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To the dynamics of the two-body problem with variable masses in the presence of reactive forces

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Abstract. We studied the problem of two spherical celestial bodies in the general case when the masses of the bodies change non-isotropically at different rates in the presence of reactive forces. The problem was investigated by methods of perturbation theory based on aperiodic motion along a quasi-conic section, using the equation of perturbed motion in the form of Newton's equations. The problem is described by the variables $a, e, i, \pi, \omega, \lambda$, which are analogs of the corresponding Keplerian elements and the equations of motion in these variables are obtained. Averaging over the mean longitude, we obtained the evolution equations of the two-body problem with variable masses in the presence of reactive forces. The obtained evolution equations have the exact analytic integral $a^3e^4 = a_0^3e_0^4 = const$.

Keywords. stars: mass loss, variable masses, two-body problem, reactive force, osculating elements, perturbation theory.

1. Introduction

Real celestial bodies are nonstationary, their masses, sizes, shapes and structures change during evolution Eggleton (2006). In this connection, we investigated the problem of two spherical celestial bodies in the general case when the masses of the bodies change nonisotropically at different rates in the presence of reactive forces Minglibayev (2012), Minglibayev et al. (2020).

2. Model description

Consider a gravitating system consisting of two celestial bodies with variable masses $m_1 = m_1(t)$, $m_2 = m_2(t)$, $(m_1 \geq m_2)$. Let us assume that the bodies are spherical with spherical distributions of masses. Let us assume that the bodies' masses decrease due to separating particles and increase due to joining (adhering) particles. In this case, the relative velocity of separating particles from the body differs from the relative velocity of joining (sticking) particles to the body. Let's consider the general case when the masses of bodies change non-isotropically at different rates $\dot{m}_1/m_1 \neq \dot{m}_2/m_2$ in the presence of reactive forces.

3. System of differential equations of secular perturbations in osculating elements

Let us consider the two-body problem with variable masses in the presence of reactive forces as the equations of perturbed aperiodic motion along a quasi-conic section in the osculating elements Minglibayev (2012). Then, after averaging over mean longitude, the equations of secular perturbations have following form

$$\dot{a} = \frac{2a^{3/2}m_0}{m\sqrt{\mu_0}}\sqrt{1-e^2}F_\tau(t), \qquad \dot{e} = -\frac{3}{2}\frac{e\sqrt{a(1-e^2)}}{\sqrt{\mu_0}}\frac{m_0}{m}F_\tau(t), \tag{3.1}$$

$$\frac{di}{dt} = -\frac{3}{2}\frac{e\sqrt{a}\cos(\pi-\Omega)}{\sqrt{\mu_0}\sqrt{1-e^2}}\frac{m_0}{m}F_n(t), \; \dot{\Omega} = -\frac{3}{2}\frac{e\sqrt{a}}{\sqrt{1-e^2}\sqrt{\mu_0}}\frac{m_0}{m}\frac{\sin(\pi-\Omega)}{\sin i}F_n(t), \tag{3.2}$$

$$\dot{\pi} = \frac{m_0}{m}\frac{\sqrt{a(1-e^2)}}{\sqrt{\mu_0}}\left\{F_r(t) - \frac{3a}{2}\frac{d^2}{dt^2}\left(\frac{m_0}{m}\right) - \frac{3e\sin(\pi-\Omega)}{2(1-e^2)}\cdot \mathrm{tg}\frac{i}{2}F_n(t)\right\}, \tag{3.3}$$

$$\dot{\lambda} = n\left(\frac{m}{m_0}\right)^2 - \frac{m_0}{m}\frac{\sqrt{a}}{\sqrt{\mu_0}}\left\{F_r(t)\left(2+e^2\right) - \left(2+3e^2\right)\frac{d^2}{dt^2}\left(\frac{m_0}{m}\right)a\right\} -$$

$$-\frac{3}{2}\frac{m_0}{m}\frac{e\sqrt{a}\sin(\pi-\Omega)}{\sqrt{\mu_0(1-e^2)}}\mathrm{tg}\frac{i}{2}F_n(t) + \frac{\sqrt{a(1-e^2)}}{1+\sqrt{1-e^2}}\frac{m_0}{m}\frac{e^2}{\sqrt{\mu_0}}\left(F_r(t) - \frac{3}{2}\frac{d^2}{dt^2}\left(\frac{m_0}{m}\right)a\right), \tag{3.4}$$

where $m = m(t) = m_1(t) + m_2(t)$, $m_0 = m(t_0) = m_1(t_0) + m_2(t_0) = const$, $\mu_0 = fm_0 = const$, $n = \sqrt{\mu_0}/a^{3/2}$, F_r, F_τ, F_n are the corresponding radial transversal and normal components of the reactive forces in the orbital coordinate system.

4. The first integral of the differential equation system of secular perturbations

Consider together the equation of the analog of the semi-major axis and the analog of eccentricity (3.1). Then, we obtain the following expression

$$3\dot{a}/a = -4\dot{e}/e, \qquad \frac{a^3}{a_0^3} = \frac{e_0^4}{e^4}, \qquad e_0 = e(t_0) = const, \qquad a_0 = a(t_0) = const. \tag{4.1}$$

The resulting integral (4.1) has a simple analytical form and can be used in the analysis of dynamic evolution of binary systems with variable masses. This integral (4.1) can be used to simplify the system of secular differential equations (3.1)–(3.4).

5. Conclusion

The obtained equations of secular perturbation and their first integral can be successfully used in the study of the dynamic evolution of the two-body problem with variable masses in the presence of reactive forces. They are convenient for describing the dynamics of separated two-body systems with variable masses. In the case of close binary systems, tidal forces should also be taken into account. These and other aspects of the two-body problem with variable masses in the presence of reactive forces will be studied in the following papers.

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