Corrigendum

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'On generalized albanese varieties for surfaces'

BY HURŞIT ÖNSIPER

Department of Mathematics, Middle East Technical University, Ankara, Turkey

Theorem 3 of the paper is incorrect. Even when $U = X$, so that $m = 0$ and $G_{um} = \mathrm{Alb}_X$, it is well known that $\prod_1^a(X) = \prod_1(\mathrm{Alb}_X)$ if and only if the Néron–Severi scheme $NS(X)$ has no torsion. Moreover, the elementary observations in Lemma 1 below show that in characteristic zero for the study of $\prod_1^a(U)$ we can replace $\varprojlim G_{um}$ simply by G_{sa}. As this lemma is not true in characteristic p, it is essential to consider $\varprojlim G_{um}$ in this case.

We keep to the notation of the paper and $\prod_{1,t}(.)$ denotes the usual topological fundamental group.

LEMMA 1. (1) *For any finite commutative group* G, *we have* $\mathrm{Ext}(G_{um}, G) = \mathrm{Ext}(G_{sa}, G)$.

(2) $\prod_1(G_{um}) = \prod_1(G_{sa})$.

Proof. (1) follows trivially from the exact sequence of groups $\mathrm{Ext}^n(\cdot, G)$ obtained from $0 \to G_a^N \to G_{um} \to G_{sa} \to 0$ together with the fact that $\mathrm{Ext}(G_a, G) = \mathrm{Ext}^2(G_a, G) = 0$.

As $\prod_1(G_a) = 0$, (2) is a consequence of the exact sequence

$$\prod_1(G_a^N) \to \prod_1(G_{um}) \to \prod_1(G_{sa}) \to 0.$$

Now we let $k = \mathbb{C}$.

LEMMA 2. *As an analytic space*

$$G_{um}^{an} \cong (H^0(X, \Omega_X(m))_{d-0}^*)/H_1(U, Z),$$

and $\alpha_{um}: U^{an} \to G_{um}^{an}$ *is given by*

$$\alpha_{um}(x) = \left(\int_{x_0}^{x} w_1, \ldots, \int_{x_0}^{x} w_r\right)$$

where x_0 *is a fixed point and* $\{w_1, \ldots, w_r\}$ *is a basis for* $H^0(X, \Omega_X(m))_{d-0}$.

Proof. We have

$$\mathrm{rank}\,(\prod_{1,t}(G_{um}^{an})) = 2q + s = \mathrm{rank}\,(H_1(U, Z))$$

(cf. [**1**]). Hence we can follow the proof of [**2**], V, proposition 11 observing that $\Omega_{um}^{inv} = H^0(X, \Omega_X(m))_{d-0}$ (see the remark after corollary 1 in the paper).

COROLLARY 1. (1) $\prod_1(G_{sa}) = (H_1(U, Z)/\text{torsion})\hat{}$.

(2) *The map* $\prod_1^a(U) \to \prod_1(G_{sa})$ *is surjective* (*cf.* [**2**], *VI, proposition* 10).

REFERENCES

[**1**] G. FALTINGS and G. WÜSTHOLZ. Einbettungen kommutativer algebraischer Gruppen und einige ihrer Eigenschaften. *J. Reine Angew. Math.* **354** (1984), 175–205.

[**2**] J. P. SERRE. *Groupes Algébriques et Corps de Classes* (Hermann, 1959).