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Testing alternate gravitational theories

E. M. Standish[1]

[1] Caltech/JPL, retired
519 Birchbark Court
Seneca, SC 29672 USA
ems @jpl.nasa.gov

Abstract. The planetary ephemerides are used to examine different suggested forms of the gravitational equations of motion which could possibly cause the observed Pioneer Anomaly. It is shown that most of the forms would be unacceptable, including that generally assumed – a constant acceleration directed toward the Sun. The tests show that three other forms could not exist within 10 au's of the Sun. Only one suggested form would be compatible with the Pioneer Anomaly affecting Saturn or any other more inward planet. Additional planetary observations in the future may possibly eliminate this form also.

Keywords. Pioneer Anomaly, Planetary Ephemerides

1. Introduction

In a previous paper (Standish, 2008), the planetary ephemerides were used to test different forms of the gravitational equations to see if any of them could possibly produce the observed effect known as the "Pioneer Anomaly", while still producing planetary ephemerides consistent with the planetary positional observations. Five different forms were considered, and for each, the addition to the equations of motion was used to generate a new ephemeris, subsequently adjusted to fit the complete set of planetary ephemeris observations currently used in the ephemeris production process. The successes or failures of the fitting processes were then judged by two criteria: 1) the reasonableness of the adjustment parameters (orbital parameters, masses of planets and asteroids, value of the au, etc.) and 2) the goodness of the fit as evidenced by plots and statistics of the residuals of the individual types of planetary observations. That paper showed that the generally-proposed form of the Pioneer Anomaly, a constant inward acceleration, could be easily ruled out; the planetary ephemerides could not be adjusted to properly fit that force. It was further shown that two other forms of the P.A. could exist, but not if the force applied to Saturn; only if the force were farther from the Sun than Saturn's orbit. The last two forms of the P.A. would be detectable if applied to Jupiter, but not if applied only outside of Jupiter.

In the present paper, the observational data set is augmented by a couple of years of available Cassini data and extensions to some Mars ranging and outer planet CCD data sets. The same process is followed as before: integrate an ephemeris with one of the five given forms of the P.A. force, applied to only planets beyond one of the three choices, 4, 6, or 11 a.u.; this gives 15 different examples in all, beside the base solution, the one without any P.A. force. The overall results are given in a Table.

2. Additions to the Data Sets

Available from the IAU Commission 4 (Ephemerides) website are the measurements fitted by planetary ephemeris improvement processes. Most importantly, for the purposes here, are

- Viking, MGS, and Odyssey ranges to Mars,
- MGS and Odyssey VLBI measurements of Mars,
- CCD observations of the outer planets, and
- Cassini orbit determination positions of Saturn.

Since the previous paper, the coverage of the MGS ranging data increased from 1999.1–2005.6 to 1999.1–2006.7, and the coverage of the Odyssey ranges, 2002.1–2005.6 to 2002.1–2008.5. The CCD observations, starting in 1996 have been extended from the end of 2006 to mid-2008. The available Cassini ODP points, not available for the previous paper, extend from late 2004 to the end of 2006. These latter data are far more accurate than any previous Saturn data and therefore provide an ability to test Saturn's orbit with a much greater sensitivity than previously possible.

3. The Forms of the Pioneer Anomaly

The generally assumed form of the Pioneer Anomaly is a constant acceleration upon a body directed toward the Sun, amounting to 8.74×10^{-10} m/sec^2 (Anderson *et al.*, 2002). The acceleration has been deduced from measurements of the Pioneer Spacecraft which have been obtained while the spacecraft were outside the orbit of Saturn. However, four other forms of the Anomaly were suggested to the author by (Laemmerzahl, 2007). Three versions of each of the five equation modifications are tried here: a version which applies to all planets farther than 4 au from the Sun (i.e., Jupiter through Pluto), a version outside of 6 au (Saturn through Pluto)., and a version outside 11 au (only Uranus, Neptune, and Pluto). The third version corresponds to the only region in the solar system in which the P.A. has actually been detected. Attempts to analyze data from spacecraft in closer regions to the Sun are ongoing (Turyshev, 2008).

The five tested modifications to the equations of motion are the following:

(*a*) 10% of the normally assumed form of the P.A., a constant acceleration directed toward the Sun (the full acceleration is 8.74×10^{-10} m/sec^2);

(*b*) $-\|\mathbf{V_{rad}}\|\mathbf{C_{P.A.}}$: an acceleration directed toward the Sun, proportional to the magnitude of the planet's heliocentric radial velocity;

(*c*) $-\mathbf{V_{rad}}\mathbf{C_{P.A.}}$: same as *(b)*, but with the same sign as the radial velocity itself (so that the acceleration alternates sign as the planet itself approaches and recedes from the Sun due to its orbital eccentricity);

(*d*) $-\mathbf{V_{rad}^2}\mathbf{C_{P.A.}}$: an acceleration directed toward the Sun, proportional to the magnitude of the heliocentric radial velocity *squared*; and

(*e*) $-\mathbf{V_{rad}}\|\mathbf{V_{rad}}\|\mathbf{C_{P.A.}}$: same as *(d)*, but with the same sign as the radial velocity itself (as in *(c)* above).

In cases *(b)–(e)*, the velocities are normalized by dividing by 3.4 au/yr, the approximate radial velocities of the Pioneer Spacecraft. Thus, in all cases, the proposed force would produce the acceleration observed upon the Pioneer Spacecraft, whose radial velocities are nearly constant, aligned pretty much with the solar direction. The effects upon the planets from the five different forms, however, vary greatly from one form to another.

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Table 1. Attempts to fit various data sources.
'X' ⇒ completely unsuccessful attempt
'o' ⇒ fair-to-poor attempt
'-' ⇒ normal fit

Form of P.A.	ρ_{min} [au]	parameter quality	Outer Pl. CCD	Mars VLBI	Mars Ranging	Cassini ODP at Saturn
No Added Force		-	-	-	-	-
10% of $\mathbf{C_{P.A.}}$	4	X	X	X	X	X
	6	X	X	X	X	X
	11	-	X	-	-	X
$-\|\mathbf{V_{rad}}\|\mathbf{C_{P.A.}}$	4	X	o	X	X	X
	6	X	o	o	X	X
	11	-	o	-	-	o
$-\mathbf{V_{rad}}\mathbf{C_{P.A.}}$	4	X	X	X	X	X
	6	o	o	o	X	X
	11	-	o	-	-	o
$-\mathbf{V_{rad}^2}\mathbf{C_{P.A.}}$	4	o	-	-	-	X
	6	-	-	-	-	X
	11	-	-	-	-	-
$-\mathbf{V_{rad}}\|\mathbf{V_{rad}}\|\mathbf{C_{P.A.}}$	4	o	-	o	o	o
	6	-	-	-	-	o
	11	-	-	-	-	-

4. Testing the Different Choices

All of the observational data sets were reduced against the base ephemeris, the one without any added force, which had been iterated to provide an optimum fit of the data. Then, each of the 15 different choices was used in 15 new iterations, providing best fits to the data sets. The final sets of observational residuals are analyzed and the adjustment parameters are examined in order to judge how well the data can be represented by each particular form and range of the force.

The results of the judging of each case are given in Table 1, where the form of the added force is shown in the first column and the minimum range over which the force applies is given in the second column. The third column shows the judgement of how reasonable the solution parameters seem to be (changes to the au, the densities of the asteroids, the masses of the planets, etc.), The final four columns show how well the data seems to be fit by that particular adjusted ephemeris. An upper case 'X' signifies a very bad fit, a lower case 'o' shows a marginal set of parameters or residuals, and a hyphen indicates that the parameters or residuals are virtually identical to those of the base solution.

• It is evident from Table 1 that the generally assumed form of the P.A., that of a constant acceleration directed toward the Sun, would completely distort the planetary ephemerides; in fact, as shown, even just 10 percent of such a force would be unacceptable. If applied only past Saturn, the data of Mars would be pretty much unaffected, but the CCD data of the outer planets and especially the Cassini data would not be fit well at all.

• The second and third forms of the P.A. could marginally exist past Saturn; they certainly could not apply to Saturn's orbit, for the Mars ranging and the Cassini data would show unacceptable signatures in their residuals.

• It is the Cassini data which rules out the fourth form of the P.A. affecting Jupiter's or Saturn's orbit. The other data sets are not much affected.

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• The planetary ephemerides can only marginally rule out the fifth form of the P.A. if it is applied to Jupiter's orbit. The data residuals are slightly worse than those of the base solution. The Cassini residuals suffer slightly if this fifth form is applied to Saturn. If applied only past Saturn, the ephemerides are virtually unaffected.

5. Conclusions

Only certain forms of the Pioneer Anomaly can exist in the solar system and still be consistent with the planetary ephemerides. Further, all but one of the forms tested here can be eliminated from having the effect applied to any of the planets out to and including Saturn. These conclusions are similar to those presented previously with the exception that the addition of the Cassini Orbit Determination normal points has provided a more sensitive test for the fourth form considered.

References

Anderson, J. D., Laing, P. A., Lau, E. L., Liu, A. S., Nieto, M. M., & Turyshev, S. G. 2002, Phys. Rev. D, 65, 082004

Laemmerzahl, C. 2007, private communication

Standish, E. M. 2008, Recent Developments in Gravitation and Cosmology, 977, 254

Turyshev, S. G. 2008, APS Meeting Abstracts, 7001