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Extreme comovements and downside/upside risk spillovers between oil prices and exchange rates

Cathy Ning¹ and Dinghai Xu²

¹Department of Economics, Toronto Metropolitan University, Toronto, Canada

²Department of Economics, University of Waterloo, Waterloo, Canada

Corresponding author: Cathy Ning; Email: qning@torontomu.ca

Abstract

This paper examines the dependence structure and risk spillovers between oil prices and exchange rates in both oil-exporting and oil-importing countries. Using a flexible dependence switching copula model, we analyze both positive and negative dependence and transitions between the dependence regimes. Additionally, we investigate the directional risk spillovers between oil and currency markets in both their downsides and upsides. Based on empirical data from 1999 to 2024 for major oil-exporting and oil-importing countries, we find that oil price–currency dependence is predominantly positive for oil-exporting countries, with infrequent transitions, but mainly negative for oil-importing countries, with frequent transitions between the two dependence regimes. These transitions often occur around crisis or war times. Furthermore, we observe that during downturns in the oil market, tail dependence between oil prices and currencies becomes more pronounced than during upturns. Our results indicate the presence of risk spillovers between oil and currency markets, with the downside spillover effects outweighing the upside ones. Moreover, we find that risk spillover is stronger from oil markets to currency markets than the reverse direction. These insights substantially enrich the existing literature and would offer valuable implications for effective risk management strategies and policymaking.

Keywords: Dependence switching copula; tail dependence; oil prices; exchange rates; downside/upside risk spillover

JEL classifications: Q40; Q47; G15; C58

1. Introduction

The comovements between oil prices and exchange rates have been an important and ongoing research topic. Oil prices have been volatile since 1999. We observed that the crude oil price has increased from below \$20 per barrel in 1999 to its highest level of \$147 per barrel in July 2008, then decreased to below \$20 in the first quarter of 2020, and increased to above \$130 in March 2022. In particular, recent events such as the pandemic, the Russian–Ukraine war, and the Israel– Hamas conflict, have contributed to increased volatility on both oil and exchange rates markets. The COVID-19 pandemic led to a significant drop in global oil demand due to reduced economic activity and travel restrictions, causing oil prices to plummet in early 2020. Exchange rates also fluctuated as central banks implemented monetary policies to stabilize economies. Geopolitical tensions, such as the Russian–Ukraine war disrupts oil supplies and hiked oil prices. Uncertainty surrounding such conflicts also led to fluctuations in exchange rates as investors reacting to geopolitical risks. Especially the Russian Ruble (RUR) plunged at the onset of the war. While the Israel–Hamas conflict may not have a direct impact on oil production and supply, it contributes to market uncertainty, leading to short-term fluctuations in oil prices and exchange rates as investors assess geopolitical risks.

Bourghelle et al. (2021) reinforce the recent spike in oil price volatility attributed to pandemic-induced shocks and uncertainty. It is also well documented in the literature that oil price volatility has significant impact on investment. For instance, Elder and Serletis (2010) find that volatility in oil prices has had a negative and statistically significant effect on investment, durables consumption, and aggregate output. Additionally, fluctuations in domestic currencies are closely tied to changes in US dollar (USD) exchange rates, given that oil prices are denominated in USD.

Theoretically, the correlation between oil prices and the US dollar denominated exchange rates could be either positive or negative. This has been analyzed in three theoretical streams in the literature. First, the law of one price which indicates that a depreciation in the US dollar could increase oil prices in the global market according to Blomberg and Harris (1995). The law asserts that when the US dollar weakens relative to domestic currencies, international buyers outside the US should be willing to pay more dollars for oil, bidding up the dollar price of oil. This indicates a negative dependence between oil prices and the US dollar. Second, by the so-called wealth effect, the dependence could be either positive or negative depending on the statuses of the roles in terms of oil importing or exporting. Based on Krugman (1983) and Golub (1983), an increase in oil prices is posited to lead to a transfer of wealth from oil importing to oil exporting countries due to the inelastic oil demand in oil importing countries.¹ This can also be observed by currency appreciation for oil-exporting countries and currency depreciation for oil-importing countries following an increase in oil prices. Lastly, the oil price-exchange rate dependence is via the trade effect channel which relates the analysis of the price of traded and non-traded goods for a country. Beckman et al. (2017) found that if the non-tradable sector of a country A is more energy intensive than the tradable one, the output price of this sector will increase relative to the output price of country B with the increase of oil prices. This implies that the currency of country A experiences a real appreciation due to higher inflation leading to a positive (negative) relationship between oil prices and currency A (B). Similar results were also found in Chen and Chen (2007) and Buetzer et al. (2016).

Empirically literature has also documented both positive and negative dependence between oil prices and the US dollar denominated exchange rates. Most earlier research supports a positive link between oil prices and the US dollar value. For example, Amano and van Norden (1998a) find higher oil prices lead to an appreciation of the US dollar in the long-run. Chen and Chen (2007) investigate the long-term relationship between real oil prices and exchange rates using a panel cointegration approach for monthly data of G7 countries. Their results suggest that real oil prices have a significant forecasting power for exchange rates and that a rise in oil prices appreciates the US dollar. Beckmann and Czudaj (2013), using a Markov switching vector error correlation model, find that in the long run, real effective appreciation of the US dollar coincides with an increase in real oil prices. More literature that documents the positive relationship between oil prices and the US dollar exchange rates can be found in Dibooglu (1996); Amano and van Norden (1998b); Bénassy-Quéré et al. (2007); Basher et al. (2012) and references therein.

More recent empirical findings support a negative association between oil prices and the US dollar. Yousefi and Wirjanto (2004) report a negative association between oil prices and the US dollar using the five OPEC countries data. Wu et al. (2012) use a t-copula-CGARCH model to measure the dependence structure between oil prices and the US dollar exchange rate. The empirical results show that the dependence structure between crude oil prices and the US dollar becomes negative and decreases continuously after 2003. Reboredo (2012) shows that an increase in oil prices is weakly associated with the US dollar depreciation using a TGARCH model. Reboredo and Castro (2013) find no significant dependence between oil prices and exchange rates during the 2008 pre-crisis period. However, contagion and negative dependence was found after the 2008 crisis. Aloui, Aissa and Nguyen (2013) find significant and symmetric negative extreme dependence between oil prices and the US dollar using a copula-GARCH approach. Jawadi et al. (2016) find a negative relationship between the US dollar and oil price using intraday data. Kima and

Jung (2018) use copula approach and find that the rise in the West Texas Intermediate (WTI) oil price returns is associated with a depreciation of the US dollar. Ji et al. (2019) use copula models and analyze the dynamic dependence between the WTI crude oil prices and the exchange rates of the US dollar and Chinese RMB. Their results show that the dependence between crude oil prices and the RMB exchange rate is weakly positive with lower tail dependence, while the dependence between crude oil prices and the US dollar is significantly negative with lower-upper and upper-lower tail dependence. Bremond et al. (2016) study the relationship between the effective exchange rate of the dollar and the oil price dynamics from 1976 to 2013. They find that the US Dollar effective exchange rate elasticity of crude oil prices is not constant across time and remains negative for 1989–2013: a depreciation of the effective exchange rate of the dollar triggers an increase of crude oil prices.

Previous research also documents time varying and nonlinear dependence between oil prices and exchange rates. Atems et al. (2015) find that exchange rates respond asymmetrically to shocks in the crude oil market depending on magnitudes and signs of the shocks. They also show that only the demand shocks lead to the depreciation of the US dollar. Bal and Rath (2015) find a nonlinear Granger causality between oil prices and exchange rates on both India and China markets. Kayalar et al. (2017) find that exchange rates of most oil exporting countries show higher oil price dependency, whereas, emerging oil importing markets are less vulnerable to price fluctuations. Significant impacts were found for the extreme market conditions such as for the global crisis and the period of sharp decrease in oil prices. De Truchis and Keddad (2016) examine the volatility dependence between the crude oil price and the exchange rates and find increasing volatility dependence around crisis using copula analysis though not much dependence in the long run using cointegration analysis. Mensi et al. (2017) study the risk spillover between oil market returns and the US exchange rate returns using a copula method. Their results show strong evidence of time varying and high average (tail) dependence between oil returns and the foreign exchange markets. There is also evidence of asymmetric systemic risks from oil to currencies and vice versa for some countries. Beckmann et al. (2020) review existing theoretical and empirical research on the relationship between oil prices and exchange rates and document that there are strong links between exchange rates and oil prices with the effects between them being strongly time varying.

Since both theoretical and empirical evidence suggests that the relationship between oil prices and the US exchange rates can exhibit both positive and negative dependence, we need a model to measure both directions of dependence and transitions between them. This is crucial for an accurate risk assessment and effective risk management strategies over time. However, the existing literature lacks such research. In this paper, we fill this gap by employing a flexible dependence switching copula model, which can well capture both directions of the dependence and transitions between them, as well as the extreme dependence at tails, a major source of systemic risk. This also allows us to examine extreme risk spillover between the oil and exchange rate markets, which is important for risk diversification and risk management. Jawadi et al. (2016) find volatility spillover from the US exchange market to the oil market using intraday data and realized volatility. We use dependence switching copula model and conditional Value-at-Risk (CoVaR) with daily data to further investigate the direction and symmetry of the extreme risk spillover between the two markets. Furthermore, Copula approach is particularly useful in modeling nonlinear dependence and tail dependence in financial markets. It is also well applied in measuring tail dependence between macroeconomic variables. For example, Serletis and Xu (2022) employ a Markov-switching copula vector error correction model to investigate the dependence structure and tail dependence between money and output.

Our contributions to the existing literature are fourfold. First, we propose the use of a flexible dependence switching copula model to capture the dynamic relationship between crude oil prices and US dollar exchange rates, accommodating both positive and negative dependence documented in the literature. In particular, the dependence switching copula model can not

only capture both directions of dependence but also identify the switches between them and when these switches occur. Our results indicate that these switches often coincide with times of crises or turbulence in either the financial markets or the oil markets. Second, our empirical results provide supportive evidence that the oil price-exchange rate dependence is time-varying and exhibits switches between positive and negative regimes. Third, the model employed is capable of capturing nonlinear dependence and the dependence at both tails under extreme market conditions. Finally, we uncover extreme risk spillover effects between the two markets through the estimation of Value-at-Risk (VaR) and downside/upside conditional Value-at-Risk (CoVaR).

Using daily oil prices and the US dollar denominated exchange rates (exchange rate per US dollar) for some major oil exporting and importing countries over the 1999–2024 period, this paper investigates the oil price-US dollar exchange rate dependence and tail dependence under four different market statuses. Our empirical analysis yields several interesting findings. First, the US dollar is on average negatively correlated with oil prices. Second, the oil price-currency dependence is positive for oil exporting countries but could be either positive or negative for oil importing countries. Third, we find evidence of dependence switches between the positive and negative regimes in the oil price-currency dependence. Notably, such switches occur more frequently in most oil importing countries compared to oil exporting countries. Fourth, we observe asymmetric (tail) dependence across all countries, indicating that significant tail dependence is present only during downturns in the oil market, not during its upturns. This suggests that extreme dependence exists primarily during oil market crashes. Lastly, we identify the existence of extreme risk spillover effects between the oil and exchange rate markets, with transmission occurring in both directions, but stronger from the oil market to currency market than the other way around. Furthermore, these risk spillovers demonstrate asymmetric characteristics, with stronger effects in the downside than the upside. Our findings are new and important for risk management and risk diversification. These findings also provide empirical support to the wealth effect theory proposed by Krugman (1983) and Golub (1983).

Our findings are also important for policymaking. First, we recommend risk management policies to account for the tail dependence and extreme risk spillovers between oil prices and exchange rates. This could involve establishing hedging strategies or contingency plans to mitigate the impact of extreme events on the economy. Second, given that downside tail dependence is stronger than upside tail dependence, policymakers may prioritize risk management strategies that focus on mitigating downside risks. Third, in light of our findings on the dynamic oil price-exchange rate dependence, policymakers may adopt flexible policy measures that can adapt to the changing nature of this dependence. This includes being prepared for both positive and negative dependence regimes and implementing policies that can mitigate the impact of sudden shifts in dependence patterns, particularly during periods of heightened geopolitical tensions or economic instability. Finally, to mitigate the impact of risk spillovers from oil to exchange rate markets, policymakers could consider diversification policies that reduce reliance on oil exports/imports and promote alternative energies and a more balanced economy.

The remainder of the paper is organized as follows. Section 2 provides the dependence switching copula model specification, its estimation method, and the upside/downside risk spillover measurements. Section 3 presents the data and the empirical results. Section 4 concludes.

2. Joint and marginal models

The dependence between oil prices and exchange rates could be either positive or negative. In addition, the dependence could change from being positive to negative and vice versa over time. To capture this, following Wang et al. (2013), we propose to use the following dependence switching copula model. The model structure is flexible to accommodate the time-varying dependence

across both positive and negative dependence regimes.² Let u_o and u_e be the cumulative distribution functions (CDFs) of oil returns (R_o) and exchange rate returns (R_e), respectively. C is the copula function that describes the dependence structure between oil prices and exchange rates.

$$C(u_{e,t}, u_{o,t}; \theta^{C^1}, \theta^{C^0} | S_t) = \begin{cases} C^1(u_{e,t}, u_{o,t}; \theta^{C^1}), & \text{if } S_t = 1 \\ C^0(u_{e,t}, u_{o,t}; \theta^{C^0}), & \text{if } S_t = 0 \end{cases}$$

where S_t is the unobserved state variable. In this paper, we define two states based on the dependence directions, that is, the positive dependence state if $S_t = 1$ and the negative dependence state if $S_t = 0$. $C^1(u_{e,t}, u_{o,t}; \theta^{C^1})$ and $C^0(u_{e,t}, u_{o,t}; \theta^{C^0})$ are two mixed copulas corresponding to the two different states. Copula $C^1(u_{e,t}, u_{o,t}; \theta^{C^1})$ describes the dependence between the oil price-exchange rate during the positive dependence state ($S_t = 1$), while $C^0(u_{e,t}, u_{o,t}; \theta^{C^0})$ captures the dependence in the negative dependence state ($S_t = 0$). θ^{C^1} and θ^{C^0} represent the parameter sets in the two copulas C^1 and C^0 , respectively. The state variable S_t follows a Markov chain with a transition probability matrix M :

$$M = \begin{bmatrix} P_{11} & 1 - P_{11} \\ 1 - P_{00} & P_{00} \end{bmatrix}$$

where P_{11} is the transition probability from a positive dependence regime (state 1) to a positive dependence state (state 1), and $(1 - P_{11})$ is the transition probability from a positive dependence state (state 1) to a negative dependence state (state 0). Similarly, P_{00} is the transition probability between negative dependence states (state 0), and $(1 - P_{00})$ is the transition probability from a negative dependence state (state 0) to a positive dependence state (state 1). In summary

$$P_{11} = \mathbb{P}(S_t = 1 | S_{t-1} = 1) \\ P_{00} = \mathbb{P}(S_t = 0 | S_{t-1} = 0).$$

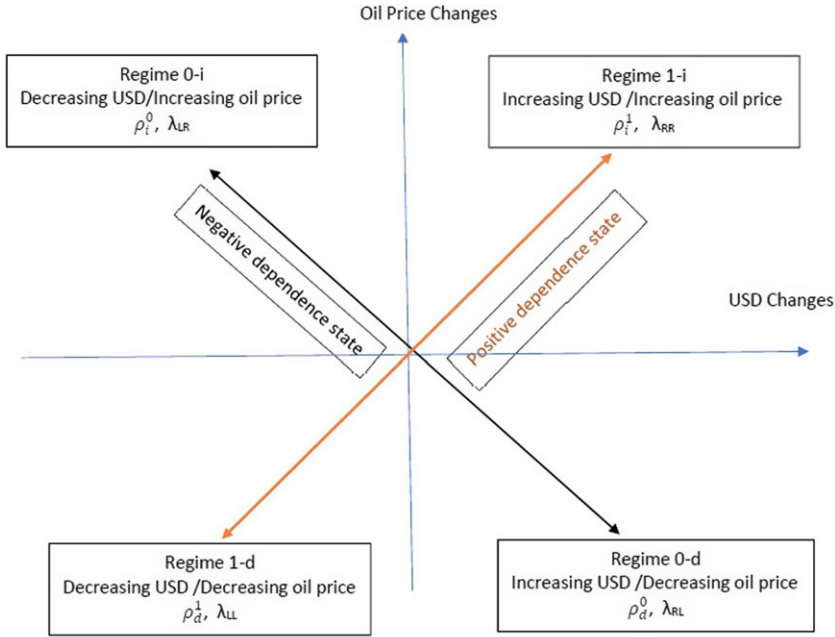
The copula function at each state is a mixture of the Clayton copula (C^C) and the survival Clayton copula (C^{SC}) as follows:

$$C^1(u_{e,t}, u_{o,t}; \theta^{C^1}) = \frac{C^C(u_{e,t}, u_{o,t}; \alpha_d^1) + C^{SC}(u_{e,t}, u_{o,t}; \alpha_i^1)}{2} \\ C^0(u_{e,t}, u_{o,t}; \theta^{C^0}) = \frac{C^C(1 - u_{e,t}, u_{o,t}; \alpha_d^0) + C^{SC}(1 - u_{e,t}, u_{o,t}; \alpha_i^0)}{2}$$

where $\theta^{C^1} = (\alpha_d^1, \alpha_i^1)$ and $\theta^{C^0} = (\alpha_d^0, \alpha_i^0)$ are copula parameters in the corresponding state ($S_t = 1, 0$), and $k = d, i$, representing oil price decreasing or increasing regime, respectively, with $\alpha_k^{S_t} \in (0, \infty)$. The Clayton copula is defined as $C^C(u_{e,t}, u_{o,t}; \alpha_{S_t,k}) = (u_{e,t}^{-\alpha_k^{S_t}} + u_{o,t}^{-\alpha_k^{S_t}} - 1)^{-\frac{1}{\alpha_k^{S_t}}}$ and the survival Clayton copula is defined as $C^{SC}(u_{e,t}, u_{o,t}; \alpha_{S_t,k}) = u_{e,t} + u_{o,t} - 1 + C^C(1 - u_{e,t}, 1 - u_{o,t}; \alpha_k^{S_t})$.

Based on the joint copula model, we can derive a set of dependence measures. The rank correlation Kendall's tau, $\tau_k^{S_t}$, can be derived via the copula parameters ($\alpha_k^{S_t}$) with $\tau_k^{S_t} = \frac{\alpha_k^{S_t}}{(2 + \alpha_k^{S_t})}$.

The linear correlation $\rho_k^{S_t}$ can also be computed using Kendall's tau with $\rho_k^{S_t} = \sin \frac{\pi}{2} \tau_k^{S_t}$. To capture the dependence at extremes, we use the tail dependence coefficients. In the positive dependence state, the left and right tail dependence coefficients can be computed as $\lambda_{LL}^1 = 2^{-\frac{1}{\alpha_d^1} - 1}$ and $\lambda_{RR}^1 = 2^{-\frac{1}{\alpha_i^1} - 1}$, respectively. They measure the probability that both exchange rates and oil



Note: 'i' and 'd' in the graph represent the status of increasing and decreasing oil returns respectively.

Figure 1. The dependence structure in four dependence regimes.

prices are extremely low together (left tail dependence measured by λ_{LL}^1) or extremely high together (right tail dependence measured by λ_{RR}^1). In the negative dependence state, the left and right tail dependence coefficients are $\lambda_{RL}^0 = 2^{-\frac{1}{\alpha_d^0}-1}$ and $\lambda_{LR}^0 = 2^{-\frac{1}{\alpha_i^0}-1}$, respectively. They measure the probability that the exchange rate changes are extremely high/low while oil returns are extremely low/high.

Furthermore, we graphically summarize the model measured dependence structure in different regimes in Figure 1. In the positive dependence state, we have two regimes. Regime 1-d represents both markets in the downturn, while regime 1-i is the case that both markets are in the upturn. In the negative dependence state, regime 0-d represents the status that the oil returns are in downturn while the US dollar exchange rates are in the upturn; and regime 0-i represents the case of rising oil returns together with the decreasing US dollar exchange rates. The dependence could transit between any two regimes of the four: between regime 1-i and 1-d with the probability of P_{11} , between regime 0-i and 0-d with the probability of P_{00} , between regime 1-i or 1-d to either regime 0-i or 0-d with the probability $1 - P_{11}$, and between regime 0-i or 0-d to either regime 1-i or 1-d with the probability $1 - P_{00}$.

To remove the heteroskedasticity and fat tails in oil and exchange rate returns data, we apply the standard filtering mechanism on the two series with GARCH(1,1)-t model as follows:

$$\begin{aligned}
 R_{i,t} &= \mu_i + \epsilon_{i,t}; \quad i = e, o \\
 h_{i,t} &= \beta_{i,0} + \beta_{i,1}\epsilon_{i,t-1}^2 + \beta_{i,2}h_{i,t-1} \\
 r_{i,t} &= \frac{\epsilon_{i,t}}{\sqrt{h_{i,t}}}; \quad r_{i,t}|I_{t-1} \sim t_{\nu_i}
 \end{aligned}$$

where I_{t-1} is the information set at time $t - 1$, and $\epsilon_{i,t}$, $h_{i,t}$, $r_{i,t}$ are the error term, the conditional variance of the error term, and the standardized residual of the error term, respectively.

To estimate the copula model, we use the classical Canonical Maximum likelihood method. In particular, we follow the following two steps in our estimation algorithm. In the first step, we estimate the marginal CDFs of the standardized residuals with empirical CDF (ECDF) of each series. The ECDF is a nonparametric estimate of the CDF based on observed data. As the sample size increases, ECDF converges to the true CDF, which is uniformly distributed. Therefore, with a large sample size as in our data, the ECDF approximates the true CDF and exhibits a uniform distribution. In contrast, parametric marginal models and marginal distributions may suffer from mis-specification, leading to discrepancies between the estimated CDF and the true CDF. Consequently, the distribution of the estimated CDF may deviate from uniformity. Thus ECDF mitigates the problem of mis-specification of marginal models and marginal distributions. It also guarantees the uniform distribution of the ECDF. In the second step, we plug the ECDF from the first step to the dependence switching copula model to estimate the joint model and the dependence parameters.³

3. Extreme risk spillover measurement

To measure the extreme risk, we construct the downside and upside VaR measures at a given confidence level, $(1 - \alpha)$, following Reboredo et al. (2016). For the downside risk, the marginal VaR at time t is defined as $Pr(r_t \leq VaR_{\alpha,t}) = \alpha$. Similarly, for the upside risk, the marginal VaR at time t is defined as $Pr(R_t \geq VaR_{1-\alpha,t}) = \alpha$. Furthermore, we focus on examining the risk spillover effect between the oil prices and exchange rates via the CoVaR measures. In this paper, we follow Adrian and Brunnermeier (2011) and Girardi and Ergun (2013) to construct the CoVaR measures under our proposed structure. In particular, the CoVaR for the exchange rate market is the VaR for exchange rate market conditional on the extreme downside movement (measured by the downside VaR, VaR-Down) of the oil market. Therefore, at any given confidence level, $(1 - \gamma)$, the downside CoVaR (CoVaR-Down) for the exchange rate market can be computed as,

$$Pr(R_{e,t} \leq CoVar_{e,\gamma,t} | R_{o,t} \leq VaR_{\alpha,t}) = \gamma$$

Similarly, the upside CoVaR (CoVaR-Up) measure given extreme upward movement of the oil market (measured by the upside VaR, VaR-Up) can be specified as,

$$Pr(R_{e,t} \geq CoVar_{e,\gamma,t} | R_{o,t} \geq VaR_{\alpha,t}) = \gamma$$

The same way, we can also obtain the downside/upside CoVaR for the oil returns by computing the VaR of the oil returns conditional on the extreme movement of the exchange rate markets.

Following Reboredo and Ugolini (2015) and Reboredo et al. (2016), the commonly used back-testing algorithm can be carried out for the constructions of the VaR and CoVaR measures.

To test for the spillover effects, following Abadie (2002); Bernal et al. (2014) and Reboredo et al. (2016), we carry out the Kolmogorov–Smirnov (KS) test to compare the cumulative distributions for CoVaR and VaR on both tails. The hypotheses for downside spillover effects are

$$H_0 : CoVaR-Down = VaR-Down$$

$$H_1 : CoVaR-Down < VaR-Down.$$

while the hypotheses for upside spillover effects are

$$H_0 : CoVaR-Up = VaR-Up$$

$$H_1 : CoVaR-Up > VaR-Up.$$

Further, we conduct a test whether the downside and upside risk spillover effects are symmetric or not. In other words, we investigate whether the downside risk spillover effects are greater than the upside spillover effects via the following hypothesis test:

$$H_0: \frac{\text{CoVaR-Down}}{\text{VaR-Down}} = \frac{\text{CoVaR-Up}}{\text{VaR-Up}}$$

$$H_1: \frac{\text{CoVaR-Down}}{\text{VaR-Down}} > \frac{\text{CoVaR-Up}}{\text{VaR-Up}}$$

where H_0 corresponds to symmetric downside and upside risk spillover effect, and H_1 indicates that the downside risk spillovers are stronger than the upside one.

To investigate if the direction of risk spillovers between the oil market and the exchange rate (EX) market is symmetric, that is, if the risk spillovers are stronger from oil markets to foreign exchange markets, we test the following hypotheses.

$$H_0: \frac{\text{CoVaR (EX|Oil)}}{\text{VaR (EX)}} = \frac{\text{CoVaR (Oil |EX)}}{\text{VaR (Oil)}}$$

$$H_1: \frac{\text{CoVaR (EX|Oil)}}{\text{VaR (EX)}} > \frac{\text{CoVaR (Oil |EX)}}{\text{VaR (Oil)}}$$

where H_0 represents symmetric risk spillovers between oil -currency markets, and H_1 means stronger risk spillovers from oil to currency markets. We examine this type of asymmetry for both downside and upside risk spillovers.

4. Results

In this paper, our exchange rate data consists of the daily US dollar exchange rates of some major oil-exporting countries including RUR, Canadian Dollar (CAD), Norwegian Krone (NOK), and oil-importing regions and countries including Euro (EUR), Japanese Yen (JPY), South Korean Won (KOW), Indian Rupee (INR), as well as the trade-weighted US dollars (TWUSD). For the daily oil data, we use the closing spot price of the WTI crude oil, which is a global benchmark for determining the prices of other light crudes. Our data covers the period from January 4, 1999 to February 1, 2024.⁴ The data start from the beginning of 1999 since the embark of Euro. The sample period spans several extreme events and turbulences including the 2001 dot-com bubble, the 2007–2008 global financial crisis, the 2009–2012 euro zone sovereign debt crisis, the COVID-19 pandemic, the Ukraine war, and the ongoing Israel–Hamas war. Figure 2 plots the WTI crude oil prices over time. The graph shows that the oil price is very volatile in our sample period.

Due to the nonstationarity of the oil price and exchange rate series, we transform both to the standard continuously compounded returns for our analysis. The returns are computed as 100 times the log difference of the consecutive prices (exchange rates). The exchange rates are expressed as local currency per US dollar. Thus an increase in the exchange rate corresponds to the appreciation of the USD.

Table 1 gives the summary statistics of the returns. Comparing the standard errors with the means, the oil price and exchange rates are very volatile (except for the Indian Rupee) as expected. The returns typically show high kurtosis, indicating fat tails in the return distribution. As a result, we use a GARCH(1,1)-t distribution model described in Section 2 to filter the returns.

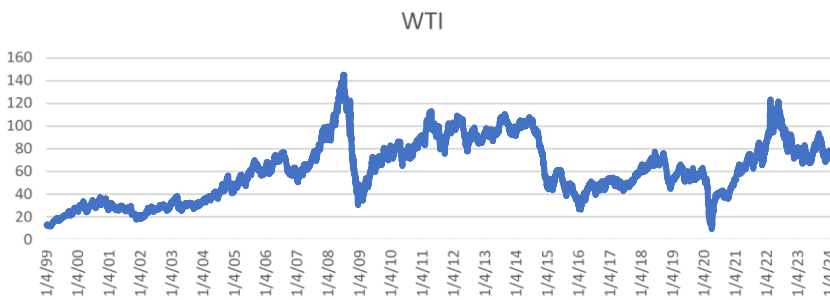
Table 2 provides the linear correlations between the oil-exchange rate returns. The correlations are generally negative with an exception of WTI-Japanese Yen. Thus, on average, there is a negative relationship between the US dollars and the WTI crude oil price. It is also interesting to note that the degrees of correlations are much higher for oil exporting currencies (RUR, CAD, NOK) than oil importing currencies (EUR, JPY, KOW, INR). This observation implies that oil

Table 1. Summary statistics of returns

	TWUSD	CAD	RUR	NOK	EUR	JPY	KOW	INR	WTI
Mean	0.0023	-0.0023	0.0229	0.0048	0.0006	0.0042	0.0020	0.0116	0.0284
S.E.	0.0042	0.0069	0.0135	0.0099	0.0077	0.0081	0.0082	0.0051	0.0369
Median	-0.0031	-0.0077	-0.0017	-0.0053	-0.0131	0.0089	-0.0127	-0.0051	0.1303
Kurtosis	7.1636	5.8420	67.7833	6.6328	4.5925	7.1377	49.0095	11.8213	84.3299
Skew	0.0218	0.1124	2.1550	0.3077	-0.0162	-0.0885	-0.5865	0.3215	-1.9868

Table 2. Correlations between oil-exchange rate returns

	TWUSD	CAD	RUR	NOK	EUR	JPY	KOW	INR
WTI	-0.2312	-0.3017	-0.2126	-0.3006	-0.1150	0.0544	-0.0654	-0.0967



Deleted the negative WTI spot oil price on April 20, 2020.

Figure 2. WTI sport oil price.

exporting countries’ currencies respond more to oil price changes than oil importing countries’ currencies.

Table 3 presents the results of dependence for all oil-exchange rate pairs. Panel A in Table 3 provides the result for 1999 to 2020. First, for the oil exporting countries, the copula coefficients are significant for all oil-exchange rate pairs in the negative dependence regimes, but not significant in the positive dependence regimes. This indicates that the dependence between the oil price and the US dollar is not only negative on average, but also negative under extreme market conditions. Specifically, for the TWUSD and the CAD exchange rate, the dependence in the negative state is asymmetric. The estimated coefficients are significant only in regime 0-d (negative dependence regime with decreasing oil price), with the estimated correlations of 0.3788 and 0.4948 and tail dependence of 0.1741 and 0.247, respectively. This indicates that only during the downturn of oil markets, in general, the US dollar appreciates while the CAD depreciates. On the other hand, when the oil price is on the upturn, there is no significant comovement between the CAD/USD exchange rates and oil prices. In the same regime of 0-d, for Russia and Norway, the oil-exchange rate dependence in the negative dependence state is symmetric for the downturn and upturn of oil prices. For Russia, the estimated negative correlation coefficients and the tail dependence are 0.4991 and 0.2495 during the oil price downturn and 0.4856 and 0.2417 when oil markets are in the upturn. It implies that Russia Ruble appreciates (depreciates) with the rising (dropping) of oil prices at both normal time and oil market booming (crashing) time. The dependence transition probabilities for the oil exporting countries are all significant and close to 1. The

Table 3. Estimation results of the dependence switching copula model

Panel A: 1999-2020								
	USTW	CAD	RUB	NOK	EUR	JPY	KRW	INR
α_d^1	0.0001 (0.8947)	0.0001 (1.0329)	0.0001 (0.9722)	0.0001 (2.6077)	0.0001 (0.9110)	0.7286 (0.1676)	25.8255 (7.5831)	1.4479 (0.5501)
α_f^1	0.0001 (0.0972)	0.0001 (0.9407)	0.0001 (0.9440)	0.0001 (1.2764)	0.0083 (0.3088)	0.0284 (0.0716)	0.0039 (3.3009)	0.2096 (0.4764)
α_d^0	0.6573 (0.1921)	0.9830 (0.2877)	0.9969 (0.2727)	0.6765 (0.3033)	0.5487 (0.2422)	0.0995 (0.0468)	0.1941 (0.0525)	0.2623 (0.0512)
α_f^0	0.6714 (0.9841)	0.8566 (2.3468)	0.9534 (0.3317)	0.7403 (0.3717)	0.8967 (0.6640)	0.0636 (0.0525)	0.0008 (0.0985)	0.0677 (0.0391)
ρ_d^1	0.0001 (0.7026)	0.0001 (0.8111)	0.0001 (0.7635)	0.0001 (2.0479)	0.0001 (0.7154)	0.4073 (0.0646)	0.9936 (0.0035)	0.6128 (0.1149)
ρ_f^1	0.0001 (0.0763)	0.0001 (0.7387)	0.0001 (0.7413)	0.0001 (1.0024)	0.0065 (0.2405)	0.0220 (0.0547)	0.0030 (2.5825)	0.1484 (0.3031)
ρ_d^0	0.3788 (0.0791)	0.4948 (0.0883)	0.4991 (0.0827)	0.3867 (0.1227)	0.3318 (0.1105)	0.0744 (0.0332)	0.1385 (0.0339)	0.1811 (0.0309)
ρ_f^0	0.3846 (0.3999)	0.4538 (0.8051)	0.4856 (0.1044)	0.4117 (0.1417)	0.4673 (0.2198)	0.0484 (0.0387)	0.0006 (0.0773)	0.0514 (0.0287)
λ_{LL}^1	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.1931 (0.0423)	0.4868 (0.0038)	0.3098 (0.0563)
λ_{RR}^1	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0183 (0.1376)
λ_{RL}^0	0.1742 (0.0537)	0.2470 (0.0510)	0.2495 (0.0475)	0.1795 (0.0824)	0.1414 (0.0788)	0.0005 (0.0015)	0.0141 (0.0136)	0.0356 (0.0184)
λ_{LR}^0	0.1781 (0.2695)	0.2226 (0.4935)	0.2417 (0.0611)	0.1960 (0.0922)	0.2308 (0.1321)	0.0000 (0.0001)	0.0000 (0.0000)	0.0000 (0.0001)
P_{11}	0.9950 (0.0361)	0.9949 (0.0074)	0.9962 (0.0027)	0.9976 (0.0406)	0.9973 (0.0121)	0.9946 (0.0033)	0.0000 (0.0000)	0.0000 (0.0000)
P_{00}	0.9944 (0.0528)	0.9968 (0.0159)	0.9964 (0.0116)	0.9990 (0.0224)	0.9948 (0.0106)	0.9976 (0.0015)	0.9826 (0.0093)	0.9206 (0.0436)
Panel B: 2021-2024								
	USTW	CAD	RUB	NOK	EUR	JPY	KRW	INR
α_d^1	0.9999 (0.9742)	0.2256 (0.5769)	0.1985 (0.3547)	0.0385 (1.4080)	0.5504 (0.3308)	0.7723 (0.3469)	0.0001 (7.7511)	0.0002 (1.1544)
α_f^1	0.1748 (0.3011)	0.0002 (0.5505)	0.0005 (0.4621)	0.0002 (0.2698)	0.2841 (0.2440)	0.0001 (0.2990)	0.3448 (15.1038)	0.2248 (1.1944)
α_d^0	0.4419 (0.1042)	0.4437 (0.1683)	1.6032 (1.2113)	0.5885 (0.2394)	0.3638 (0.1584)	0.1450 (0.1283)	0.1988 (0.9136)	0.4836 (11.0601)
α_f^0	0.3428 (0.1350)	0.9369 (0.2306)	0.0002 (0.0205)	0.7506 (0.1806)	0.2810 (0.1388)	0.1962 (0.1273)	0.1169 (7.5218)	0.0069 (6.2322)
ρ_d^1	0.5000 (0.2945)	0.1585 (0.3613)	0.1413 (0.2282)	0.0296 (1.0640)	0.3325 (0.1507)	0.4238 (0.1284)	0.0001 (6.0870)	0.0002 (0.9064)

Table 3. (Continued)

Panel B: 2021-2024								
	USTW	CAD	RUB	NOK	EUR	JPY	KRW	INR
ρ_i^1	0.1259 (0.1984)	0.0001 (0.4323)	0.0004 (0.3627)	0.0001 (0.2119)	0.1942 (0.1441)	0.0001 (0.2348)	0.2289 (8.4014)	0.1581 (0.7485)
ρ_d^0	0.2804 (0.0527)	0.2813 (0.0850)	0.6434 (0.2244)	0.3496 (0.1052)	0.2394 (0.0865)	0.1060 (0.0871)	0.1415 (0.5877)	0.3011 (5.3717)
ρ_i^0	0.2278 (0.0753)	0.4804 (0.0737)	0.0002 (0.0161)	0.4156 (0.0682)	0.1923 (0.0823)	0.1398 (0.0821)	0.0866 (5.2536)	0.0054 (4.8612)
λ_{LL}^1	0.2500 (0.1688)	0.0231 (0.1819)	0.0152 (0.0950)	0.0000 (0.0000)	0.1419 (0.1074)	0.2038 (0.0822)	0.0000 (0.0000)	0.0000 (0.0000)
λ_{RR}^1	0.0095 (0.0648)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0436 (0.0913)	0.0000 (0.0000)	0.0670 (5.8978)	0.0229 (0.3752)
λ_{RL}^0	0.1042 (0.0385)	0.1048 (0.0621)	0.3245 (0.1060)	0.1540 (0.0738)	0.0744 (0.0617)	0.0042 (0.0177)	0.0153 (0.2452)	0.1193 (3.9094)
λ_{LR}^0	0.0662 (0.0527)	0.2386 (0.0434)	0.0000 (0.0000)	0.1986 (0.0441)	0.0424 (0.0517)	0.0146 (0.0335)	0.0013 (0.5067)	0.0000 (0.0000)
P_{11}	0.9475 (0.0381)	0.9217 (0.1069)	0.9854 (0.1559)	0.9186 (0.0996)	0.9598 (0.0354)	0.9898 (0.0090)	0.7979 (15.6461)	0.9324 (1.5020)
P_{00}	0.9951 (0.0042)	0.9908 (0.0215)	0.9615 (0.0677)	0.9954 (0.0122)	0.9915 (0.0080)	0.9930 (0.0067)	0.9541 (0.9032)	0.9578 (2.7115)

Estimates are bolded for significance at 5% level. The numbers under the estimates are standard errors. The superscripts and subscripts of "1, 0, d, i, R, L" denote positive and negative dependence state, decreasing and increasing oil prices, right and left tail dependence, respectively.

results indicates that the transitions between the positive and negative dependence regimes are unlikely.

Second, for the oil-importing countries except for the EUR, the estimated copula parameters α_d^1, α_d^0 and correlation coefficients ρ_d^1, ρ_d^0 are significant, indicating the significant dependence in both the positive and negative dependence regimes when the oil market is in the downturn. That is, when the oil market is in the downturn, the oil importing countries' currencies co-move with the oil market. The dependence in the positive state is much larger than that in the negative state. For example, the oil price-Japanese Yen/USD exchange rate correlation ρ_d^1 and ρ_d^0 are 0.4073 and 0.0744, respectively, indicating that Japanese yen appreciates largely with the decrease of the oil price in the 1-d regime (positive dependence state and the oil price decreasing regime), while depreciates slightly in the 0-d regime (negative dependence state and the oil price is in the downturn regime). Interestingly, when oil markets are in the upturn, there is no dependence between the two markets for oil importing countries. For the tail dependence, λ_{LL}^1 is significant but λ_{RR}^1 is not, indicating when oil markets are in the downturn, the US dollar depreciates while the oil importing country's currencies appreciate, which is consistent with the wealth effect view of the oil price-exchange rate relationship. However, this positive comovement between the US dollar (verses the oil importing countries' currencies) and oil prices during oil market downturn does not hold when the oil market is in the upturn. Thus, for oil importing countries, the oil price-US dollar (USD) exchange rate dependence at extremes is asymmetric in the sense that they move down together during the oil market downturn but do not move up together during the oil market upturn. In the negative dependence regime, there is no significant extreme or tail dependence for the oil importing countries. One exception among oil-importing currencies is the euro. Its exchange rate dependence on oil price is more similar to that of the USD. We attribute

this to the fact that the eurozone’s demand for oil is elastic, unlike other oil-importing countries. Consequently, the euro itself is not as dependent on the oil market as other oil-importing countries. Therefore, the EUR/USD exchange rate-oil price dependence is primarily influenced by the relationship between the denominator currency USD and oil price.

Panel B of Table 3 presents the dependence results for the period of 2020–2024, which includes the pandemic, the Russian–Ukraine war, and the Israel–Hamas war. Consistent with the results before this period, there is a significant positive dependence between the currencies of the oil-exporting countries and oil prices, reflected in significant values of the parameters for state 0 (negative dependence state between the oil price-exchange rate per USD, which is a positive dependence state between the oil price and the foreign currency) . However, for the RUB, this positive dependence does not hold when the oil price increases. Neither the correlation coefficient ρ_i^0 nor the tail dependence λ_{LR}^0 are significant, with a value close to zero, while they were significant at 0.4856 and 0.2417, respectively, in the previous period. This is consistent with what was observed during the Russian–Ukrainian war: the RUB depreciated due to the onset of the war, despite the increase in oil prices and Russia being an oil-exporting country. It seems that, during this time, the negative impact of the war on Russia’s currency value cancels out the positive impact of the oil price. For oil-importing countries, there is a significant negative dependence

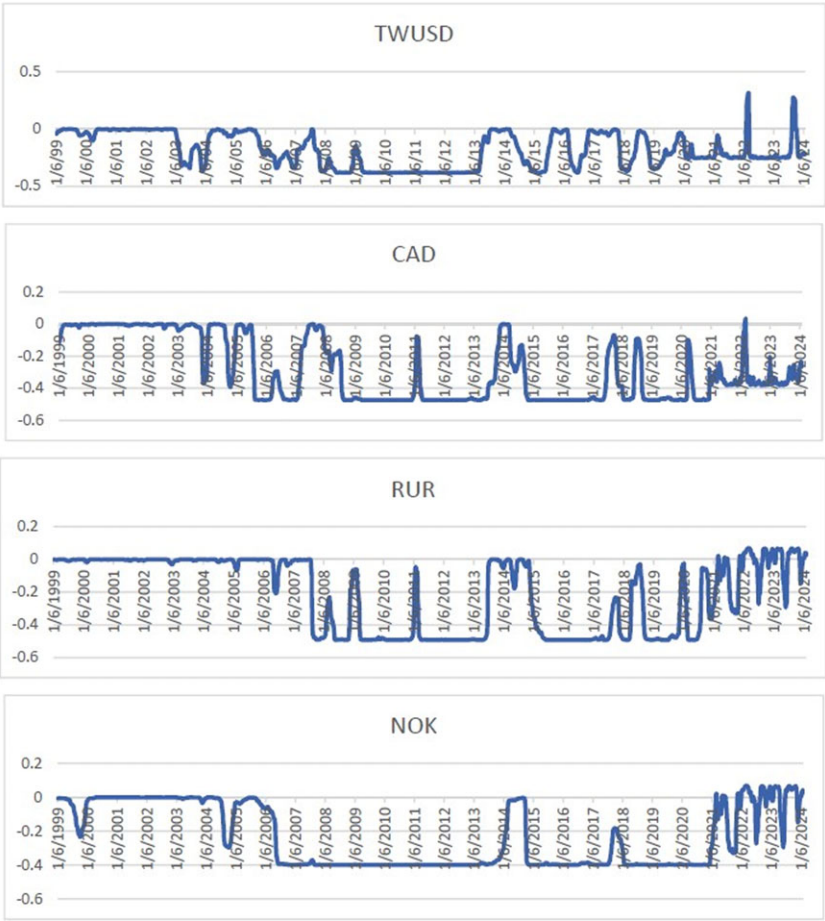


Figure 3. Smoothing correlations.

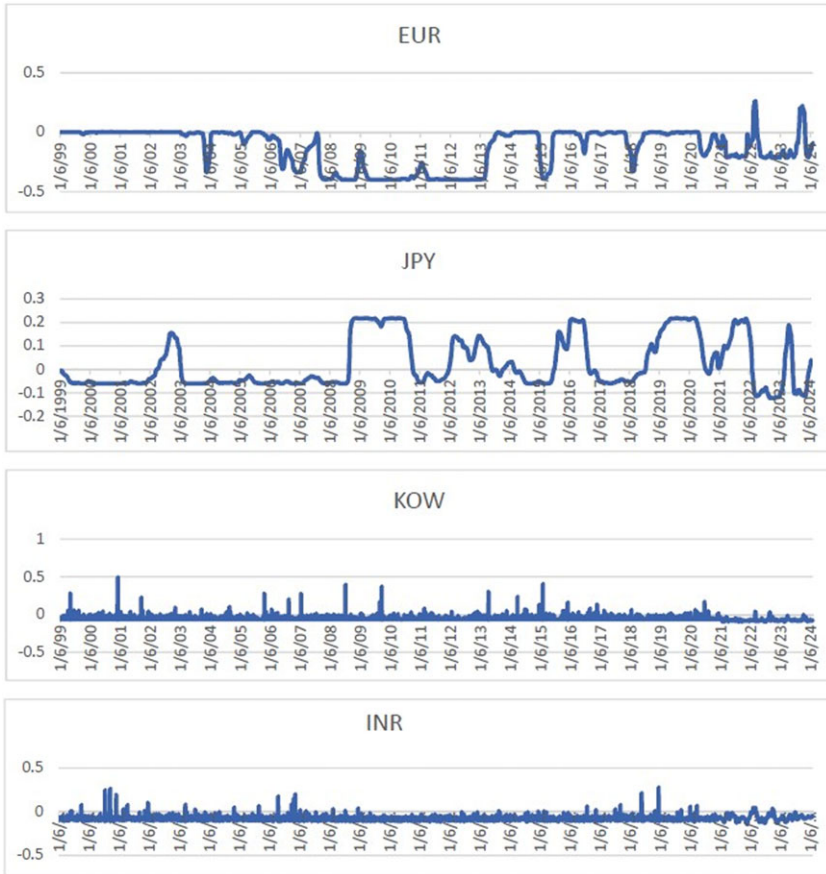


Figure 3. (Continued)

between currencies such as EUR and JPY and oil prices, but this negative dependence does not hold for KOW and INR anymore. The insignificant dependence between oil prices and KOW or INR could be related to the decreased demand for oil during the pandemic in these countries, as well as other factors such as monetary policies and economic status impacting the movements of their currencies more than oil prices.

Furthermore, it is important to highlight information gleaned from Table 3. Many instances reveal that while the correlations, which signify average dependence during regular market phases, lack statistical significance, there is a significant presence of tail dependence, which reflects interdependence during exceptionally turbulent market conditions. This implies that the oil price-exchange rate dependence may not be linear. For risk management and risk diversification, the dependence is most relevant during the periods of extreme market conditions, particularly during the market downturns. Hence, it is crucial to employ copula models to capture nonlinear dependence.

Figure 3 illustrates the smoothed correlations. All correlations change signs over time, particularly during financial crises or periods of geopolitical tension. For oil-exporting countries, the correlations between oil prices and the USD exchange rates are either negative or around 0, mostly remaining in the negative regime, especially after 2007. This indicates a negative dependence between oil prices and the USD exchange rate, which is equivalent to a positive correlation between the currencies of oil-exporting countries and oil prices. It's worth noting that around the

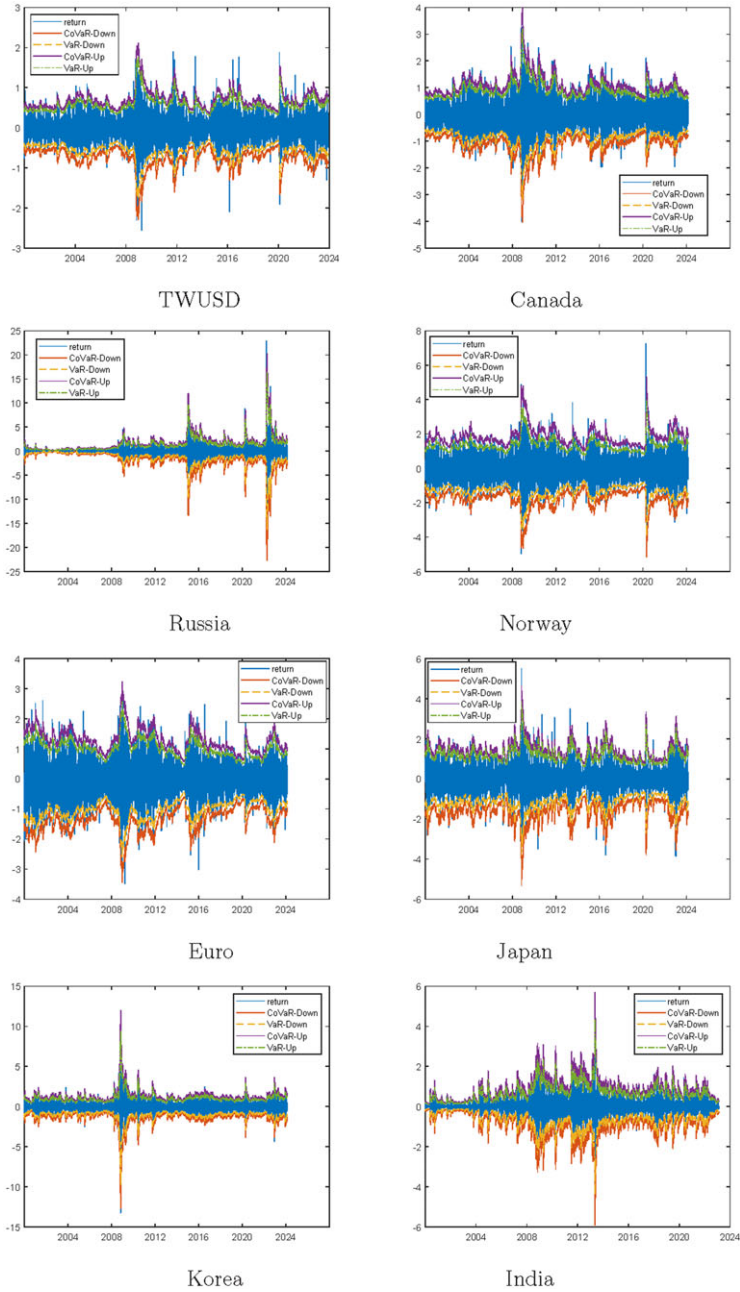


Figure 4. Downside and upside VaR and CoVaR for exchange rate markets.

onset of the Russian–Ukrainian war in February 2022 and the Israel–Hamas conflict in October 2023, the correlation between oil prices and the TWUSD exchange rate increases significantly. This is likely due to the safe-haven effect of the USD during wartime. The positive correlations during wars are observed across all exchange rates in terms of USD, indicating a stronger appreciation of the USD compared to other currencies during such periods.

Table 4. Descriptive statistics for VaRs and tests of risk spillovers from oil to foreign exchange markets

	VaR-Down	CoVaR-Down	KS-CoVaRDown	VaR-Up	CoVaR-Up	KS-CoVaRUp	KS-Ratio
TWUSD	-0.5806 (0.1945)	-0.7582 (0.2538)	0.3737 [0.0000]	0.5804 (0.1928)	0.6869 (0.2284)	0.2509 [0.0000]	1.3061 [0.0000]
CAD	-0.9062 (0.3247)	-1.1663 (0.4182)	0.3841 [0.0000]	0.8927 (0.3214)	1.1260 (0.4052)	0.3530 [0.0000]	1.2869 [0.0000]
RUR	-1.3544 (1.5304)	-1.7824 (2.0058)	0.1601 [0.0000]	1.4248 (1.5569)	1.8193 (1.9952)	0.1477 [0.0000]	1.3238 [0.0000]
NOK	-1.3706 (0.4063)	-1.8097 (0.5360)	0.4523 [0.0000]	1.3793 (0.4060)	1.7111 (0.5040)	0.3684 [0.0000]	1.3204 [0.0000]
EUR	-1.0398 (0.3120)	-1.3304 (0.3991)	0.3293 [0.0000]	1.0297 (0.3082)	1.3193 (0.3950)	0.3323 [0.0000]	1.2796 [0.0000]
JPY	-1.1748 (0.3691)	-1.6595 (0.5208)	0.4546 [0.0000]	1.1919 (0.3718)	1.5499 (0.4838)	0.2657 [0.0000]	1.4127 [0.0000]
KOW	-1.0690 (0.6698)	-1.4124 (0.8845)	0.3552 [0.0000]	1.0818 (0.6756)	1.4545 (0.9087)	0.3040 [0.0000]	1.3213 [0.0000]
INR	-0.6674 (0.4374)	-0.9108 (0.5943)	0.2202 [0.0000]	0.6752 (0.4282)	0.8842 (0.5630)	0.2079 [0.0000]	1.3682 [0.0000]

Notes: Standard errors for VaR and CoVaR are reported in parenthesis. P-Values for the Kolmogorov-Smirnov (KS) test statistic are presented in the square brackets. KS-CoVaR/Down: "H0: CoVar/Down = VaR/Down; H1: CoVar/Down < VaR/Down". KS-CoVaR/Up: "H0: CoVar/Up = VaR/Up; H1: CoVar/Up > VaR/Up". KSRatio: "H0: CoVaR-Down/VaR-Down = CoVaR-Up/VaR-Up; H1: CoVaR-Down/VaR-Down > CoVaR-Up/VaR-Up."

The graphs for oil-importing countries exhibit different patterns. For the Japanese yen (JPY), the correlations are close to 0, but there are many periods of positive correlations between oil prices and the JPY/USD exchange rate, indicating a negative correlation between the Japanese yen and oil prices. For Korea and India, the correlations between oil prices and the TWUSD exchange rate frequently alternate between large positive and weak negative regimes, suggesting an overall positive correlation between oil prices and the USD exchange rate, supporting the notion of a negative correlation between the currencies of oil-importing countries and oil prices. An exception among oil-importing countries is the eurozone. Overall, the correlation between the oil price and the euro exchange rate follows a similar pattern to that of the oil price against the TWUSD. It appears that the correlation between the oil price and the euro exchange rate is largely influenced by the US dollar in the denominator of the exchange rate, with limited impact from the euro in the numerator. This could also be related to the fact that the demand for oil in the eurozone is elastic,⁵ resulting in a limited effect of oil prices on the euro.

We present the temporal dynamics of the upside and downside VaR and CoVaR at 95% confidence levels ($\alpha = 0.05$ and $\gamma = 0.05$) for each foreign exchange market in Figure 4. Figure 4 shows that the VaR and CoVaR (VaR for exchange rates conditional on the oil prices) follow the same trend on all foreign exchange rate markets. That is, in general, the downside CoVaR (CoVaR-Down) is systematically lower than the downside VaR measure (VaR-Down), while the upside CoVaR (CoVaR-Up) is systematically higher than the upside VaR measure (VaR-Up). This implies that there might exist a systematic oil market spillover impact to the foreign exchange rate market in our empirical sample. To see if the spillover effects are statistically significant, we performed the KS test described in section 2.

The results for risk spillovers from oil markets to exchange rate markets are presented in Table 4. The p-values of the KS test are all close to 0. Thus the null hypothesis of no spillover effects is rejected. Therefore, there are significant downside/upside risk spillovers from oil markets to exchange rate markets. From Figure 4, we observe that the VaR and CoVaR values are

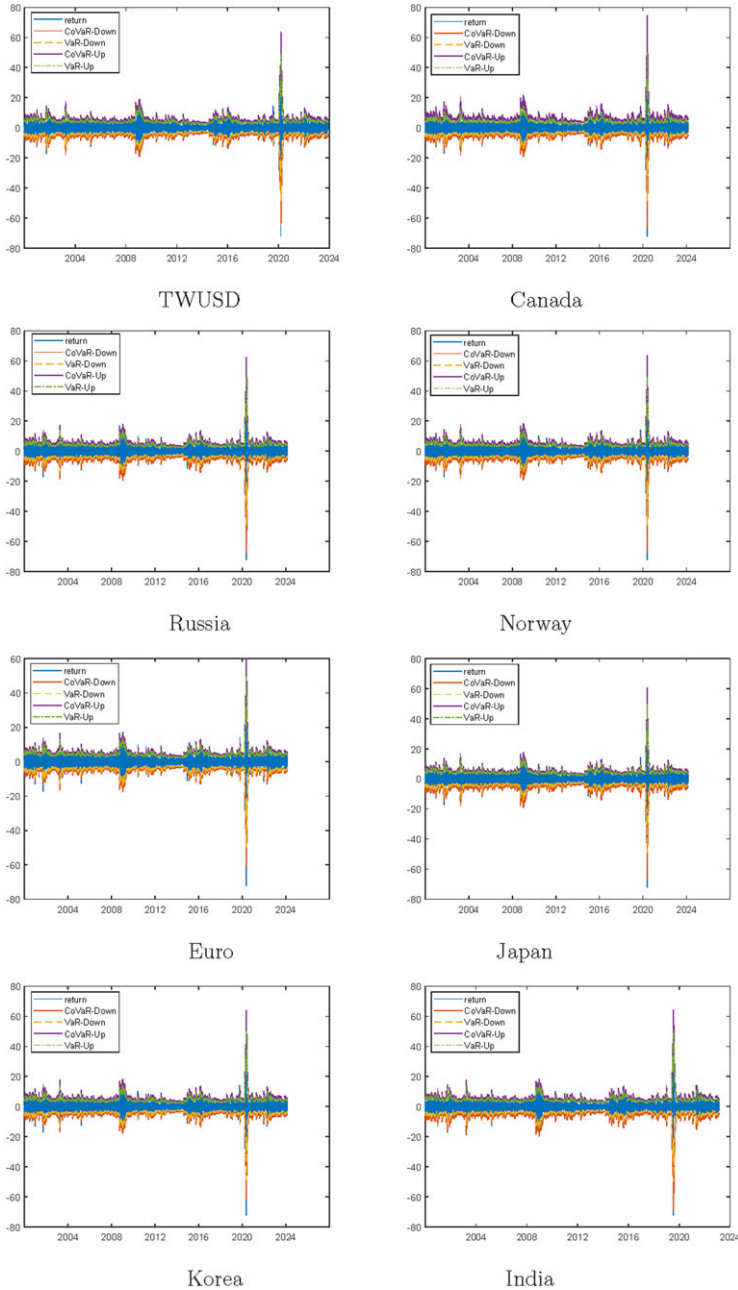


Figure 5. Downside and upside VaR and CoVaR for oil markets.

significantly higher during the 2008 global financial crisis (except for Russia and India) and the COVID-19 pandemic in 2020, indicating increased risk during these crisis periods. Additionally, we note a substantial spike in VaR and CoVaR values around 2022 for the RUR, reflecting the heightened risk faced by the Ruble during the Russian–Ukraine war.

The dynamics of VaR and CoVaR for the oil markets are plotted in Figure 5. Figure 5 resembles Figure 4 in that the CoVaR (VaR for oil prices conditional on exchange rates) generally lies outside

Table 5. Descriptive statistics for VaRs and tests for risk spillovers from foreign exchange to oil markets

	VaR-Down	CoVaR-Down	KS-CoVaRDown	VaR-Up	CoVaR-Up	KS-CoVaRUp	KS-Ratio
TWUSD	-4.5893 (2.2339)	-4.7718 (2.3222)	0.0646 [0.0000]	4.7122 (2.2659)	4.8042 (2.7895)	0.3036 [0.0000]	1.0398 [0.0000]
CAD	-4.5030 (2.1826)	-4.6568 (2.2566)	0.0551 [0.0000]	4.6155 (2.2106)	4.7767 (2.2283)	0.0568 [0.0000]	1.0342 [0.0000]
RUR	-4.5735 (2.2167)	-4.6480 (2.2526)	0.0278 [0.0183]	4.5628 (2.1853)	4.5805 (2.1939)	0.0086 [0.0978]	1.0163 [0.0000]
NOK	-4.5633 (2.2113)	-4.8507 (2.2101)	0.0861 [0.0000]	4.6229 (2.2073)	4.7549 (2.8268)	0.0467 [0.0000]	1.0638 [0.0000]
EUR	-4.5026 (2.1824)	-4.6231 (2.2404)	0.0430 [0.0000]	4.6272 (2.2162)	4.8056 (2.3022)	0.0346 [0.0000]	1.0268 [0.0000]
JPY	-4.5923 (2.2256)	-4.7838 (2.3178)	0.0666 [0.0000]	4.5699 (2.1886)	4.8088 (2.3037)	0.0604 [0.0000]	1.0417 [0.0000]
KOW	-4.6035 (2.2308)	-4.6826 (2.2544)	0.0187 [0.2387]	4.5714 (2.1891)	4.5950 (2.1330)	0.0426 [0.0000]	1.0172 [0.0000]
INR	-4.7088 (2.2625)	-4.9595 (2.3822)	0.0832 [0.0000]	4.7542 (2.2571)	4.3558 (2.0669)	0.1385 [0.0000]	1.0533 [0.0000]

Notes: Standard errors for VaR and CoVaR are reported in parentheses. P-Values for the Kolmogorov–Smirnov (KS) test statistic (KS-CoVaR-Down, KS-CoVaR-Up, KS-Ratio) are presented in the square brackets. KS-CoVaRDown: “H0: CoVaR-Down = VaR-Down; H1: CoVaR-Down < VaR-Down”. KS-CoVaR-Up: “H0: CoVaR-Up = VaR-Up; H1: CoVaR-Up > VaR-Up”. KS-Ratio: “H0: CoVaR-Down/VaR-Down = CoVaR-Up/VaR-Up; H1: CoVaR-Down/VaR-Down > CoVaR-Up/VaR-Up.”

of the VaR range. Specifically, the downside CoVaR tends to be lower than the downside VaR, while the upside CoVaR tends to be higher than the upside VaR. This pattern indicates evidence of risk spillover effects from the exchange rate market to the oil market. Furthermore, a significant spike in VaRs and CoVaRs is observed around 2020, coinciding with the onset of the pandemic, indicating a substantial increase in risk in the oil market during that period.

We provide the results for the spillover effects from exchange rate markets to oil markets in Table 5. The KS test results in column 4 and 7 of Table 5 confirm significant downside and upside risk spillover effects from exchange rates to oil markets for all countries except for Korea during oil market downside and Russia during oil market upside.

The last column in Tables 4 and 5 shows the results of KS-ratio test for the downside/upside asymmetry of the risk spillover effects. The close to zero p values for the test statistic in the last column of both tables (except for the direction from currency to oil markets for CAD, RUR and KOW in Table 5) give strong evidence that the oil to currency risk spillover effects are asymmetric in the sense that the downside risk spillover effects are greater than the upside risk spillover effects. This is consistent with our finding of the asymmetric tail dependence with the lower tail dependence is stronger than the upper tail dependence.

In Table 6, we present the test results for the relative strength of the two directions of risk spillovers. For the downside, values in column 2 consistently surpass those in column 3, implying a more pronounced transmission of risks from oil markets to foreign exchange markets in the downside. The formal KS test in column 4 confirms that we reject the null hypothesis of equal strength of the two directions of risk spillovers, in favor of the alternative that the risk spillovers are stronger from oil markets to foreign exchange markets than in the opposite direction. Regarding the upside risk spillovers, presented in the last three columns of Table 6, the results mirror those of the downside. Hence, for both the downside and upside, the risk spillovers are stronger from oil markets to foreign exchange markets than in the opposite direction.

Table 6. Risk spillover direction tests

	Downside Oil to Ex	Downside Ex to Oil	Downside KS direction	Upside Oil to Ex	Upside Ex to Oil	Upside KS direction
TWUSD	1.3061 (0.0037)	1.0398 (0.0081)	1.0012 [0.0000]	1.1835 (0.0225)	1.0295 (0.0450)	1.0012 [0.0000]
CAD	1.2869 (0.0020)	1.0117 (0.0024)	1.0000 [0.0000]	1.2614 (0.0018)	1.0029 (0.0056)	1.0021 [0.0000]
RUR	1.3238 (0.0163)	1.0455 (0.0001)	1.0002 [0.0000]	1.2716 (0.0099)	1.0029 (0.0000)	1.0001 [0.0000]
NOK	1.3204 (0.0282)	1.0192 (0.0012)	1.0010 [0.0000]	1.2405 (0.0219)	1.2188 (0.0533)	1.0002 [0.0000]
EUR	1.2796 (0.0011)	1.0268 (0.0052)	1.0000 [0.0000]	1.2812 (0.0119)	1.0385 (0.0734)	1.0000 [0.0000]
JPY	1.4127 (0.0044)	1.0417 (0.0085)	1.0011 [0.0000]	1.3002 (0.0328)	1.0522 (0.0108)	1.0021 [0.0000]
KOW	1.3213 (0.0021)	1.0079 (0.0156)	1.0002 [0.0000]	1.3444 (0.0221)	1.0756 (0.0243)	1.0001 [0.0000]
INR	1.3682 (0.0080)	1.0052 (0.0000)	1.0020 [0.0000]	1.3068 (0.0060)	1.0108 (0.0003)	1.0021 [0.0000]

Notes: Standard errors for CoVaR /VaR are reported in parentheses. P-Values for the directional Kolmogorov–Smirnov (KS) test statistics are presented in the square brackets. KS direction test hypotheses: H0: CoVaR (EX|Oil)/VaR(EX) = CoVaR (Oil|EX)/VaR(Oil); H0: CoVaR (EX|Oil)/VaR(EX) > CoVaR (Oil|EX)/VaR(Oil) for downside or upside risk spillover.

5. Conclusion

This paper applies a flexible dependence switching copula model to investigate the dependence between exchange rates and oil prices over time. In general, we observe a negative correlation between oil prices and the US dollar. However, this relationship varies depending on whether the exchange rate involves the USD against currencies of oil-exporting or importing countries, as well as the market conditions of the oil industry.

Our findings reveal that currencies of oil-exporting countries are generally positively correlated with oil prices, with infrequent shifts between positive and negative dependence regimes. Conversely, currencies of oil-importing countries exhibit the opposite pattern, with more frequent shifts between these regimes. Additionally, we observe that the dependence between oil prices and exchange rates exists primarily during oil market downturns.

Furthermore, significant tail dependence is detected between oil prices and USD exchange rates, with lower tail dependence being greater than upper tail dependence for most countries. This asymmetric extreme dependence is evident only during oil market downturns. Notably, our empirical analysis identifies substantial risk spillover effects from oil markets to USD exchange rate markets and vice versa, with stronger risk spillover from oil markets to currency markets than in the reverse direction. Moreover, downside risk spillovers outweighing upside ones.

These findings contribute significantly to understanding the dynamics of oil price-exchange rate dependence and have crucial implications for risk management, investment strategies, and policymaking.

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Notes

- 1 One exception is that oil demand was elastic in EU, an oil importing region, especially after the start of the war in Ukraine. We thank one of the referees for the insight.
- 2 Including a third state of no dependence could be interesting to consider. Our model incorporates the no dependence state as a special case when the dependence parameters are 0. To ensure the model estimation is tractable and to keep the numbers of parameters manageable, we have followed the literature and explicitly included two states: the positive and negative dependence states.
- 3 In this paper, our estimation algorithm is based on the procedure in Wang et al. (2013). For the details of the estimation, please refer to Wang et al. (2013).
- 4 The exchange rates data are from DataStream, while the crude oil price data are from the website of the US Energy Information Administration https://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm, whose data are sourced from Thomson Reuters. On April 20, 2020, the WTI price was negative. This date was excluded since the log of the negative price would be a complex.
- 5 We once again thank one of the referees for providing valuable insight into the elastic nature of oil demand in the eurozone.

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