CONVECTIVE MOTIONS AS AN INDICATOR OF SOLAR STRUCTURE

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Most stars contain regions which are convectively unstable and one of the more daunting tasks facing astrophysics today is to find a satisfactory theoretical formulation of turbulent energy transport in stars. Various theories have been proposed, such as the mixing-length formalism and its extensions, and it would be most useful if one could test the accuracy of such models in view of their importance in the theory of stellar structure and evolution.

Fortunately, the outer layers of the sun are convectively unstable and the sun is near enough for some of its surface characteristics, such as granulation and supergranulation, to be observed. Granules have an average cell diameter of 2,000 kms, horizontal and vertical velocities of the order of 1 km/sec, an intensity modulation of 15%, a temperature difference of about 900° K between ascending and descending currents and an average lifetime of approximately 20 minutes. On the other hand, supergranules have an average horizontal extent of 30,000 kms, horizontal and vertical velocities of .3 to .5 km/sec, have no observable intensity modulation and a lifetime of approximately 20 hours.

Such a wealth of information should allow us to probe the structure of the sun's outer layers and ultimately to test the validity of existing models. Unfortunately, in order to do this one must be able to model fairly accurately large-scale convective motions in a highly stratified compressible layer.

Numerous attempts have been made to study thermal convection within the Boussinesq and even the anelastic approximation but, since the depth of the convective layer is very much larger than the pressure scale height, a satisfactory model of granulation and supergranulation should be based on the fully compressible equations. Such equations, for layers with polytropic structure, were derived some time ago (Van der Borght 1977) within the framework of the one-mode approximation and have lately (Van der Borght and Fox 1983) been integrated in an attempt to model granulation. The results are very encouraging and

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velocities of the order of 0.97 km/sec, an intensity modulation of 16.37% and an e-folding time of 6 minutes are obtained in a medium for which the Prandtl number is 0.2.

Unfortunately, such models make it necessary to fit a polytropic structure to the model and requires the introduction of average values for the buoyancy, eddy diffusivity and eddy viscosity with a resultant loss in accuracy. The depth dependence of these quantities is ignored and the models are not accurate enough to enable us to compare the accuracy of competing models.

The basic hydrodynamic equations can be written (Van der Borght 1980).

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho x_i) = 0 \tag{1}$$

Equation of motion:

$$\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j + \delta_{ij} p - P_{ij}) + \delta_{i3} g\rho = 0 \tag{2}$$

where the viscous tensor P_{ij} is defined as follows

$$P_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_\ell}{\partial x_\ell}\right) \tag{3}$$

and μ is the viscosity.

Energy equation:

$$\frac{\partial}{\partial t}[\rho(E + \tfrac{1}{2}u_\ell u_\ell)] + \frac{\partial}{\partial x_j}\left\{\rho u_j\left(E + \frac{u_\ell u_\ell}{2} + gz\right) + pu_j - K\frac{\partial T}{\partial x_j} - u_i P_{ij}\right\} = 0 \tag{4}$$

where E is the internal energy per unit mass and K is the conductivity.

State equation:

$$p = R\eta\rho T \tag{5}$$

where η is the inverse of the molecular weight.

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For a given model (e.g. Böhm 1963, Kohl 1966) the values and variation with depth of the conductivity K, internal energy E and inverse molecular weight η are known. The only quantity in the above equations which is left unspecified is the eddy viscosity μ. One could, for instance, assume that the viscosity is constant or that the kinematic viscosity is constant (models A and B of Graham and Moore, 1978). It would be even better if the exact values were given by the theory of turbulence, if one were available. In any case, as mentioned above, enough observed characteristics are available to distinguish not only between the accuracy of various models but also to establish the best law of variation of viscosity with depth.

Solving the full three-dimensional equations is out of the question for the moment, due mainly to the numerical complexities. But, since granulation and supergranulation exhibit a periodic structure, it is to be expected that the one-mode approximation would yield fairly accurate results. The fully compressible single mode equations which take into account the variation with depth of the degree of ionization, thermal diffusivity, eddy kinematic viscosity and buoyancy have been derived but space prevents us from giving them in this paper. With such equations the characteristics of a particular model can be fully taken into account and the resultant thermal convection can be studied in detail without the need of ad-hoc assumptions, except for the value and depth dependence of the turbulent kinematic viscosity.

Trial integrations of this complicated system of differential equations have been carried out and the results are very encouraging. They confirm earlier results based on the polytropic approximation (Van der Borght and Fox 1983) and show how sensitive the results are to the value adopted for the Prandtl number. The adoption of a more accurate variation with depth of the buoyancy leads to more realistic distributions of the velocity and the temperature perturbation. It seems likely that work of this kind will not only be useful in comparing models of convective regions but will help in our understanding of the turbulent processes in such regions.

References

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DISCUSSION

R. Cayrel: Do your computations allow you to estimate the amount of overshooting above the granulation layer?

Van der Borght: Yes, the upper boundary for the numerical integrations can be set well above the unstable region.