NUMERICAL EFFECTS OF GRAVITATIONAL LIGHT DEFLECTION ON THE DETERMINATION OF THE EQUINOX AND EQUATOR

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Observations of the sun and major and minor planets made by transit circle telescopes are used to determine positions of the equinox and the celestial equator and, by repeated observing programs, the motions of these fiducial references. Long series of such absolute observations, when combined into catalogs such as the FK5, yield a fundamental coordinate system which is an observational approximation to an ideal, dynamically defined coordinate system. In such a system the equinox, for example, is defined implicitly by the right ascensions (at mean epoch) and the proper motions of the stars included in the catalog system, together with the adopted constant of precession. It may be noted that independent, highly accurate determinations of the latter quantity thus help to improve the fundamental proper motion system.

In principle the use of a moving zero point such as the equinox is perfectly acceptable if the motion is sufficiently well known, however, in practice, there remain serious problems involving both observational accuracy and the theory of the involved motions. Considering that the FK5, for example, depends upon observations of the sun and stars to m=9.5 one has a dynamic range of 10^{14} in apparent brightness, it is not surprising that systematic errors do occur. Current work towards the establishment of an extra-galactic reference frame at both radio and optical wavelengths avoids many problems and promises to provide a highly accurate system. A major problem is the extension of such a system to brighter objects. Although in this case the information flow is reversed; i.e., from fainter to brighter, rather than vice versa as in the classical case, it still remains to overcome problems with a dynamic range of perhaps 10^{6} in apparent brightness. Ultimate limitations may involve source structure and its evolution at various wavelengths.

In any event, the fundamental optical reference system is founded upon observations of solar system objects, and such observations, significantly improved, are likely to continue in order to help investigate systematic effects in both the dynamical and extra-galactic systems, and to refine and improve our knowledge of the solar system itself.

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Those solar system objects which move more rapidly provide more information and hence observations of Mercury, Venus and the Sun provide the bulk of the weight in any solution for the equator and/or the equinox. Historically the gravitational deflection of light has not been accounted for when reducing these observations since the former has been small in comparison to the precision of the latter.

There are two ways in which the light deflection can influence the observational determination of the equinox. The first, a direct effect, is the apparent displacement of the observed object, and the second, an indirect effect, is due to the necessity of applying a DAY/NIGHT correction to observations of the Sun, Mercury and Venus in order to place these observations on the stellar (nighttime) system. This correction is determined by observing selected stars as both daytime and nighttime objects and modeling the systematic differences between the day and night results. Since day stars are observed both before and after local noon, and to the north and south of the sun, the already small deflection effect tends to cancel. On the other hand, day star observations are notoriously unpredictable and magnitude selection effects are likely to occur. Although the direct effect is also small, there is no simple cancellation tendency and indeed there are obviously correlations between the deflections and the positions of the observed objects and the positions of the observers in earth orbit. It is conceivable that these correlations could lead to enhanced systematic effects. For all of these reasons it appeared worthwhile to investigate the effect which the gravitational deflection of light could have upon the determination of the equinox and equator using actual observational data.

To this end we chose to use the results of observations made with the Washington Six-inch Transit Circle during the period 1963-71. These have been published as the $W5_{50}$ catalog (Hughes and Scott) [1]. The various procedures which were used in the original discussion, with no relativistic corrections, were at hand so that a rediscussion, including the gravitational deflection of light, could be easily accomplished.

The angular displacements of objects observed at coordinates, α and δ, with the sun at, α_s and δ_s, were computed from the formulae given on page B-17 of the 1984 Astronomical Almanac [2] and also see Murray [3]:

$$\Delta\alpha = \mu \sec \delta \cos\delta_s \sin(\alpha - \alpha_s)$$

$$\Delta\delta = \mu [\sin\delta \cos\delta_s \cos(\alpha - \alpha_s) - \sin\delta_s \cos\delta]$$

$$\mu = \theta/\sin D$$

For stars,

$$\theta = 0''.00407\ (1 - \cos D)/\sin D$$

and for planets,

$$\theta = 0''.00407\ [\sin L/(1 + \cos L)]$$

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where,

θ = Apparent angle of deflection, always positive, radially outward from the sun.

D = the angle, at the earth, between the object and the sun.

L = the angle, at the sun, between the object and the earth.

The positions of the sun required in these expressions were derived from approximate expressions based upon the principles given on page C-24 of the 1984 Astronomical Almanac [2]. Similarly, expressions for cos D and sin D are given on page S-20 of the same publication. The angle, L, follows from,

$$\tan L = r_p \, [(\sin D/r_s) - \cos D]$$

where r_p and r_s represent the distances, earth-planet and earth-sun, respectively.

In the $W5_{50}$ there were 62 clock stars which were observed both day and night. An analysis of the differences, Night-Day, gave the corrections shown in Table 1 (which appears as Table 11 in the $W5_{50}$). Note that the correction to the right ascensions was found to be a function of the apparent time only.

The magnitude of the individual light deflection corrections for the night observations never exceeded $0^s.0004$ in right ascension and $0''.0008$ in declination. The mean values were considerably smaller. The individual corrections for the day star observations reached a magnitude of $0^s.003$ and $0''.05$. Such large values were attained by the dozen or so observations of day stars made within 10° of the sun. Due to the tendency towards cancellation mentioned earlier however, the group values, when subjected to the same analysis which produced Table 1, gave essentially the same results as given in that table. Specifically, the largest corrections never exceeded $0^s.0005$ and $0''.005$ and in one or two cases led to rounding differences of 1 unit in the last place given in Table 1. Thus the indirect effect was found to be negligible and corrections for it were not applied to the observations of Mercury, Venus and the Sun. Hence the solar observations are unaffected.

It may be remarked that the day corrections were found from an analysis of 2,592 right ascension observations and 2,558 declination observations of the day stars listed in table 4 of the introduction to the $W5_{50}$. The observations were made up to 4.5 hours before and after apparent noon.

The direct effect upon Mercury and Venus can easily amount to several centi-arc seconds. The maximum value found for Venus, for example, was $0''.068$.

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Table 1. – Night Minus Day Corrections ($W5_{50}$)

Declination	Correction Decl.	Apparent Time	Correction R.A.	Correction Decl.
	"	h	s	"
-30^o	-0.47	8.5	-0.012	-0.04
-25	-0.45	9.0	-0.010	-0.04
-20	-0.43	9.5	-0.008	-0.05
-15	-0.36	10.0	-0.007	-0.04
-10	-0.29	10.5	-0.006	-0.04
- 5	-0.20	11.0	-0.007	-0.03
0	-0.11	11.5	-0.007	-0.01
+ 5	-0.04	12.0	-0.008	0.00
+10	+0.05	12.5	-0.008	+0.01
+15	+0.07	13.0	-0.009	+0.03
+20	+0.08	13.5	-0.010	+0.04
+25	+0.15	14.0	-0.010	+0.04
+30	+0.20	14.5	-0.009	+0.04
		15.0	-0.008	+0.04
		15.5	-0.007	+0.05

Table 2 – Definition of the Unknowns

This set corresponds nearly to Sets IV and VI of Brouwer and Clemence. Primed quantities refer to the earth, unprimed to a planet.

$X1 = \Delta\delta_o$ Correction to the equator

$X2 = \Delta M' + \Delta\psi_3$ Correction to the mean anomaly plus $\Delta\psi_3$

$X3 = \Delta\varepsilon$ Correction to the obliquity

$X4 = -\Delta\alpha_o \sin\varepsilon$ Correction to the equinox x sin ε

$X5 = e'\Delta\psi_3$ Eccentricity times $\Delta\psi_3$

$X6 = \Delta e'$ Correction to eccentricity

where $\Delta\psi_3 = -\Delta\alpha_o \cos\varepsilon + \Delta\pi'$

$\Delta\pi'$ is longitude of perihelion

$X7 = \Delta M_o + \Delta r$ Correction to mean anomaly + Δr

$X8 = \Delta p$, $X9 = \Delta q$, $X10 = 2e\,\Delta r$, $X11 = 10\,\Delta a/a$, $X12 = 2\,\Delta e$

$\Delta I = \Delta p \cos\omega - \Delta q \sin\omega$, $\Delta\Omega \sin I = \Delta p \sin\omega + \Delta q \cos\omega$;
$\Delta\omega + \Delta\Omega \cos I = \Delta r$

where I is the inclination, Ω, the longitude of the node; and, ω the angle, node to perihelion.

X13 = Semidiameter correction in α, X14 = Semidiameter correction in δ.

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Table 3 - Comparison of Classical and Relativistic Solutions for Corrections to Orbital Parameters, Equator, Equinox, and Semi-diameter.

MERCURY (with Phase Corrections Applied)

Unit = Seconds of Arc

	Classical		Relativistic	
	M.E., UNIT WEIGHT = 0.769		M.E., UNIT WEIGHT = 0.770	
	410 Observations			
X 1	-0.26979	±0.064(ME)	-0.26991	±0.064(ME)
X 2	-0.44472	0.098	-0.44461	0.098
X 3	-0.01938	0.109	-0.01830	0.109
X 4	-0.27992	0.084	-0.27928	0.084
X 5	-0.20747	0.112	-0.20669	0.112
X 6	0.00267	0.106	0.00252	0.106
X 7	-1.28726	0.394	-1.28422	0.394
X 8	-0.26455	0.260	-0.26021	0.261
X 9	-0.14440	0.383	-0.14375	0.384
X10	0.28876	0.748	0.29158	0.749
X11	0.00997	0.025	0.00975	0.025
X12	-0.39692	0.779	-0.39089	0.780
X13	0.05150	0.073	0.04445	0.073
X14	-0.08724	0.105	-0.09029	0.105

VENUS (with Phase Corrections Applied)

	Classical		Relativistic	
	M.E., UNIT WEIGHT = 0.790		M.E., UNIT WEIGHT = 0.791	
	1208 Observations			
X 1	-0.25596	±0.034(ME)	-0.25636	±0.034(ME)
X 2	-0.56459	0.032	-0.56348	0.032
X 3	-0.15134	0.047	-0.15090	0.047
X 4	-0.11493	0.044	-0.11382	0.044
X 5	-0.08893	0.066	-0.08821	0.066
X 6	-0.10889	0.068	-0.10773	0.068
X 7	0.28126	0.057	0.28326	0.057
X 8	-0.03013	0.066	-0.02964	0.066
X 9	0.28508	0.076	0.28375	0.076
X10	0.05506	0.215	0.05359	0.215
X11	-0.00067	0.013	-0.00071	0.013
X12	-0.25473	0.211	-0.24911	0.211
X13	0.06758	0.024	0.06491	0.024
X14	-0.11267	0.034	-0.11321	0.034

The observational and resulting normal equations may be cast into various forms. The set of unknowns used here (see Table 2) corresponds nearly with Set IV and Set VI given by Brouwer and Clemence [4]. The particular quantities of interest here are:

$$X1 = \Delta\delta_o$$

and $$X4 = -\Delta\alpha_o \sin\varepsilon$$

where $\Delta\alpha_o$ and $\Delta\delta_o$ are the corrections to the assumed equinox and equator, respectively, and ε is the obliquity.

From the results given in Table 3 one finds for Mercury;

$$(-\Delta\alpha_o \sin\varepsilon)_{REL.} - [-(\Delta\alpha_o \sin\varepsilon)]_{CLASS.} =$$

$$0''27992 + 0''27928 = -0''00064$$

$$D(\Delta\alpha_o) = -0''0016$$

and for Venus, $D(\Delta\alpha_o) = -0''0028$

Similarly, for Mercury, $D(\Delta\delta_o) = -0''0001$

and for Venus, $D(\Delta\delta_o) = +0''0004$

These differences are completely negligible in comparison to the precisions of the quantities given in Table 2, and it is unlikely that the neglect of gravitational light bending has done any real harm to the accuracy of the FK5 equinox. On the other hand, the effects are systematic at the milliarcsecond level and if the precision of planetary observations were to improve by one order of magnitude it would be necessary to include the light bending in the reduction procedures. Whether or not such an improvement in direct planetary observations can be made is unknown at this time. Optical interferometric observations of "point" sources such as minor planets may contribute to the dynamical coordinate system with greatly improved precision, and in this case the light bending would be routinely applied even if smaller than that for day objects.

References

[1] Hughes, J.A. and Scott, D.K.: 1982, Publ. of the U.S. Naval Observatory, Second Series, Vol. XXIII, Part III, pg. 165ff.

[2] The Astronomical Almanac for the year 1984 issued by the Nautical Almanac Office, U.S. Naval Observatory and Her Majesty's Nautical Almanac Office, Royal Greenwich Observatory.

[3] Murray, A.: 1981, Mon. Not. Roy. Ast. Soc. 195, 639.

[4] Brouwer, D. and Clemence, G.M.: 1961, Methods of Celestial Mechanics (Academic Press: New York and London).

DISCUSSION

Branham : since the equations used by you for calculating the deflection are straightforward, why not apply them systematically ?

Hughes : you are correct, and we shall apply the deflection corrections routinely in the future, just as with stellar apparent places.

Murray : do you think that the neglect of the deflection has had any effect upon any numerical integrations using observational data ?

Hughes : I doubt very much that it has since more serious systematic error sources exist.

Seidelmann : problems with phase effects, particularly with Venus, cause many problems with accuracy.

Hughes : that is so and I hope that our present limb modeling using the image dissector micrometer in New Zealand will help solve this serious problem.