [\exp(10^{-6}\ t_{AGB}) - 1] (M_{CO}^{start} - 0.5)$. Since the maximum core mass is 1.4 M_O and since, from the early studies, M_{CO} must exceed ~ 0.95 M_O if s-process isotopes are produced in the solar system distribution, one has that the maximum time which a star can spend producing s-process isotopes in this distribution is ~ 7 x 10^5 yr. The abundance of s-process isotopes relative to solar is produced in a convective shell about 200 (Truran and Iben

1977). Assuming that dredge-up brings up a fraction λ of the mass that has been added to the He-N zone between pulses, the total amount of dredged-up material is approximately $\Delta M_{dredge} \sim \lambda \, \Delta M_{CO}$, and λ is on the order of ~ 1/3 when $M_{CO} \lesssim 0.95$.

Let us next assume that the rate of star formation varies with mass according to $\frac{d}{dM}\left(\frac{dN}{dt}\right) = 1.3 \frac{1}{M^{2.3}}$, where we have normalized to a total birthrate of 1 star per year over the interval $M = 1 \rightarrow \infty$. A measure of the contribution of all stars (still assuming no mass loss) to the Galactic nucleosynthesis of solar system s-process isotopes may now be written as

$$\Delta_s \sim 200 \; \lambda \; 1.3 \left\{ \int_1^{1.4} \frac{dM}{M^{2.3}} (M-0.95) + \int_{1.4}^{8} \frac{dM}{M^{2.3}} \, 0.45 \cong 68 \; \lambda \sim 23. \right.$$

If mass were not lost, all stars initially more massive than 1.4 M_0 would become supernovae and thus the SN rate would be on the order of $\nu_{SNI} = 1.3 \int_{1.4}^{8} \frac{dM}{M^{2.3}} = 0.58 \text{ yr}^{-1}$, which is approximately 60 times the observed SNI rate. It is clear that most real stars of initial mass less than 8 M_0 must lose their hydrogen-rich envelopes before their cores reach the Chandrasekhar mass.

Before pursuing quantitatively the consequences of mass loss let us explore just a bit further the demands of Galactic nucleosynthesis. Type I supernovae are thought to be the consequence of binary star evolution, they occur at the rate $\nu_{SNI} \sim 10^{-2}$ yr^{-1} (see, e.g., Iben and Tutukov 1984), and they are thought to produce at least 0.8 M_0 of ^{56}Fe in the explosion (see, e.g., Woosley, Axelrod, and Weaver, 1984, Nomoto 1984). Since the overabundance (relative to solar) of ^{56}Fe in one gram of pure ^{56}Fe is ~ 1000x, a measure of the ^{56}Fe production rate in the Galaxy is

$$\Delta_{Fe} \sim 1000 \times 0.01 \times 0.8 \sim 8.$$

In the absence of mass loss, all intermediate mass stars of initial mass larger than 1.4 M_0 would also become SNeI and contribute 0.8 M_0 of ^{56}Fe each, leading to $\Delta_{Fe} \sim 58 \times 8 = 464$. Thus, we have an essential paradox: the actual ^{56}Fe production rate is a factor of about 60 smaller than it would be if all intermediate mass stars of initial mass larger than 1.4 M_0 were to lose no mass until they become supernovae, and yet, the estimated rate of production of solar system s-process isotopes in the absence of mass loss is only four times larger than the actual rate of ^{56}Fe production by SNeI with binary star progenitors. That is, abolishing single star progenitors of SNeI also abolishes the source of s-process isotopes in the solar system distribution.

We are now in a position to make the paradox even more dramatic. Assuming that mass is lost by real AGB stars at some fraction of the rate given by the semi-empirical Reimers expression (1975), it may be shown that the typical lifetime of an AGB star is on the order of $\sim 10^6$ yr (Fusi-Pecci and Renzini 1976, Iben and Truran 1978, Renzini and Voli 1981). The paucity of real AGB stars brighter than $M_{BOL} \sim -6$ means that the actual lifetime of an AGB star with $M_{BOL} < -6$ must be much less than 10^6 yr and, from a comparison of the number of LPV's and Cepheids in the Magellanic Clouds one may estimate $t_{AGB} \lesssim 10^5$ yr for $M_{BOL} < -6$.

Note that, when $M_{CO} \sim 0.95$, the quiescent hydrogen-burning luminosity is $L \sim 6 \times 10^4$ (0.95-0.5), corresponding to $M_{BOL} \sim -6.3$. Thus, when M_{CO} exceeds ~ 0.95, the core can grow no more than $\Delta M_{CO} < 10^5 \times 10^{-6}$ (0.95-0.5) < 0.05 before the hydrogen-rich envelope evaporates. Being generous and assuming that all stars that develop an initial core mass as large as $M_{CO}^{start} \sim 0.9$ (corresponding to an initial main sequence mass of $\sim 5\ M_{\odot}$) produce s-process isotopes in the solar system distribution we have that

$$\Delta_s' \lesssim 200\ \lambda \times 0.05 \times 1.3 \int_5^8 \frac{dM}{M^{2.3}} \sim 0.2.$$

Since $\Delta_s' \ll \Delta_{Fe}$, we might conclude that, if the cross section for the $^{22}Ne(\alpha,n)^{25}Mg$ reaction is correct and if the early estimates of the maximum temperatures achieved in the convective helium-burning shell are correct, then either there is another process than the one we have envisioned for the Galaxy-wide production of s-process isotopes in the solar system distribution, or the solar system distribution of these isotopes has been produced by an isolated massive AGB star which injected its nucleosynthesis products into the matter out of which the Sun was born. This latter interpretation is consistent with the recent arguments of Olive and Schramm (1982).

An alternative interpretation is that the cross section for the $^{22}Ne(\alpha,n)^{25}Mg$ reaction has been significantly underestimated. Suppose, for example, that the $^{22}Ne(\alpha,n)^{25}Mg$ reaction goes to completion in all AGB stars with progenitor main sequence masses in the range $(1.5-3)M_{\odot}$ and that they produce s-process isotopes in the solar system distribution as core mass increases, on average, by $\sim 0.3\ M_{\odot}$. Then

$$\Delta_s'' \sim 200 \times 1/3 \times 0.3 \times 1.3 \int_{1.5}^{4} \frac{dM}{M^{2.3}} \sim 8.5.$$

The proximity of this number to Δ_{Fe} allows one to argue that the solar system distribution of s-process isotopes is equivalent to the Galactic one. To the best of the author's knowledge there has been no effort on the part of the experimental physicist to address the possibility that the $^{22}Ne(\alpha,n)^{25}Mg$ cross section at 30 keV has been underestimated by one or two orders of magnitude or even by a factor of 2!

Still another possibility is that the maximum temperatures reached at the base of the convective shell have been underestimated by a considerable amount. Certainly, these maximum temperatures have been rising as the total number of thermal pulses has slowly increased over the past few years (see the discussions by Becker 1981 and by Iben and

Renzini 1983). For example, from a selection of the data available before 1976, Iben and Truran (1978) constructed the approximation $T^{max}_{CSB} \sim [3.1 + 2.85 (M_{CO}-0.96)] \times 10^8$ K. It has now become clear, however, that, even for core masses as small as $\sim 0.65\ M_\odot$, T^{max}_{CSB} exceeds 3×10^8 K after only a dozen or so pulses (Iben 1983b, Becker 1983) and that the maximum temperatures achieved after 20 or so pulses at larger core masses have still not reached asymptotic values (Becker 1983, Chieffi and Iben 1983). Considering the fact that, at $T \sim 3 \times 10^8$ K, the $^{22}Ne(\alpha,n)^{25}Mg$ reaction rate is proportional to $\sim T^{23.4}$, it is conceivable that, even with the currently accepted cross section, M^{crit}_{CO} may drop from the value $M^{crit}_{CO} \sim 0.95\ M_\odot$ suggested by data available five years ago. That is, only a ten percent increase in T^{max}_{CSB} will lead to a factor of 10 increase in the neutron producing reaction rate, and this might translate into $M^{crit}_{CO} \sim 0.70\ M_\odot$. It is abundantly clear that progress in answering the crucial questions about nucleosynthesis in AGB model stars has been hampered by the lack of sufficient computer power in the hands of interested scientists.

Implication of the Paucity of Bright AGB Stars for the Nitrogen Abundance in Planetary Nebulae. The observed short lifetime of high luminosity AGB stars also has ramifications for the abundance of nitrogen in planetary nebulae. The first attempt to account for the observed relationship between N and He in planetary nebulae assumed that the effects of third dredge-up episodes could be neglected (Kaler, Iben, and Becker 1979) and found reasonable agreement between observation and theory. In this picture, most of the correlation between N and He is due to the effect of the second dredge-up episode which is experienced by massive intermediate mass stars. However, during the thermally pulsing AGB phase, third dredge-up episodes cause the abundance of nitrogen in the convective envelope to decrease, since there is no ^{14}N in the dredged-up material (Iben and Truran 1978, Becker and Iben 1980), and this decrease will persist unless enough of the simultaneously dredged-up ^{12}C is converted into ^{14}N by burning at the base of the convective envelope during the interpulse phase (Renzini and Voli 1981, Becker and Iben 1980).

The fact that there are no carbon stars brighter than $M_{BOL} \sim -6$ has on occasion been attributed to the effective burning of dredged-up carbon. However, in the absence of such burning, it still requires $\sim 5 \times 10^5$ yr for surface C to exceed surface O (Iben and Truran 1978, Becker and Iben 1980, Renzini and Voli 1981, Iben 1981) and, since the observations reduce the available lifetime to only $\sim 10^5$ yr, the absence of bright carbon stars may not be invoked as a demonstration of burning. It may very well be that the observed correlation between N and He in planetary nebulae is, after all, due almost entirely to the second dredge-up!

Activation of the $^{13}C(\alpha,n)^{16}O$ Neutron Source in Low Mass, Low Z AGB Stars. Although the maximum temperatures in the convective shells of low mass thermally pulsing AGB stars can reach 300×10^6 K when $M_{CO} \gtrsim 0.65\ M_\odot$, the short duration of the high temperature phase prevents more than about a percent of the ^{22}Ne from being burned (Becker 1981, Iben 1982, 1983b), provided, of course, that the currently used α-capture cross section is correct. However, even if this cross section were ten times larger than currently used, burning would be incomplete and the solar-system distribution of s-process isotopes would not result (Truran and Iben 1977).

Another potential source of neutrons is the ^{13}C(α,n)^{16}O reaction (Cameron 1955) and, ever since the early work of Schwarzschild and Härm (1967) and of Sanders (1967), it has been assumed that somehow hydrogen may be injected into the ^{12}C-rich convective helium-burning shell and that the reaction ^{12}C(p,γ)^{13}N($\beta^+\nu$)^{13}C will produce the ^{13}C that can then immediately act as a neutron source. Subsequent calculations have not provided support for this envisioned scenario (see, in particular, the entropy argument of Iben 1976, 1982), but another sequence of events has been found to occur in low-mass, low-metallicity AGB stars (Iben and Renzini 1983). In this sequence, semiconvective mixing during the dredge-up phase brings small amounts of hydrogen into the outer edge of the ^{12}C rich region <u>after</u> shell convection has died down. When hydrogen-burning is rekindled, the major result of burning in the region of initially small hydrogen abundance and large ^{12}C abundance is ^{13}C. Then, <u>after</u> the extended quiescent hydrogen-burning phase, when a new helium shell flash is triggered, the <u>already prepared</u> ^{13}C is engulfed by the convective shell. The effective rate of release of neutrons by the ^{13}C(α,n)^{16}O reaction is governed more by the rate at which ^{13}C enters the convective shell than by the cross section for the α-capture reaction (Iben 1983a). The neutrons are released when the convective shell has attained only half of its maximum size and the temperature at the base of the shell is only about 150 x 10^6 K.

This process appears to work only in AGB models of low total mass, low core mass, and low Z (Iben 1983a,b), but considerably more theoretical exploration is required before its dependences on all parameters have been properly elucidated. As of this writing, it would appear that the ^{13}C neutron source will be highly variable from one AGB model to another and this should be reflected in a diversity in the distribution of neutron-rich isotopes from one real AGB star to the next.

REFERENCES

Becker, S.A. 1981, in <u>Physical Processes in Red Giants</u>, eds. I. Iben, Jr. and A. Renzini (Dordrecht: Reidel), p. 141.

_____ 1983, in progress.

Becker, S.A. and Iben, I. Jr. 1979, Ap. J., <u>232</u>, 831.

_____ 1980, Ap. J., <u>237</u>, 111.

Blanco, V.M., Frogel, J.A., and McCarthy, M.F. 1981, P.A.S.P., <u>93</u>, 532.

Blanco, B.M., McCarthy, M.F., and Blanco, V.M. 1978, Nature, <u>271</u>, 638.

Blanco, V.M. and McCarthy, M.F. 1983, preprint.

Blanco, V.M., McCarthy, M.F., and Blanco, B.M. 1980, Ap. J., *242*, 938.

Cahn, J.H. 1980, Space Sci. Rev., *27*, 457.

Cameron, A.G.W. 1955, Ap. J.,*121*, *144*.

Carbon, D.F., Langer, G.E., Butler, D., Kraft, R.P., Suntzeff, N.B., Kemper, E., Trezger, C.F., and Romanishin, W. 1983, Ap. J., *49*, 207.

Chieffi, A. and Iben, I. Jr. 1983, in progress.

Cohen, J.G. 1982, Ap. J., *258*, 143.

Cohen, J.G., Frogel, J.A., Persson, S.E., and Elias, J.H. 1981, Ap. J., *249*, 481.

Frogel, J. and Blanco, V.M. 1984, in *Observational Tests of Stellar Evolution Theory*, eds. A. Maeder and A. Renzini (Dordrecht: Reidel), p. 175.

Frogel, J. and Cohen, J.G. 1982, Ap. J., *253*, 580.

Frogel, J. A., Cohen, J. G., Persson, S. E., and Elias, J. H. 1981, in *Physical Processes in Red Giants*, eds. I. Iben, Jr. and A. Renzini (Dordrecht: Reidel), p. 159.

Frogel, J. and Whitford, A.E. 1982, Ap. J. Lett., *259*, L7.

Fusi-Pecci, F. and Renzini, A. 1976, A. and Ap., *46*, 447.

Iben, I. Jr. 1975a, Ap. J., *196*, 525.

______ 1975b, Ap. J., *196*, 549.

______ 1976, Ap. J., *208*, 165.

______ 1977, Ap. J., *217*, 788.

______ 1981, Ap. J., *246*, 278.

______ 1982, Ap. J., *260*, 821.

______ 1983a, Ap. J. Lett., *275*, ______.

______ 1983b, in progress.

Iben, I. Jr. and Renzini, A. 1982, Ap. J. Lett., *263*, L188.

______ 1983, Ann. Rev. Ast. and Ap., *21*, 271.

______ 1984, Physics Reports, in press.

Iben, I. Jr. and Truran, J.W. 1978, Ap. J., *230*, 980.

Kaler, J. B., Iben, I. Jr., and Becker, S. A. 1978, Ap. J. Lett., *224*, L63.

Kraft, R. P. 1979, Ann. Rev. A. and Ap., *17*, 309.

Kraft, R. P., Suntzeff, N. B., Langer, G. E., Carbon, D. F., Trefzger, Ch. F., Fried, E., and Stone, R. P. S. 1982, P.A.S.P., *94*, 55.

Iben, I. Jr. and Tutukov, A. V. 1984, Ap. J. Suppl., Feb. 1 issue.

Little-Marenin, I.R. and Little, S.J. 1979, A. J., *84*, 1374.

Lloyd-Evans, T. 1983, M.N.R.A.S., *205*, ______.

McClure, R.D. 1983a, Ap. J., *268*, 264.

______ 1983b, preprint.

McClure, R.D., Fletcher, J.M., and Nemec, J.M. 1980, Ap. J. Lett., *238*, L35.

Nomoto, K. 1984, in *Stellar Nucleosynthesis*, eds. C. Chiossi and A. Renzini (Dordrecht: Reidel).

Olive, K. and Schramm, D.N. 1982, Ap. J., *257*, 276.

Reimers, D. 1975, Mem. Soc. R. Sci. Liege, 6^e Ser., *8*, 369.

Renzini, A. and Voli, M. 1981, A. and Ap., *94*, 175.

Richer, H.B. 1981, Ap. J., *243*, 744.

Sanders, R. H. 1967, Ap. J., *150*, 971.

Schwarzschild, M. and Harm, R. 1967, Ap. J., *150*, 961.

Searle, L., Wilkinson, A., and Bagnulo, W.G. 1980, Ap. J., 239, 803.
Sweigart, A. V. and Mengel, J. G. 1979, Ap. J., 229, 624.
Truran, J.W. and Iben, I. Jr. 1977, Ap. J., 216, 797.
Whitford, A.E. and Rich, M. 1983, Ap. J., in press.
Wood, P.R. 1981, in Physical Processes in Red Giants, eds. I. Iben, Jr. and A. Renzini (Dordrecht: Reidel), p. 135.
Wood, P.R., Bessel, M.S., and Fox, M.W. 1983, Ap. J., 272, 99.
Woosley, S. E., Axelrod, T. S., and Weaver, T. A. 1984, in Stellar Nucleosynthesis, eds. C. Chiosi and A. Renzini (Dordrecht: Reidel), p. 263.

DISCUSSION

Blanco: You referred to the difference in the ratio of carbon and M-type stars found when one compares the LMC with the SMC. This ratio was defined by Blanco, McCarthy and Blanco as the proportion between the number of carbon stars with sufficiently strong near-infrared CN bands to be detected in unwidened 2350 Å/mm spectra and the number of M6 or later stars classified in similar spectra on the basis of TiO-band strengths. The C/M ratio defined in this way was found by us to be about 2 in the LMC and about 15 in the SMC. These ratios may be found to be somewhat different by other observers depending on how they judge the strength of the relevant molecular bands. The important thing, however, is that, regardless of who determines the C/M ratios, an appreciable difference is found between the two clouds, and if the galactic nuclear bulge is included, a much smaller value for C/M, namely about 10^{-3}. If earlier M-type stars than M6 are included in computing the C/M ratios of the clouds, and we have done this down to type M2, C/M becomes smaller in each cloud and the difference between the LMC and the SMC becomes smaller. This is indeed what is expected from AGB theory.

Wing: As you pointed out, recent work on the clusters of the Magellanic Clouds has shown clearly that the brightest carbon stars can be associated with the brightest and coolest members of an intermediate-age population. But the conclusion that very old clusters do not contain carbon stars may need to be qualified. In such clusters, M stars have been found at the tip of the giant branch, but there may be carbon stars at fainter absolute magnitudes and bluer colors. Carbon stars like that-the R stars - do occur in the direction of the galactic center, and towards the galactic poles, and in ω Centauri; they would appear at V = 17 or fainter in the clouds.

Iben: I agree, but these are a different kind of carbon stars, not to be confused with thermally pulsing AGB stars. They may rather result from mass exchange in binary systems.