PRECESSION-NUTATIONS AND TIDAL POTENTIAL

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1. Basic Equations

We have previously established (Melchior and Georis, 1968) the equations connecting the development of precession-nutation with the tesseral part of the tidal potential. These equations are:

$$\begin{aligned} \sin\theta\Delta\psi &= -E_{☾}\sum_i \frac{\omega}{\Delta\omega_i}[A_i + A_{-i}]\sin(\Delta\omega_i t) \\ \Delta\theta &= -E_{☾}\sum_i \frac{\omega}{\Delta\omega_i}[A_i - A_{-i}]\cos(\Delta\omega_i t) \end{aligned} \tag{1}$$

where

ω_i, A_i respectively represent the frequencies and amplitudes of the tidal waves, ω the Earth's rotation velocity,

$$\Delta\omega_i = \omega_i - \omega \quad \text{and} \quad \Delta\omega_i = -\Delta\omega_{-i}$$

$$E_{☾} = \frac{3}{2}\frac{f\mu}{c^3}\frac{C-A}{C}\frac{1}{\omega^2} \sim 0''.0164 \quad \text{(see later)}.$$

These equations mathematically express two interesting theorems:

THEOREM I: The frequency of a nutation may directly be deduced from the frequency of the corresponding tide by simple subtraction of the 'sidereal frequency' (15°.041/h UT).

THEOREM II: Two waves having frequencies symmetric to the sidereal frequency form only one and the same wave of nutation; the sum of their amplitudes gives the semi-major axis and their difference the semi-minor axis of the nutation ellipse.

2. Proposal for a Systematic Codification of Nutation Tables

One can easily verify the non-absolutely rational construction of the IAU nutation tables. A few arguments are negative while the greatest part of them are positive; the corresponding amplitudes have been changed in sign in the $\Delta\psi$ development where the sine of these arguments appears.

Negative argument waves are marked by an X in Table I. In addition to this, terms are listed in a completely arbitrary order, taking no account of their period.

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Doodson's tidal potential development uses Brown's tables with following arguments in the order of decreasing rate of change.

τ	lunar time $= H+180°$ ($H=$ Moon's hour angle)
s	mean longitude of the Moon
h	mean longitude of the Sun
p	mean longitude of the Moon's perigee
$N'=-N$	longitude (with change of sign) of the Moon's node
p_s	longitude of the perihelion.

If t is the mean solar time,

$$\tau + s = t + h = T \text{ sidereal}.$$

Every tidal wave receives an argument of the form

$$a\tau + bs + ch + dp + eN' + fp_s$$

where

$a=1$ for diurnal tesseral waves

$-4<b, c, d, e, f< +4$ (a few waves excepted).

Doodson introduces a very practical and efficient classification of the numberless tidal waves in associating with them a code-number noted as

$$a, b+5, c+5; d+5, e+5, f+5$$

(with X for 10 and E for 11).

For example, a wave whose argument is

$$\tau - 2s + 2h - p \quad \text{becomes} \quad 137.455$$

(Table II lists the 156 principal tesseral wave arguments).

We propose to the Commission the adoption of these variables (rather than the anomalies) and the same *code-number notation*. The argument of a nutation can be found by subtracting the sidereal frequency ($\tau+s$), say 110 000 from the Doodson's higher argument of the couple of associated tides.

The higher tidal argument systematically should be the first of the two associated waves in Table I if astronomers did not use negative arguments for only a few waves. For all of these (X), the second tidal argument is the highest.

This kind of codification allows an automatic classification of the nutation waves in decreasing period order; the IAU Table classification is very confused as can be seen in column 'Argument of Nutation' of Table I.

The new classification authorizes a division into five groups of nutations characterized by the first code number corresponding to five period ranges (cf. Tables VA, VB).

Also, this codification allows a quite natural input of nutation tables into computers. The code number restitutes the angular argument without any difficulty.

Moreover this codification links together in one and the same spirit tidal and nutation theories allowing a better understanding of the fundamental link between these two problems.

3. Comparison Between Woolard's (Nutations) and Doodson's (Tides) Developments

Both of these authors have developed the luni-solar potential in different ways.

Woolard looks for all nutation waves whose amplitudes in $\sin\theta\Delta\psi$ or in $\Delta\theta$ are greater than $0\rlap{.}''0002$.

Doodson seeks all tidal waves whose amplitudes A_i are greater than 0.000 10 G_1 where G_1 is a geodesic coefficient:

$$G_1 = G \sin 2\phi = \tfrac{3}{4}\mu \frac{gr^2a^2}{c^3} \sin 2\phi$$

These two limits do not absolutely correspond to the same terms because, as can be seen from (1), amplitudes (A_i, A_{-i}) of tidal waves must be multiplied by an integration factor $\omega/\Delta\omega_i$ in order to give the nutation amplitudes.

Waves with a period very near to one sidereal day are much enlarged in nutation because $\Delta\omega_i \approx 0$.

On the other hand, non-negligible tidal waves practically disappear in nutation because their frequencies sensibly deviate from the sidereal frequency (large $\Delta\omega_i$).

Table I: This table reproduces, in the usual order, the list of IAU nutation waves. Their arguments are given in terms of Doodson's variables (S, H, P, N, PS, corre-

TABLE I
IAU Nutation development

Argument of Nutation			Tidal Arguments		$\Delta\psi$	$\sin\theta\cdot\Delta\psi$	$\Delta\theta$
N	55.565	X ..	165.545	165.565	−17.2327	−6.8584	9.2100
2N	55.575	X .	165.535	165.575	0.2088	0.0831	−0.0904
2H	57.555	..	167.555	163.555	−1.2729	−0.5066	0.5522
H − PS	56.554	..	166.554	164.556	0.1261	0.0502	*
H + PS	56.556	.	166.556	164.554	0.0214	0.0085	−0.0093
3H − PS	58.554	..	168.554	162.556	−0.0497	−0.0198	0.0216
2H − N	57.565	..	167.565	163.545	0.0124	0.0049	−0.0066
2H − 2PS	57.553	..	167.553	163.557	0.0016	0.0006	*
4H − 2PS	59.553	.	169.553	161.557	−0.0015	−0.0006	0.0007
2P − N	55.765		165.765	165.345	0.0045	0.0018	−0.0024
H + N − PS	56.544		166.544	164.566	−0.0015	−0.0006	0.0008
H − N − PS	56.564	.	166.564	164.546	0.0010	0.0004	0.0005
2H − 2P − N	57.365		167.365	163.745	0.0005	0.0002	0.0003
H − N + PS	56.566		166.566	164.544	−0.0005	−0.0002	0.0003
2PS − N	55.567		165.567	165.543	−0.0004	−0.0002	0.0002
2H − 2P + N	57.345		167.345	163.765	0.0004	0.0002	−0.0002
3H − PS − N	58.564		168.564	162.546	0.0003	0.0001	−0.0002
2P	55.755		165.755	165.355	−0.0003	−0.0001	0.0002
2H − 2P	57.355	..	167.355	163.755	0.0045	0.0018	*
−2H + 2N	57.575	X ..	163.535	167.575	0.0021	0.0009	*

Table I (Continued)

Argument of Nutation				Tidal Arguments		$\Delta\psi$	$\sin\theta\cdot\Delta\psi$	$\Delta\theta$
−2P + 2N	55.775	X		165.335	165.775	0.0010	0.0004	
H − P	56.455			166.455	164.655	−0.0003	−0.0001	
−P + PS	55.654	X		165.456	165.654	−0.0002	−0.0001	
2S	75.555		..	185.555	145.555	−0.2037	−0.0811	0.0884
S − P	65.455		..	175.455	155.655	0.0675	0.0269	*
2S − N	75.565		..	185.565	145.545	−0.0342	−0.0136	0.0183
3S − P	85.455		..	195.455	135.655	−0.0261	−0.0104	0.0113
S + P	65.655		..	175.655	155.455	0.0114	0.0045	−0.0050
2S − 2H	73.555		..	183.555	147.555	0.0060	0.0024	*
−S + 2H − P	63.655	X	..	157.455	173.655	−0.0149	−0.0059	*
S − P + N	65.445		..	175.445	155.665	0.0058	0.0023	−0.0031
−S + P + N	65.465	X	..	155.645	175.465	−0.0057	−0.0023	0.0030
3S − 2H + P	83.655		..	193.655	137.455	−0.0052	−0.0021	0.0022
3S − P − N	85.465		..	195.465	135.645	−0.0044	−0.0018	0.0023
4S − 2H	93.555		..	1X3.555	127.555	−0.0032	−0.0013	0.0014
2H + S − P	67.455		..	177.455	153.655	0.0026	0.0010	−0.0011
4S − 2P	95.355		..	1X5.355	125.755	−0.0026	−0.0010	0.0011
S + P − N	65.665		..	175.665	155.445	0.0019	0.0008	−0.0010
−S + 2H − P − N	63.645	X	..	157.465	173.645	−0.0014	−0.0006	−0.0007
−S + 2H − P + N	63.665	X	..	157.445	173.665	−0.0013	−0.0005	0.0007
3S − 2H − P − N	83.465		.	193.465	137.645	−0.0009	−0.0004	0.0005
2S + H − PS	76.554		.	186.554	144.556	0.0007	0.0003	−0.0003
2S − H + PS	74.556		.	184.556	146.554	−0.0006	−0.0002	0.0003
5S − 2H − P	X3.455		..	1E3.455	117.655	−0.0006	−0.0002	0.0003
2S + 2H − 2P	77.355		.	187.355	143.755	0.0006	0.0002	−0.0002
−2S + 2H − N	73.545	X	..	147.565	183.545	0.0006	0.0002	0.0003
S + 2H − P − N	67.465		.	177.465	153.645	0.0005	0.0002	−0.0003
−2S + 2H + N	73.565	X	..	147.545	183.565	−0.0005	−0.0002	0.0003
4S − 2H − N	93.565		..	1X3.565	127.545	−0.0005	−0.0002	0.0003
4S − 2P − N	95.365		..	1X5.365	125.745	−0.0004	−0.0002	0.0002
2S − 2P	75.355		..	185.355	145.755	0.0028	0.0011	*
2S − 2N	75.575		..	185.575	145.535	0.0025	0.0010	*
−S + 3H − P − PS	62.656	X	..	158.454	172.656	−0.0007	−0.0003	*
−3S + 2H + P	83.455	X	..	137.655	193.455	−0.0006	−0.0002	*
−S + H + P − PS	64.456	X	..	156.654	174.456	−0.0004	−0.0002	
−2S + 3H − PS	72.556	X	..	148.554	182.556	−0.0004	−0.0002	
−S + H	64.555	X	..	156.555	174.555	0.0004	0.0002	*
−S − P + 2N	65.675	X	..	155.435	175.675	0.0004	0.0002	*
3S − P − 2N	85.475		..	195.475	135.635	0.0003	0.0001	*
S + H − P − PS	66.454		..	176.454	154.656	−0.0003	−0.0001	*
3S − H − P + PS	84.456		.	194.456	136.654	−0.0003	−0.0001	
−2S + 2P + N	75.365	X	.	145.745	185.365	−0.0002	−0.0001	
−S + 2H + P − N	63.445	X	.	157.665	173.445	−0.0002	−0.0001	
2S − 2P + N	75.345		.	185.345	145.765	0.0002	0.0001	
3S − 3H + P + PS	82.656		.	192.656	138.454	−0.0002	−0.0001	
4S − 3H + PS	92.556		.	1X2.556	128.554	−0.0002	−0.0001	
S − P + 2N	65.435		.	175.435	155.675	−0.0002	−0.0001	
3S + H − P − PS	86.454		.	196.454	134.656	0.0002	0.0001	
5S − 3P	X5.255		.	1E5.255	115.855	−0.0002	−0.0001	

X	Negative argument (X = 10, E = 11, if appearing in place of digits)
.	Component appearing in Doodson's tides development
*	Symmetrical waves of *K*1/precession/and of equal amplitudes – no nutation in obliquity

TABLE II

Diurnal tides – Doodson's development

P	Precession
N	IAU nutation terms
W	Woolard's nutation terms
*	Symmetrical waves of $K1$/precession/and of equal amplitudes – no nutation in obliquity
M	Lunar waves
S	Solar waves
3	Waves deriving from 3th order potential

		Argument			Frequency ω_i	Amplitude	Remarks
1	M	105.955			−11.76553679	0.00011	
2	M	107.755			−11.83839039	0.00046	
3	M	109.555			−11.91124400	0.00028	
4	M	115.755	3		−12.30526965	−0.00010	
5	M	115.845			−12.30770508	0.00021	
6	M	115.855	N		−12.30991149	0.00108	
7	M	117.555	3		−12.37812325	−0.00010	
8	M	117.645			−12.38055867	0.00053	
9	M	117.655	N		−12.38276508	0.00278	
10	M	118.654			−12.42383176	0.00021	
11	M	119.445			−12.45341229	0.00010	
12	M	119.455			−12.45561970	0.00054	
13	M	124.756			−12.81321951	−0.00013	
14	M	125.645	3		−12.84743793	−0.00023	
15	M	125.655	3		−12.84964434	−0.00058	
16	M	125.745	N		−12.85207977	0.00180	
17	M	125.755	N	2Q1	−12.85428618	0.00955	E/O1
18	M	126.556			−12.88607310	−0.00016	
19	M	126.655			−12.89071297	−0.00011	
20	M	126.754			−12.89535285	0.00015	
21	M	127.455	3		−12.92249795	−0.00011	
22	M	127.545	N		−12.92493337	0.00218	
23	M	127.555	N	SIG1	−12.92713978	0.01153	V/01
24	M	128.544			−12.96600005	0.00014	
25	M	128.554	N		−12.96820646	0.00079	
26	M	129.355	W		−12.99999339	0.00035	
27	M	133.855	W		−13.32580728	−0.00023	
28	M	134.656	N		−13.35759420	−0.00061	
29	M	135.435	W		−13.38496439	−0.00028	
30	M	135.545	3		−13.39181263	−0.00084	
31	M	135.555	3		−13.39401904	−0.00211	
32	M	135.635	N		−13.39424805	−0.00042*	
33	M	135.645	N		−13.39645446	0.01360	
34	M	135.655	N	Q1	−13.39866087	0.07216	E/O1
35	M	135.755	3		−13.40330271	−0.00013	
36	M	135.855	W		−13.40794455	−0.00019*	
37	M	136.456			−13.43044780	−0.00013	
38	M	136.555	W		−13.43508767	−0.00039	
39	M	136.644			−13.43752113	0.00011	
40	M	136.654	N		−13.43972754	0.00068	
41	M	137.445			−13.46930807	0.00258	

Table II (Continued)

		Argument			Frequency ω_i	Amplitude	Remarks
42	M	137.455	N	RO1	−13.47151448	0.01371	EV/01
43	M	137.555	3		−13.47615631	−0.00018	
44	M	137.655	N		−13.48079814	−0.00078*	
45	M	137.665	W		−13.48300455	0.00024	
46	M	138.444			−13.51037475	0.00011	
47	M	138.454	N		−13.51258116	0.00064	
48	M	139.455	W		−13.55365176	−0.00014	
49	M	143.535	W		−13.85648548	−0.00017	
50	M	143.745	W		−13.86797556	−0.00020	
51	M	143.755	N		−13.87018197	−0.00113	
52	M	144.546	W		−13.89976249	−0.00015	
53	M	144.556	N		−13.90196890	−0.00130	
54	M	145.455	3		−13.93839374	0.00012	
55	M	145.535	N		−13.93862275	−0.00218*	
56	M	145.545	N		−13.94082916	0.07105	
57	M	145.555	N	O1	−13.94303557	0.37689	
58	M	145.645	3		−13.94547099	0.00016	
59	M	145.655	3		−13.94767740	−0.00108	
60	M	145.665	3		−13.94988381	0.00014	
61	M	145.755	N		−13.95231924	−0.00243*	
62	M	145.765	N		−13.95452565	−0.00040	
63	M	146.544	W		−13.98189583	0.00012	
64	M	146.554	N		−13.98410224	0.00115	
68	M	147.555	N	TO1	−14.02517284	−0.00491*	V/K1M
65	M	147.355	W		−14.01588917	−0.00021	
66	M	147.455	3		−14.02053101	−0.00021	
67	M	147.545	N		−14.02296643	0.00014	
69	M	147.565	N		−14.02737925	0.00107	
70	M	148.554	N		−14.06623952	−0.00033*	
71	M	152.656	W		−14.37348998	−0.00014	
72	M	153.645	N		−14.41235025	−0.00063	
73	M	153.655	N		−14.41455666	−0.00278	
74	M	154.656	N		−14.45562726	0.00015*	
75	M	155.435	N		−14.48299745	0.00017*	
76	M	155.445	N		−14.48520386	−0.00197	
77	M	155.455	N		−14.48741027	−0.01065	
78	M	155.545	3		−14.48984569	0.00098	
79	M	155.555	3		−14.49205210	−0.00661	
80	M	155.565	3		−14.49425851	0.00086	
81	M	155.645	N		−14.49448752	0.00085	
82	M	155.655	N	M1	−14.49669393	−0.02964*	E/K1M
83	M	155.665	N		−14.49890034	−0.00594	
84	M	155.675	N		−14.50110675	0.00017	
85	M	156.555	N		−14.53312073	0.00016*	
86	M	156.654	N		−14.53776060	−0.00018*	
87	M	157.445	N		−14.56734113	0.00016	
88	M	157.455	N	KI1	−14.56954754	−0.00566*	EV/K1M
89	M	157.465	N		−14.57175395	−0.00124	
90	M	158.454	N		−14.61061422	−0.00024*	
91	S	161.557	N		−14.87679800	0.00042	
92	S	162.556	N	PI1	−14.91786469	0.01029	E/P1

Table II (Continued)

Argument					Frequency ω_i	Amplitude	Remarks
Resonance frequency J.V.					−14.9387894		R model
93	M	163.535	N		−14.95451854	0.00014*	
94	M	163.545	N		−14.95672495	−0.00199	
95	SM	163.555	N	P1	−14.95893136	0.17584	
96	S	163.557	N		−14.95893528	−0.00011*	
97	M	163.755	N		−14.96821503	−0.00026*	
98	S	164.554	N		−14.99999804	−0.00147	
99	S	164.556	N	S1	−15.00000196	−0.00423*	E/K1S
100	M	165.455	3		−15.03642680	−0.00036	
101	M	165.545	N		−15.03886222	0.01050	
102	MS	165.555	P	K1	−15.04106863	−0.53050	Sidereal day
103	M	165.565	N		−15.04327504	−0.07182	
104	M	165.575	N		−15.04548145	0.00154	
105	M	165.655	3		−15.04571046	−0.00013	
Resonance frequency mol.					−15.0732651		Model 1
Resonance frequency mol.					−15.0736125		Model 2
Resonance frequency			J.V.		−15.0747606		CP model
106	S	166.554	N	PSI1	−15.08213530	−0.00423*	E/K1S
Resonance frequency			J.V.		−15.1016841		R model
107	M	167.355	N		−15.11392223	−0.00026*	
108	S	167.553	N		−15.12320198	−0.00011*	
109	S	167.555	N	FI1	−15.12320590	−0.00756	
110	M	167.565	N		−15.12541231	0.00029	
111	M	167.575	N		−15.12761872	0.00014*	
112	S	168.554	N		−15.16427258	−0.00044	
113	M	172.656	N		−15.47152304	−0.00024*	
114	M	173.445	N		−15.50109965	−0.00017	
115	M	173.645	N		−15.51038331	0.00018	
116	M	173.655	N	TT1	−15.51258972	−0.00566*	EV/K1M
117	M	173.665	N		−15.51479613	−0.00112	
118	M	173.765	3		−15.51943797	−0.00089	
119	M	174.456	N		−15.54437666	−0.00018*	
120	M	174.555	N		−15.54901653	0.00016*	
121	M	175.445	N		−15.58323692	0.00087	
122	M	175.455	N	J1	−15.58544333	−0.02964*	E/K1M
123	M	175.465	N		−15.58764974	−0.00587	
124	M	175.475	W		−15.58985615	0.00013	
125	M	175.555	3		−15.59008516	−0.00241	
126	M	175.655	N		−15.59472699	0.00046	
127	M	175.665	N		−15.59693340	0.00029	
128	M	175.675	N		−15.59913981	0.00017*	
129	M	176.454	N		−15.62651000	0.00015*	
130	M	177.455	N		−15.66758060	0.00012	
131	M	182.556	N		−16.01589774	−0.00032*	
132	M	183.545	N		−16.05475801	−0.00016	
133	M	183.555	N	SO1	−16.05696442	−0.00492*	V/K1M
134	M	183.565	N		−16.05917083	−0.00096	
135	M	185.355	N		−16.12981802	−0.00240*	
136	M	185.365	N		−16.13202443	−0.00048	
137	M	185.455	3		−16.13445986	−0.00040	
138	M	185.465	3		−16.13666627	−0.00016	

Table II (Continued)

		Argument			Frequency ω_i	Amplitude	Remarks
139	M	185.555	N	OO1	-16.13910169	-0.01623	
140	M	185.565	N		-16.14130810	-0.01039	
141	M	185.575	N		-16.14351451	-0.00218*	
142	M	185.585	W		-16.14572092	-0.00014	
143	M	191.655	W		-16.52848550	-0.00015	
144	M	193.455	N		-16.60133912	-0.00078*	
145	M	193.465	N		-16.60354553	-0.00015	
146	M	193.655	N		-16.61062278	-0.00059	
147	M	193.665			-16.61282919	-0.00038	
148	M	195.255	W		-16.67419271	-0.00019*	
149	M	195.455	N	NU1	-16.68347639	-0.00311	E/OO1
150	M	195.465	N		-16.68568280	-0.00199	
151	M	195.475	N		-16.68788921	-0.00042*	
152	M	1X3.555	N		-17.15499748	-0.00050	
153	M	1X3.565	N		-17.15720389	-0.00032	
154	M	1X5.355	N		-17.22785108	-0.00041	
155	M	1X5.365	N		-17.23005749	-0.00027	
156	M	1E3.455	N		-17.69937218	-0.00012	

(X = 10, E = 11, if appearing in place of digits)

Remarks

E	Elliptic
EV	Evection
V	Variation
R model	Roche's model
CP model	Central particle model

sponding to s, h, p, N, p_s) and in Doodson's code-number. One can easily find the arguments of the two associated tides (written in Doodson's codified notation).

Arguments already represented in the Doodson's tidal table are marked with a point.

Negative argument nutations are indicated by an X.

It must be seen that a few pairs of corresponding tides have exactly the same amplitude: $A_i = -A_{-i}$. If so, from (1), $\Delta\theta$ is theoretically zero; we have noted such pairs by an * to indicate that this zero is not an approximative one.

Table II. This table reproduces Doodson's diurnal tides development. Waves corresponding to a term appearing in the IAU nutation development are noted with an N and a few waves given in Woolard's development by a W.

Table III. This table gives in sidereal and mean solar days, the *period* of each nutation associated with a diurnal tesseral tidal wave. This period can easily be deduced from

$$P = \frac{\omega}{\omega_i - \omega} = \frac{\omega}{\Delta\omega_i}$$

(diurnal waves deduced from the third-order potential are eliminated).

TABLE III

	Argument				Amplitude	Frequency ω_i	Nutation Period	
							Sid. day	Solar day
1	M	105.955			0.00011	−11.76553679	4.591947	4.579409
2	M	107.755			0.00046	−11.83839039	4.696403	4.683580
3	M	109.555			0.00028	−11.91124400	4.805722	4.792600
5	M	115.845			0.00021	−12.30770508	5.502769	5.487744
6	M	115.855	N		0.00108	−12.30991149	5.507214	5.492177
8	M	117.645			0.00053	−12.38055867	5.653453	5.638016
9	M	117.655	N		0.00278	−12.38276508	5.658145	5.642696
10	M	118.654			0.00021	−12.42383176	5.746926	5.731235
11	M	119.445			0.00010	−12.45341229	5.812622	5.796751
12	M	119.455			0.00054	−12.45561970	5.817584	5.801700
13	M	124.756			−0.00013	−12.81321951	6.751385	6.732951
16	M	125.745	N		0.00180	−12.85207977	6.871240	6.852478
17	M	125.755	N	2Q1	0.00955	−12.85428618	6.878173	6.859392
18	M	126.556			−0.00016	−12.88607310	6.979628	6.960571
19	M	126.655			−0.00011	−12.89071297	6.994688	6.975590
20	M	126.754			0.00015	−12.89535285	7.009814	6.990674
22	M	127.545	N		0.00218	−12.92493337	7.107801	7.088393
23	M	127.555	N	SIG1	0.01153	−12.92713978	7.115219	7.095792
24	M	128.544			0.00014	−12.96600005	7.248468	7.228676
25	M	128.554	N		0.00079	−12.96820646	7.256183	7.236371
26	M	129.355	W		0.00035	−12.99999339	7.369188	7.349067
27	M	133.855	W		−0.00023	−13.32580728	8.768966	8.745023
28	M	134.656	N		−0.00061	−13.35759420	8.934539	8.910144
29	M	135.435	W		−0.00028	−13.38496439	9.082199	9.057400
32	M	135.635	N		−0.00042*	−13.39424805	9.133398	9.108460
33	M	135.645	N		0.01360	−13.39645446	9.145651	9.120680
34	M	135.655	N	Q1	0.07216	−13.39866087	9.157938	9.132932
36	M	135.855	W		−0.00019*	−13.40794455	9.209997	9.184850
37	M	136.456			−0.00013	−13.43044780	9.338677	9.313178
38	M	136.555	W		−0.00039	−13.43508767	9.365658	9.340085
39	M	136.644			0.00011	−13.43752113	9.379870	9.354259
40	M	136.654	N		0.00068	−13.43972754	9.392795	9.367148
41	M	137.445			0.00258	−13.46930807	9.569567	9.543438
42	M	137.455	N	RO1	0.01371	−13.47151448	9.583019	9.556854
44	M	137.655	N		−0.00078*	−13.48079814	9.640039	9.613717
45	M	137.665	W		0.00024	−13.48300455	9.653690	9.627331
46	M	138.444			0.00011	−13.51037475	9.826307	9.799477
47	M	138.454	N		0.00064	−13.51258116	9.840491	9.813623
48	M	139.455	W		−0.00014	−13.55365176	10.112207	10.084597
49	M	143.535	W		−0.00017	−13.85648548	12.697351	12.662682
50	M	143.745	W		−0.00020	−13.86797556	12.821718	12.786709
51	M	143.755	N		−0.00113	−13.87018179	12.845897	12.810804
52	M	144.546	W		−0.00015	−13.89976249	13.178820	13.142836
53	M	144.556	N		−0.00130	−13.90196890	13.204347	13.168293
55	M	145.535	N		−0.00218*	−13.93862275	13.643362	13.606110
56	M	145.545	N		0.07105	−13.94082916	13.670722	13.633395
57	M	145.555	N	O1	0.37689	−13.94303557	13.698192	13.660790
61	M	145.755	N		−0.00243*	−13.95231924	13.814996	13.777275
62	M	145.765	N		−0.00040	−13.95452565	13.843049	13.805252
63	M	146.544	W		0.00012	−13.98189583	14.200769	14.161995

Table III (Continued)

	Argument			Amplitude	Frequency ω_i	Nutation Period	
						Sid. day	Solar day
64	M 146.554	N		0.00115	−13.98410224	14.230413	14.191558
65	M 147.355	W		−0.00491*	−14.01588917	14.671644	14.631584
67	M 147.545	N		−0.00021	−14.02296643	14.773633	14.733294
68	M 147.555	N	TO1	0.00014	−14.02517284	14.805720	14.765293
69	M 147.565	N		0.00107	−14.02737925	14.837946	14.797432
70	M 148.554	N		−0.00033*	−14.06623952	15.429441	15.387312
71	M 152.656	W		−0.00014	−14.37349898	22.530781	22.469262
72	M 153.645	N		−0.00063	−14.41235025	23.923379	23.858058
73	M 153.655	N		−0.00278	−14.41455666	24.007631	23.942080
74	M 154.656	N		0.00015*	−14.45562726	25.691844	25.621694
75	M 155.435	N		0.00017*	−14.48299745	26.951882	26.878291
76	M 155.445	N		−0.00197	−14.48520386	27.058862	26.984980
77	M 155.455	N		−0.01065	−14.48741027	27.166696	27.092519
81	M 155.645	N		0.00085	−14.49448752	27.518456	27.443319
82	M 155.655	N	M1	−0.02964*	−14.49669393	27.629992	27.554550
83	M 155.665	N		−0.00594	−14.49890034	27.742435	27.666686
84	M 155.675	N		0.00017	−14.50110675	27.855797	27.779738
85	M 156.555	N		0.00016*	−14.53312073	29.611439	29.530587
86	M 156.654	N		−0.00018*	−14.53776060	29.884420	29.802822
87	M 157.445	N		0.00016	−14.56734113	31.750465	31.663772
88	M 157.455	N	KI1	−0.00566*	−14.56954754	31.899036	31.811938
89	M 157.465	N		−0.00124	−14.57175395	32.049005	31.961497
90	M 158.454	N		−0.00024*	−14.61061422	34.942303	34.846895
91	S 161.557	N		0.00042	−14.87679800	91.562737	91.312731
92	S 162.556	N	PI1	0.01029	−14.91786469	122.082691	121.749353
93	M 163.535	N		0.00014*	−14.95451854	173.784552	173.310044
94	M 163.545	N		−0.00199	−14.95672495	178.330713	177.843793
95	SM163.555	N	P1	0.17584	−14.95893136	183.121117	182.621116
96	S 163.557	N		−0.00011*	−14.95893528	183.129856	182.629832
97	M 163.755	N		−0.00026*	−14.96821503	206.456079	205.892364
98	S 164.554	N		−0.00147	−14.99999804	366.224800	365.224848
99	S 164.556	N	S1	−0.00423*	−15.00000196	366.259758	365.259710
101	M 165.545	N		0.01050	−15.03886222	6816.987155	6798.373824
102	MS165.555	P	K1	−0.53050	−15.04106863	∞	∞
103	M 165.565	N		−0.07182	−15.04327504	−6816.987155	−6798.373824
104	M 165.575	N		0.00154	−15.04548145	−3408.493577	−3399.186912
106	S 166.554	N	PSI1	−0.00423*	−15.08213530	−366.259758	−365.259710
107	M 167.355	N		−0.00026*	−15.11392223	−206.456079	−205.892364
108	S 167.553	N		−0.00011*	−15.12320198	−183.129856	−182.629832
109	S 167.555	N	FI1	−0.00756	−15.12320590	−183.121117	−182.621116
110	M 167.565	N		0.00029	−15.12541231	−178.330713	−177.843793
111	M 167.575	N		0.00014*	−15.12761872	−173.784552	−173.310044
112	S 168.554	N		−0.00044	−15.16427258	−122.082681	−121.749343
113	M 172.656	N		−0.00024*	−15.47152304	−34.942303	−34.846895
114	M 173.445	N		−0.00017	−15.50109965	−32.695770	−32.606496
115	M 173.645	N		0.00018	−15.51038331	−32.049005	−31.961497
116	M 173.655	N	TT1	−0.00566*	−15.51258972	−31.899036	−31.811938
117	M 173.665	N		−0.00111	−15.51479613	−31.750465	−31.663772
119	M 174.456	N		−0.00018*	−15.54437666	−29.884420	−29.802822
120	M 174.555	N		0.00016*	−15.54901653	−29.611439	−29.530587

Table III (Continued)

Argument				Amplitude	Frequency ω_i	Nutation Period	
						Sid. day	Solar day
121	M 175.445	N		0.00087	−15.58323692	−27.742435	−27.666686
122	M 175.455	N	J1	−0.02964*	−15.58544333	−27.629992	−27.554550
123	M 175.465	N		−0.00587	−15.58764974	−27.518456	−27.443319
124	M 175.475	W		0.00013	−15.58985615	−27.407818	−27.332983
126	M 175.655	N		0.00046	−15.59472699	−27.166696	−27.092519
127	M 175.665	N		0.00029	−15.59693340	−27.058862	−26.984980
128	M 175.675	N		0.00017*	−15.59913981	−26.951882	−26.878291
129	M 176.454	N		0.00015*	−15.62651000	−25.691844	−25.621694
130	M 177.455	N		0.00012	−15.66758060	−24.007631	−23.942080
131	M 182.556	N		−0.00032*	−16.01589774	−15.429441	−15.387312
132	M 183.545	N		−0.00016	−16.05475801	−14.837946	−14.797432
133	M 183.555	N	SO1	−0.00492*	−16.05696442	−14.805720	−14.765293
134	M 183.565	N		−0.00096	−16.05917083	−14.773633	−14.733294
135	M 185.355	N		−0.00240*	−16.12981802	−13.814996	−13.777275
136	M 185.365	N		−0.00048	−16.13202443	−13.787055	−13.749411
139	M 185.555	N	OO1	−0.01623	−16.13910169	−13.698192	−13.660790
140	M 185.565	N		−0.01039	−16.14130810	−13.670722	−13.633395
141	M 185.575	N		−0.00218*	−16.14351451	−13.643362	−13.606110
142	M 185.585	W		−0.00014	−16.14572092	−13.616111	−13.578933
143	M 191.655	W		−0.00015	−16.52848550	−10.112207	−10.084597
144	M 193.455	N		−0.00078*	−16.60133912	−9.640039	−9.613717
145	M 193.465	N		−0.00015	−16.60354553	−9.626426	−9.600141
146	M 193.655	N		−0.00059	−16.61062278	−9.583019	−9.556854
147	M 193.665			−0.00038	−16.61282919	−9.569567	−9.543438
148	M 195.255	W		−0.00019*	−16.67419271	−9.209997	−9.184850
149	M 195.455	N	NU1	−0.00311	−16.68347639	−9.157938	−9.132932
150	M 195.465	N		−0.00199	−16.68568280	−9.145651	−9.120680
151	M 195.475	N		−0.00042*	−16.68788921	−9.133398	−9.108460
152	M 1X3.555	N		−0.00050	−17.15499748	−7.115219	−7.095792
153	M 1X3.565	N		−0.00032	−17.15720389	−7.107801	−7.088393
154	M 1X5.355	N		−0.00041	−17.22785108	−6.878173	−6.859392
155	M 1X5.365	N		−0.00027	−17.23005749	−6.871240	−6.852478
156	M 1E3.455	N		−0.00012	−17.69937218	−5.658145	−5.642696

(X = 10, E = 11, if appearing in place of digits)

4. Calculation of Nutation Amplitudes from the Static Tidal Development

Equations (1) permit a very simple calculation of these amplitudes, the coefficient $E_☾$ must be determined from the precession constant $(C-A)/C$.

Here appears the incoherence of our system of fundamental constants. It is well-known that adopting 1/81.30 for the relative Moon's mass and the adopted IAU value for precession, one finds for the nutation constant

$$N = 9''.222 \quad \text{(cf. Table IV)}$$

while IAU admits

$$N = 9''.2100 .$$

If one uses the proposed value 50″.40 for the precession constant, one will find

$$N = 9''.2272, \text{ (cf. Table IV)}$$

the discrepancy being emphasized.

We have calculated two models of nutation tables from the tidal tables in taking respectively

$$E_{☾} = 0''.0164120 \quad \text{giving} \quad N = 9''.2100 \quad \text{(Table VA)}$$
$$E_{☾} = 0''.0164427 \quad \text{giving} \quad N = 9''.2272 \quad \text{(Table VB)}$$

Comparing Tables VA and I, one ascertains a small disagreement in the semi-annual solar nutation 57.555 and a small divergency in the ellipticity of principal nutation 55.565. All the other waves are perfectly identical. We shall explain later these two small discrepancies.

5. Comparison Between Nutation Tables and Observations. Ellipticity of the Principal Nutation

(a) Table VI presents first a set of 'theoretical' coefficients following several authors. One can notice a certain number of internal contradictions due to the fact that these authors have adopted different values of fundamental constants (precession, Moon's mass).

TABLE IV

P	μ^{-1}	N	K
50.37000	81.3000	9.221661	9.221858
50.37500	81.3000	9.222576	9.222774
50.38000	81.3000	9.223492	9.223689
50.38500	81.3000	9.224407	9.224604
50.39000	81.3000	9.225322	9.225520
50.39500	81.3000	9.226238	9.226435
50.40000	81.3000	9.227153	9.227351
50.40500	81.3000	9.228069	9.228266
50.40000	81.2900	9.227508	9.227705
50.40000	81.2950	9.227331	9.227528
50.40000	81.3000	9.227153	9.227351
50.40000	81.3050	9.226976	9.227173
50.40000	81.3100	9.226798	9.226996
50.40000	81.3150	9.226621	9.226819
50.40000	81.3200	9.226444	9.226641
50.40000	81.3250	9.226266	9.226464
50.40000	81.3300	9.226089	9.226287
50.40000	81.3350	9.225912	9.226109
50.40000	81.3400	9.225734	9.225932

N	Newcomb's formula
K	Kulikov's formula

TABLE VA

$E_{☾} = 0.0164120$

			Arg.	Tides	Period	$\sin\theta\cdot\Delta\psi$	$\Delta\theta$	Nut. arg
9	156	M	117.655	1E3.455	5.658145	−0.000247	0.000269	X3.455
16	155	M	125.745	1X5.365	6.871240	−0.000172	0.000233	95.365
17	154	M	125.755	1X5.355	6.878173	−0.001031	0.001124	95.355
22	153	M	127.545	1X3.565	7.107801	−0.000216	0.000291	93.565
23	152	M	127.555	1X3.555	7.115219	−0.001288	0.001404	93.555
32	151	M	135.635	195.475	9.133398	0.000125	0.000000	85.475
33	150	M	135.645	195.465	9.145651	−0.001742	0.002340	85.465
34	149	M	135.655	195.455	9.157938	−0.010378	0.011313	85.455
36	148	M	135.855	195.255	9.209997	0.000057	0.000000	85.255
41	147	M	137.445	193.665	9.569567	−0.000345	0.000464	83.665
42	146	M	137.455	193.655	9.583019	−0.002063	0.002249	83.655
44	144	M	137.655	193.455	9.640039	0.000246	0.000000	83.455
48	143	M	139.455	191.655	10.112207	0.000048	0.000001	81.655
55	141	M	145.535	185.575	13.643362	0.000976	0.000000	75.575
56	140	M	145.545	185.565	13.670722	−0.013609	0.018272	75.565
57	139	M	145.555	185.555	13.698192	−0.081081	0.088379	75.555
61	135	M	145.755	185.355	13.814996	0.001095	−0.000006	75.355
67	134	M	147.545	183.565	14.773633	0.000198	0.000266	73.565
68	133	M	147.555	183.555	14.805720	0.002388	0.000002	73.555
69	132	M	147.565	183.545	14.837946	−0.000221	0.000299	73.545
70	131	M	148.554	182.556	15.429441	0.000164	−0.000002	72.556
73	130	M	153.655	177.455	24.007631	0.001048	−0.001142	67.455
74	129	M	154.656	176.454	25.691844	−0.000126	0.000000	66.454
75	128	M	155.435	175.675	26.951882	−0.000150	0.000000	65.675
76	127	M	155.445	175.665	27.058862	0.000746	−0.001003	65.665
77	126	M	155.455	175.655	27.166696	0.004543	−0.004953	65.655
81	123	M	155.645	175.465	27.518456	0.002267	0.003034	65.465
82	122	M	155.655	175.455	27.629992	0.026881	0.000000	65.455
83	121	M	155.665	175.445	27.742435	0.002308	−0.003100	65.445
85	120	M	156.555	174.555	29.611439	−0.000155	0.000000	64.555
86	119	M	156.654	174.456	29.884420	0.000176	0.000000	64.456
87	117	M	157.445	173.665	31.750465	0.000500	0.000666	63.665
88	116	M	157.455	173.655	31.899036	0.005926	0.000000	63.655
89	115	M	157.465	173.645	32.049005	0.000557	−0.000746	63.645
90	113	M	158.454	172.656	34.942303	0.000275	0.000000	62.656
92	112	S	162.556	168.554	122.082681	−0.019735	0.021498	58.554
93	111	M	163.535	167.575	173.784552	−0.000798	0.000000	57.575
94	110	M	163.545	167.565	178.330713	0.004975	−0.006673	57.565
95	109	SM	163.555	167.555	183.121117	−0.505745	0.551187	57.555
96	108	S	163.557	167.553	183.129856	0.000661	0.000000	57.553
97	107	M	163.755	167.355	206.456079	0.001761	0.000000	57.355
99	106	S	164.556	166.554	366.259758	0.050853	0.000000	56.554
101	103	M	165.545	165.565	6816.987155	6.860505	9.209993	55.565

(X = 10, E = 11, if appearing in place of digits)

TABLE VB

$E_{\leftmoon} = 0.0164427$

			Arg.	Tides	Period	$\sin\theta\cdot\Delta\psi$	$\Delta\theta$	Nut. arg.
9	156	M	117.655	1E3.455	5.658145	−0.000247	0.000269	X3.455
16	155	M	125.745	1X5.365	6.871240	−0.000172	0.000233	95.365
17	154	M	125.755	1X5.355	6.878173	−0.001033	0.001126	95.355
22	153	M	127.545	1X3.565	7.107801	−0.000217	0.000292	93.565
23	152	M	127.555	1X3.555	7.115219	−0.001290	0.001407	93.555
32	151	M	135.635	195.475	9.133398	0.000126	0.000000	85.475
33	150	M	135.645	195.465	9.145651	−0.001745	0.002344	85.465
34	149	M	135.655	195.455	9.157938	−0.010397	0.011334	85.455
36	148	M	135.855	195.255	9.209997	0.000057	0.000000	85.255
41	147	M	137.445	193.665	9.569567	−0.000346	0.000465	83.665
42	146	M	137.455	193.655	9.583019	−0.002067	0.002253	83.655
44	144	M	137.655	193.455	9.640039	0.000247	0.000000	83.455
48	143	M	139.455	191.655	10.112207	0.000048	0.000001	81.655
55	141	M	145.535	185.575	13.643362	0.000978	0.000000	75.575
56	140	M	145.545	185.565	13.670722	−0.013635	0.018306	75.565
57	139	M	145.555	185.555	13.698192	−0.081233	0.088544	75.555
61	135	M	145.755	185.355	13.814996	0.001097	−0.000006	75.355
67	134	M	147.545	183.565	14.773633	0.000199	0.000267	73.565
68	133	M	147.555	183.555	14.805720	0.002393	0.000002	73.555
69	132	M	147.565	183.545	14.837946	−0.000222	0.000300	73.545
70	131	M	148.554	182.556	15.429441	0.000164	−0.000002	72.556
73	130	M	153.655	177.455	24.007631	0.001050	−0.001144	67.455
74	129	M	154.656	176.454	25.691844	−0.000126	0.000000	66.454
75	128	M	155.435	175.675	26.951882	−0.000150	0.000000	65.675
76	127	M	155.445	175.665	27.058862	0.000747	−0.001005	65.665
77	126	M	155.455	175.655	27.166696	0.004551	−0.004962	65.655
81	123	M	155.645	175.465	27.518456	0.002271	0.003040	65.465
82	122	M	155.655	175.455	27.629992	0.026931	0.000000	65.455
83	121	M	155.665	175.445	27.742435	0.002312	−0.003106	65.445
85	120	M	156.555	174.555	29.611439	−0.000155	0.000000	64.555
86	119	M	156.654	174.456	29.884420	0.000176	0.000000	64.456
87	117	M	157.445	173.665	31.750465	0.000501	0.000668	63.665
88	116	M	157.455	173.655	31.899036	0.005937	0.000000	63.655
89	115	M	157.465	173.645	32.049005	0.000558	−0.000748	63.645
90	113	M	158.454	172.656	34.942303	0.000275	0.000000	62.656
92	112	S	162.556	168.554	122.082681	−0.019772	0.021539	58.554
93	111	M	163.535	167.575	173.784552	−0.000800	0.000000	57.575
94	110	M	163.545	167.565	178.330713	0.004984	−0.006685	57.565
95	109	SM	163.555	167.555	183.121117	−0.506692	0.552218	57.555
96	108	S	163.557	167.553	183.129856	0.000662	0.000000	57.553
97	107	M	163.755	167.355	206.456079	0.001765	0.000000	57.355
99	106	S	164.556	166.554	366.259758	0.050948	0.000000	56.554
101	103	M	165.545	165.565	6816.987155	6.873338	9.227222	55.565

(X = 10, E = 11, if appearing in place of digits)

TABLE VI

Principal Nutations Amplitudes

(1) Theory				Rigid Earth			
		$\Delta\theta$			$\Delta\psi \sin\theta$		$\cos 2\theta$
	$\cos\Omega$	$\cos 2$☾	$\cos 2$○	$\sin\Omega$	$\sin 2$☾	$\sin 2$○	$\cos\theta$
Spencer Jones	9.2272						
Jeffreys	9.2262	0.0945	0.5528	6.8594	0.0875	0.5068	0.74347
Fedorov	9.2200	0.0884	0.5520	6.8690	0.0812	0.5070	0.74501
Molodensky	9.2232	0.0944	0.5558	6.8672	0.0876	0.5104	0.74456
Taradia	9.2274	0.0894	0.5503	6.8720	0.0820	0.5048	0.74474
Tables	9.2100	0.0884	0.5522	6.8584	0.0811	0.5066	0.74467
						1850	0.74459
						1875	0.74466
						1900	0.74474
						1950	0.74488
		Ratio of Doodson's amplitudes					0.74489
(2) Theory				Liquid core models			
Jeffr. Vic. R	9.2187	0.0971	0.5403	6.8491	0.0897	0.4883	0.74296
Jeffr. Vic. CP	9.2015	0.0972	0.5734	6.8260	0.0896	0.5232	0.74184
Molodensky 1	9.1963	0.0969	0.5770	6.8325	0.0899	0.5274	0.74296
Molodensky 2	9.1997	0.0965	0.5745	6.8369	0.0895	0.5255	0.74317
(3)				Observations			
Przybyllok	9.2069						
Spencer Jones	9.2066						
Morgan	9.206	0.0980					
Clemence	9.2070						
Fedorov-Jeffr.	9.1980	0.0949	0.5780	6.8530	0.0918	0.5330	0.74505
Fedorov 1967	9.1974	0.0965	0.578	6.8437	0.0934	0.533	0.74510
Evtouchenko		0.099					
Taradia	9.1970			6.8476			0.74455
Popov			0.578			0.533	
Wako A			0.5673			0.5209	
Wako B			0.5716			0.5252	

Moreover, the ratio of the principal nutation ellipse axes is fairly variable. This ratio whose value from the static theory is

$$\frac{\cos 2\theta}{\cos\theta}$$

corresponds to $\theta(1875)$ in the IAU nutation table and to $\theta(1950)$ in the tidal tables (Doodson says he has adopted the 1900 value but it seems that this small discrepancy is finally due to truncation errors in the calculation). That is the reason why in Table VA one finds 6.″8605 instead of 6.″8584 (IAU) for the ellipse semi-minor axis.

(b) Observations give a systematically weaker N value:

$$N \sim 9''\!.20$$

while other significant disagreements appear for the semi-annual and semi-monthly nutations.

These discrepancies are attributed to dynamical effects of the Earth's liquid core.

6. Dynamical Effects of the Earth's Liquid Core

Tesseral forces of diurnal tides generate the precession-nutation torque which tends to rotate the equator towards the ecliptic.

Once applied to the liquid core, this torque tends to create core motions with respect to the Earth's mantle in which our reference axes are fixed.

The hydrodynamical theory (Poincaré) shows the existence of resonance frequencies. They were calculated for a few models firstly by Jeffreys and Vicente (1957) and by Molodensky (1961) afterwards.

These frequencies are indicated in Table II. They are very near to the solar wave ψ_1 (166.554) corresponding to the *annual nutation* 56.554.

The central line of the tidal spectrum is the 165.555 one (K_1) corresponding to a sidereal day; pairs of symmetric lines with respect to this central one generate nutations.

The displacement of the resonance line with respect to the central one produces a dissymmetric distortion on the A_i amplitudes; then the effect of resonance is different for A_i and A_{-i}.

If so, the nutation ellipse axes $(A_i + A_{-i})$, $(A_i - A_{-i})$ are modified in different proportions and their ratio is no longer $\cos 2\theta / \cos\theta$ as can be seen in the liquid core models list of Table VI. One will never have rigorously

$$A_i = A_{-i}$$

and never nutations purely in longitude. The most important case will be evidently the annual nutation one 56.554 which because of the near proximity of the resonance line ($\psi_1 = 166.554$) will have a non-negligible obliquity component.

7. Experimental Proofs of Dynamical Effects of the Liquid Core by Earth Tides Measurements

Earth tide measurements have been considerably developed since 1959. Using continuous-recording apparatus, it has been possible to obtain observational series of 1000 to 2000 days, the analysis of which was conclusive for the problem considered here.

In tidal phenomena, principal waves (of maximum amplitudes) are not the same as those in nutation, because of the factor $\omega / \Delta\omega_i$ in Equations (1).

One can actually isolate the following waves with certitude:

K_1	165.555	associated with	55.555	precession
P_1	163.555	associated with	53.555	semi-annual nutation
O_1	145.555	associated with	75.555	semi-monthly nutation
Q_1	135.655	associated with	85.455	9-day nutation

and one searches for

ψ_1	166.554	associated with	56.554	annual nutation.

It is known that the used parameters in Earth tides interpretation, as in the Chandlerian polar motion, are Love's numbers h, k and l (Melchior, 1966).

Performed measurements are interpreted in amplitude ratio form (observed to theoretical amplitude); they give linear combinations of Love's numbers:

one for the horizontal component	$\gamma = 1 + k - h$
one for the vertical component	$\delta = 1 + h - \frac{3}{2}k$
one for the fundamental astronomy	$\Lambda = 1 + k - l$

Table VII presents results of the liquid core resonance effect for all principal waves. Calculations were made for two Earth models following Molodensky's theory:

resonance factor	β
Love's number	h and k
combinations	γ and δ

Waves at the two tide spectrum extremities (Q_1, O_1, OO_1, ν_1) are little affected by resonance effect and closely correspond to the purely static theory. Figure 1 illustrates these results.

Experimental results are summarized in Table VIII. They are based on more than 50000 days of tide recordings in the best conditions. Special attention has been drawn to the problem of instrumental calibration.

For the vertical component (North American and Askania), recording gravimeters calibrated on traditional gravimetric bases are used.

For horizontal components, quartz horizontal pendulums Verbaandert-Melchior are provided, which are supplied with an automatic calibration system related by interferometric measurements to the green line of mercury.

The resonance effect is most striking in horizontal components for

$$k \approx \tfrac{1}{2}h$$

and then

$$\gamma \approx 1 - \frac{h}{2} \qquad \delta \approx 1 + \frac{h}{4}.$$

Horizontal pendulum measurements are thus more favorable in the study of dynamical effects of the core.

TABLE VII
Molodensky's Theory

		A	Frequency ω_i	$\Delta\omega_i = \omega_i - \omega$	$\omega/\Delta\omega_i$	β Mod. 1	β Mod. 2
Q1	135.655	0.07216	−13.39866087	1.64240777	9.157938	5.26	5.00
O1	145.555	0.37689	−13.94303557	1.09803307	13.698193	7.08	6.50
M1	155.655	−0.02964	−14.49669393	0.54437471	27.629992	12.45	12.06
PI1	162.556	0.01029	−14.91786469	0.12320395	122.082681	42.23	41.20
P1	163.555	0.17554	−14.95893136	0.08213728	183.121117	56.86	55.50
S1	164.556	−0.00423	−15.00000196	0.04106668	366.259758	87.89	85.73
	165.545	0.01050	−15.03886223	0.00220641	6816.987155	185.30	180.17
K1	165.555	−0.53050	−15.04106864	0.00000000	∞	197.92	192.30
	165.565	−0.07182	−15.04327505	−0.00220641	−6816.987155	212.41	206.20
	165.575	0.00154	−15.04548147	−0.00441283	−3408.485856	229.21	222.94
PS1	166.554	−0.00423	−15.08213530	−0.04106666	−366.259758	717.52	728.04
FI1	167.555	−0.00756	−15.12320590	−0.08213726	−183.121117	−125.17	−124.06
	168.554	−0.00044	−15.16427259	−0.12320395	−122.082681	−61.91	−61.22
J1	175.455	−0.02964	−15.58544333	−0.54437469	−27.629992	−10.87	−10.86
OO1	185.555	−0.01623	−16.13910169	−1.09803305	−13.698193	−4.45	−4.54
NU1	195.455	−0.00311	−16.68347639	−1.64240775	−9.157938	−2.44	−2.57

	Model 1				Model 2			
	h	k	γ	δ	h	k	γ	δ
Q1	0.621	0.307	0.686	1.160	0.615	0.300	0.685	1.165
O1	0.618	0.305	0.687	1.160	0.614	0.300	0.686	1.166
PI1	0.601	0.297	0.696	1.155	0.599	0.292	0.693	1.161
P1	0.594	0.294	0.700	1.153	0.593	0.288	0.695	1.161
S1	0.579	0.286	0.707	1.150	0.580	0.281	0.701	1.158
	0.533	0.263	0.730	1.138	0.540	0.259	0.719	1.151
K1	0.527	0.260	0.733	1.137	0.535	0.256	0.721	1.151
	0.521	0.256	0.735	1.137	0.529	0.252	0.723	1.151
PSI1	0.959	0.478	0.520	1.242	0.928	0.475	0.547	1.215
FI1	0.680	0.337	0.657	1.174	0.670	0.331	0.661	1.173
J1	0.626	0.310	0.684	1.161	0.621	0.304	0.683	1.165
OO1	0.623	0.308	0.685	1.161	0.619	0.303	0.684	1.165
Stat.	0.621	0.307	0.686	1.160				

Argument		Nutation Period		Molodensky		n/n_0 Jeffreys	
		M. Sid. day	M. Sol. day	Mod. 1	Mod. 2	Mod. 1	Mod. 2
Q1	135.655	9.157938	9.132933	1.0137	1.0121		
O1	145.555	13.698193	13.660791	1.0254	1.0214	1.0269	1.0266
M1	155.655	27.629992	27.554550	1.0369	1.0353		
PI1	162.556	122.082692	121.749353	1.0389	1.0372		
P1	163.555	183.121117	182.621117	1.0359	1.0344	1.0350	0.9707
S1	164.556	366.259758	365.259710	1.0286	1.0273		
	165.545	6816.987155	6798.373824	1.0037	1.0032	1.0036	1.0012
K1	165.555	∞	∞				
	165.565	−6816.987155	−6798.373824	0.9962	0.9964	0.9964	0.9989
	165.575	−3408.485856	−3399.179212	0.9918	0.9921		
PSI	166.554	−366.259758	−365.259710	1.2472	1.2448		
FI1	167.555	−183.121117	−182.621117	1.0884	1.0857	1.0895	1.1420
	168.554	−122.082692	−121.749353	1.0678	1.0814		
J1	175.455	−27.629992	−27.554550	1.0687	1.0663		
OO1	185.555	−13.698193	−13.660791	1.0798	1.0772	1.0768	1.0670
NU1	195.455	−9.157938	−9.132933	1.0924	1.0897		

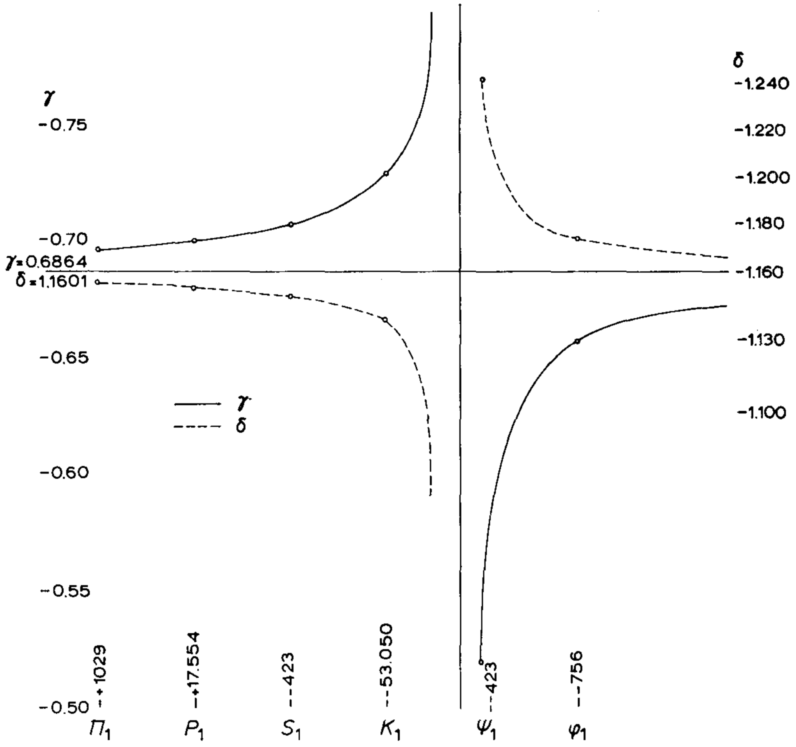

Fig. 1.

TABLE VIII

Earth tides			Experimental results	
Waves	Factor A.M.	$\gamma = 1 + k - h$ W.M.	Factor A.M.	$\delta = 1 + h - \frac{3}{2}k$ W.M.
K1	0.7422	0.7501	1.1406	1.1507
P1	0.7068	0.7167	1.1699	1.1664
O1	0.6785	0.6752	1.1522	1.1676
Q1	0.6519	0.6374	1.1573	1.1684

Stations:

Sclaigneaux	Dourbes	Kanne	Uccle	Luxembourg	Strasbourg
Walferdange	Pribram	Tiefenort	Frankfurt	Bonn	Genova
Graz	Dannemora	Lohja	Trieste	Resina	Stockholm
			Talgar	Kyoto	Austin

14.104 days 13.811 days

A.M. Arithmetic mean W.M. Weighted mean

Results published in Table VIII incontrovertibly carry one of the neatest confirmations of theoretical model calculations.

The wave ψ_1 which is most perturbed by resonance unluckily has a very weak amplitude. Four long observational series, by the best instruments (to our knowledge) – 2 gravimeters and 2 horizontal pendulums – confirm the predicted coefficients for models but the mean quadratic error is so considerable that results lose their meaning.

8. Resonance Effect on Nutations

The third part of Table VII gives the amplificational factor n/n_0 for each circular nutation associated with the principal tidal waves.

One restores elliptic nutations by recombination of two circular nutations with the same period, but of opposite sense.

Such a calculation was carried out in Tables IXA and IXB according to the two Molodensky's models; in each case, two calculations were executed corresponding to the following values:

$$E_{☾}=0''.0164120 \quad \text{and} \quad E_{☾}=0''.0164427$$
$$N_0=9''.2100 \qquad\qquad N_0=9''.2272$$

In the most interesting case,

$$E_{☾}=0''.0164427$$

corresponding to

$$P=50''.400$$
$$\mu^{-1}=81.30$$

one verifies that, in each model, the nutation constant is brought back to the value

$$N=9''.2014$$

which is perfectly consistent with observational results.

In longitude, one nevertheless finds an amplitude of 6″.840 a little weaker than the deduced observational value.

One remarks elsewhere in Table VI that the axes ratio, sensibly changed in liquid core models, does not seem altered in experimental results.

For the short-period nutations, we note the following points:

(a) *semi-annual nutation*

New amplitudes (0″.572, 0″.523) agree very well with experimental results. Alteration with respect to the statical amplitudes (0″.552, 0″.507) is important.

(b) *semi-monthly nutation*

Jeffreys-Vicente and Molodensky have applied the resonance coefficient to already

TABLE IXA

Resonance Effect on Nutations – Molodensky's Model 1

		Tidal arg.			Frequency	Amplitude	Period
34	M	135.655	N	Q1	−13.39866087	0.07315	9.157938
149	M	195.455	N	NU1	−16.68347639	−0.00340	9.157938
57	M	145.555	N	O1	−13.94303557	0.38646	13.698192
139	M	185.555	N	OO1	−16.13910169	−0.01753	13.698192
82	M	155.655	N	M1	−14.49669393	−0.03073	27.629992
122	M	175.455	N	J1	−15.58544333	−0.03168	27.629992
112	S	168.554	N		−15.16427258	−0.00047	122.082681
92	S	162.556	N	PI1	−14.91786469	0.01069	122.082681
95	SM	163.555	N	P1	−14.95893136	0.18215	183.121117
109	S	167.555	N	FI1	−15.12320590	−0.00823	183.121117
99	S	164.556	N	S1	−15.00000196	−0.00435	366.259758
106	S	166.554	N	PSI1	−15.08213530	−0.00528	366.259758
101	M	165.545	N		−15.03886222	0.01054	6816.987155
103	M	165.565	N		−15.04327504	−0.07155	6816.987155

$E_{☾} = 0.0164120$ Rigid Earth 9.2100

			Tidal arg.		Period	$\sin\theta \cdot \Delta\psi$	$\Delta\theta$	Arg.
34	149	M	135.655	195.455	9.157938	−0.010483	0.011505	85.455
57	139	M	145.555	185.555	13.698192	−0.082940	0.090822	75.555
82	122	M	155.655	175.455	27.629992	0.028300	0.000430	65.455
112	92	S	168.554	162.556	122.082681	−0.020477	−0.022360	58.554
95	109	SM	163.555	167.555	183.121117	−0.522696	0.572164	57.555
99	106	S	164.556	166.554	366.259758	0.057886	0.005590	56.554
101	103	M	165.545	165.565	6816.987155	6.825822	9.184261	55.565

$E_{☾} = 0.0164427$ Rigid Earth 9.2272

34	149	M	135.655	195.455	9.157938	−0.010503	0.011526	85.455
57	139	M	145.555	185.555	13.698192	−0.083096	0.090992	75.555
82	122	M	155.655	175.455	27.629992	0.028353	0.000431	65.455
112	92	S	168.554	162.556	122.082681	−0.020515	−0.022402	58.554
95	109	SM	163.555	167.555	183.121117	−0.523674	0.573235	57.555
99	106	S	164.556	166.554	366.259758	0.057994	0.005600	56.554
101	103	M	165.545	165.565	6816.987155	6.838591	9.201441	55.565

modified amplitudes (0.″0945 and 0.″0875 in stead of 0.″0884 and 0.″0811); this explains the higher coefficients obtained by them with respect to Tables IXA, IXB (0.″0970 and 0.″0897 against 0.″0910 and 0.″0831).

(c) *annual nutation*

This nutation associated with ψ_1 and S_1 waves lies nearest to resonance. Its amplitude is thus strongly modified: from (0.″0502, 0.″0000) to (0.″0579, 0.″0056). We must therefore introduce an annual nutation in obliquity.

TABLE IXB

Resonance Effect on Nutations – Molodensky's Model 2

		Tidal arg.			Frequency	Amplitude	Period
34	M	135.655	N	Q1	−13.39866087	0.07303	9.157938
149	M	195.455	N	NU1	−16.68347639	−0.00339	9.157938
57	M	145.555	N	O1	−13.94303557	0.38496	13.698192
139	M	185.555	N	OO1	−16.13910169	−0.01748	13.698192
82	M	155.655	N	M1	−14.49669393	−0.03069	27.629992
122	M	175.455	N	J1	−15.58544333	−0.03161	27.629992
112	S	168.554	N		−15.16427258	−0.00048	122.082681
92	S	162.556	N	PI1	−14.91786469	0.01067	122.082681
95	SM	163.555	N	P1	−14.95893136	0.18189	183.121117
109	S	167.555	N	FI1	−15.12320590	−0.00821	183.121117
99	S	164.556	N	S1	−15.00000196	−0.00434	366.259758
106	S	166.554	N	PSI1	−15.08213530	−0.00527	366.259758
101	M	165.545	N		−15.03886222	0.01053	6816.987155
103	M	165.565	N		−15.04327504	−0.07156	6816.987155

$E_{☾}=0.0164120$ Rigid Earth 9.2100

			Tidal arg.		Period	$\sin\theta\cdot\Delta\psi$	$\Delta\theta$	Arg.
34	149	M	135.655	195.455	9.157938	−0.010466	0.011485	85.455
57	139	M	145.555	185.555	13.698192	−0.082614	0.090474	75.555
82	122	M	155.655	175.455	27.629992	0.028250	0.000417	65.455
112	92	S	168.554	162.556	122.082681	−0.020416	−0.022340	58.554
95	109	SM	163.555	167.555	183.121117	−0.521975	0.571323	57.555
99	106	S	164.556	166.554	366.259758	0.057766	0.005590	56.554
101	103	M	165.545	165.565	6816.987155	6.828060	9.184261	55.565

$E_{☾}=0.0164427$ Rigid Earth 9.2272

34	149	M	135.655	195.455	9.157938	−0.010486	0.011507	85.455
57	139	M	145.555	185.555	13.698192	−0.082769	0.090643	75.555
82	122	M	155.655	175.455	27.629992	0.028303	0.000417	65.455
112	92	S	168.554	162.556	122.082681	−0.020455	−0.022382	58.554
95	109	SM	163.555	167.555	183.121117	−0.522951	0.572392	57.555
99	106	S	164.556	166.554	366.259758	0.057874	0.005600	56.554
101	103	M	165.545	165.565	6816.987155	6.840832	9.201441	55.565

9. Conclusions: Answers to Questions Proposed in the Preliminary Programme for the Heidelberg Colloquium

SPECIFY THE THEORETICAL RELATIONSHIPS BETWEEN THE PRECESSIONAL CONSTANTS AND OTHER ASTRONOMICAL OR GEOPHYSICAL CONSTANTS

The $H\equiv(C-A)/C$ value deduced from the constant of precession and the Moon's mass does not permit us to construct an accurate table of the nutations.

It is necessary to introduce the resonance parameter β, expressing the dynamical effects of the Earth's liquid core.

It is also necessary to adopt an ordered and systematic nutation classification corresponding to the tidal one in order to assure an easy comparison with geophysical phenomena.

ARE THE CURRENT THEORY AND ADOPTED CONSTANTS OF NUTATION OF ADEQUATE ACCURACY?

No. Indeed the effects of the liquid core considerably alter the principal nutations coefficients, especially the annual nutation one.

One term of annual nutation in obliquity must be added to the development.

WHEN IS AN IMPROVED BASIS FOR NUTATION LIKELY TO BE AVAILABLE?

We estimate that the tesseral Earth tides measurements associated with nutations will provide necessary experimental confirmation of theories on the dynamical effects of the Earth's liquid core.

WHAT ARE THE PROS AND CONS OF ADOPTING AN IMPROVED BASIS FOR NUTATION?

Pros: (1) For a very long time, the nutation constant 9.″2100 has been unsatisfactory and the desire to modify it is already an old one. The value 9.″2014 would be satisfactory from theoretical and experimental points of view.

(2) It is recommended to modify the semi-annual and semi-monthly nutation coefficients in the sense of an increase because such a modification still satisfies theory and experience. This will permit us to reduce the residuals in time and latitude observations whose spectral analysis reveals the presence of the corresponding periods.

(3) It is necessary to modify the annual nutation in obliquity. This will reduce also the residuals of annual period.

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