ON THE STABILITY OF MAGNETIC FLUX TUBES IN THE EQUATOR OF A STAR

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Abstract. We consider the linear stability of a toroidal flux tube lying in the equatorial plane of a differentially rotating star and investigate its dependence on superadiabaticity, magnetic field strength, and gradient of angular velocity.

Key words: MHD stability – Magnetic flux tubes – Dynamo theory – Sun

1. Introduction

A fundamental question in relation with the dynamo mechanism is whether magnetic flux can be stored in the solar convection zone for a time sufficiently long to permit the operation of the solar dynamo. We set out from the hypothesis that the magnetic field in the convection zone is concentrated in individual flux tubes separated by nearly field-free plasma. Parker (1975) pointed out that magnetic buoyancy of flux tubes in thermal equilibrium might lead to a rapid loss of magnetic flux. In order to avoid this problem, some authors have proposed that magnetic flux is stored within the overshoot region below the convection zone (see, e.g., Moreno–Insertis, 1992). Here we consider a toroidal flux tube lying in the equator of a differentially rotating star and investigate the effects of the rotationally induced forces in determining whether the equilibrium is stable or unstable. Both axisymmetric and non-axisymmetric perturbations about the equilibrium configuration are considered, as well as differential rotation and different rotation rates between the flux tube and the surrounding medium. Our work is an extension of the stability analysis performed by van Ballegooijen (1983). Our treatment is not restricted to small differences between the rotation rates of the flux tube and its environment and it consistently includes the rotationally induced forces.

2. Model and basic equations

We use a frame of reference rotating with angular velocity Ω (equal to the rotation rate of the matter inside the equilibrium flux tube) with origin in the center of the star. We employ cylindrical coordinates (r, ϕ, z) and denote $\{\mathbf{e}_r, \mathbf{e}_\phi, \mathbf{e}_z\}$ the corresponding unit vectors. In the following, the subscripts "e" and "i" stand for quantities outside the flux tube (*external medium*) and inside (*internal medium*), respectively. If the star rotates differentially with angular velocity $\Omega_e(r)$, the external velocity field in the equatorial plane is $\mathbf{v}_e(r) = r[\Omega_e(r) - \Omega]\mathbf{e}_\phi$.

The equation for the external medium in stationary equilibrium ($\partial/\partial t \equiv 0$) is:

$$\rho_e(\mathbf{v}_e \cdot \nabla)\mathbf{v}_e = -\mathbf{grad}\, p_e + \rho_e\left[\mathbf{g} - \mathbf{\Omega} \wedge (\mathbf{\Omega} \wedge \mathbf{r})\right] + 2\rho_e \mathbf{v}_e \wedge \mathbf{\Omega}\,, \tag{1}$$

which, for an axially symmetric equilibrium, reduces to:

$$\frac{\partial p_e}{\partial r} = \rho_e(\mathbf{g} \cdot \mathbf{e}_r + r\Omega_e^2) = -\rho_e(g - r\Omega_e^2)\,. \tag{2}$$

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The motion of the flux tube is described within the framework of the ***thin flux tube approximation*** for curved flux tubes (Spruit, 1981; Ferriz–Mas and Schüssler, 1993). Denote equilibrium quantities with "0". The application of the momentum equation to a flux tube whose unperturbed path is a circumference of radius r_0 ("toroidal flux ring") yields the following equilibrium condition:

$$\frac{v_A^2}{r_0 g_0} - \left(\frac{\rho_{e0}}{\rho_{i0}} - 1\right)\left(1 - \frac{r_0 \Omega_{e0}^2}{g_0}\right) + \frac{r_0}{g_0}(\Omega_{e0}^2 - \Omega^2) = 0\,, \tag{3}$$

where $v_A \stackrel{\text{def}}{=} B_0/\sqrt{4\pi\rho_{i0}}$ is the Alfvén speed in the equilibrium flux tube, and $\Omega_{e0} \stackrel{\text{def}}{=} \Omega_e(r_0)$. This equation expresses a balance among curvature force, buoyancy force, and rotationally induced forces.

3. Linearization and dispersion relation

Consider perturbations contained in the equatorial plane; it can be shown that perturbations in latitude are decoupled from these. Introduce the unperturbed arc-length s_0 as Lagrangian coordinate ($s_0 = r_0\phi_0$), and call $\boldsymbol{\xi}$ the Lagrangian displacement vector: $\mathbf{r}(s_0, t) = \mathbf{r}_0 + \boldsymbol{\xi}(s_0, t)$. The thin flux tube equations are linearized about the equilibrium configuration, and Fourier components of the form $\boldsymbol{\xi} \sim \exp(i\omega t + im\phi_0)$ are considered. It proves useful to introduce a length unit, $H \stackrel{\text{def}}{=} p_{i0}/(g_0\rho_{i0})$, and a time unit, $\tau = \sqrt{2}\,H/v_A$. The frequency ω and the angular velocities Ω, Ω_{e0} are cast in dimensionless form: $\tilde{\omega} = \tau\omega$, $\tilde{\Omega} = \tau\Omega$, $\tilde{\Omega}_{e0} = \tau\Omega_{e0}$. Also, $f \stackrel{\text{def}}{=} H/r_0$ is a measure of the curvature of the unperturbed flux tube.

The dispersion relation for perturbations within the equator is of the form

$$\tilde{\omega}^4 + d_2\,\tilde{\omega}^2 + d_1\,\tilde{\omega} + d_0 = 0\,. \tag{4}$$

In the case $\beta \stackrel{\text{def}}{=} 8\pi p_{i0}/B_0^2 \gg 1$ (which is a good approximation for the deep parts of the convection zone of cool stars like the Sun), the coefficients of the dispersion relation take a particularly simple form:

$$\begin{aligned} d_2 &= 2(\sigma - 1 - 2m^2)f^2 + \frac{4}{\gamma}f - \frac{2}{\gamma}\left(\frac{1}{\gamma} - \frac{1}{2}\right) + \beta\delta + \\ &\quad +(\sigma - 1)(\tilde{\Omega}_{e0}^2 - \tilde{\Omega}^2) - 4\tilde{\Omega}^2 - 4q\,\tilde{\Omega}_{e0}^2\,, \\ d_1 &= 16mf\left(f - \frac{1}{2\gamma}\right)\tilde{\Omega}\,, \\ d_0 &= -2m^2f^2\left[2(\sigma + 3 - m^2)f^2 - \frac{4}{\gamma}f + \frac{1}{\gamma} + \beta\delta\ + (\sigma - 1)\,(\tilde{\Omega}_{e0}^2 - \tilde{\Omega}^2) - 4q\tilde{\Omega}_{e0}^2\right], \end{aligned}$$

where $q \stackrel{\text{def}}{=} r_0\tilde{\Omega}_e'(r_0)/(2\Omega_{e0})$. The prime denotes the derivative with respect to radius. The parameter $\sigma \stackrel{\text{def}}{=} [d\log g(r)/d\log r]_{r_0}$ expresses the dependence of the acceleration of gravity with depth ($\sigma \simeq -1.82$ for the bottom of the solar convection zone). The superadiabaticity is defined as $\delta = \nabla - \nabla_{ad}$, where $\nabla = [d\log T/d\log p]_{r_0}$ and ∇_{ad} is the corresponding adiabatic value. The dispersion relation (4) differs from that given by van Ballegooijen (1983) [cf. his Eq. (51)] since our treatment is not restricted to small differences between Ω and Ω_{e0}.

4. Stability

A mode is ***unstable*** if $\mathrm{Im}(\omega) < 0$; otherwise, the perturbation does not grow. A discussion of the perturbations perpendicular to the equatorial plane (i.e., in latitude) can be found in Moreno–Insertis *et al.* (1992); these perturbations give rise to the ***poleward slip instability*** (Spruit and van Ballegooijen, 1982).

4.1. Axisymmetric modes: $m = 0$

The dispersion relation is $\tilde{\omega}^2(\tilde{\omega}^2 + d_2) = 0$. The stability criterion is $d_2 < 0$, i.e.,

$$\beta\delta < 2(1-\sigma)f^2 - \frac{4}{\gamma}f + \frac{2}{\gamma}\left(\frac{1}{\gamma} - \frac{1}{2}\right) + (1-\sigma)(\tilde{\Omega}_{e0}^2 - \tilde{\Omega}^2) + 4\tilde{\Omega}^2 + 4q\,\tilde{\Omega}_{e0}^2 \,. \quad (5)$$

4.2. Non-axisymmetric modes: $m \geq 1$

A necessary and sufficient condition for stability is (Ferriz–Mas and Schüssler, 1993):

$$-\frac{4}{27}d_0\left(d_2^2 - 4d_0\right)^2 + d_1^2\left(\frac{d_2^3}{27} - \frac{4}{3}d_0 d_2 + \frac{d_1^2}{4}\right) < 0 \,. \quad (6)$$

Flux tube equilibria with $\Omega \neq \Omega_{e0}$ are possible in the equatorial plane [see Eq. (3)]. A positive difference $\Omega_{e0} - \Omega$ (the matter in the flux tube rotates more slowly than the surrounding matter) exerts a stabilizing influence, while more rapid internal rotation favors instability. Differential rotation exerts a stabilizing effect for $q > 0$ (rotation rate increasing outward), and destabilizing for $q < 0$; it plays a crucial role in determining the stability of non-axisymmetric modes, but not in the stability of axisymmetric modes, for which the term $4\tilde{\Omega}^2$ is dominant (see also van Ballegooijen, 1983; Moreno–Insertis *et al.*, 1992).

As an application of criterion, we show in Fig. 1 the stability diagram on the (B_0, δ)–plane for the modes $m = 1$, using values typical for the bottom of the solar convection zone. The dotted regions indicate stability. We have taken $q = 0.06$ (consistent with helioseismological results) and $\Omega_{e0} = \Omega$. For field strengths higher than $\simeq 1.1 \cdot 10^5$ G, the regions of stable and unstable equilibria are no longer separated by a single dividing line (curve of marginal stability). A second region of stability appears (region II) which extends into the superadibatically stratified part of the convection zone and is connected to the main stability region I by a common boundary point. It should be noted that the "island-like" stability region II is a consequence of having included rotation, and is not merely a consequence of the particular values of the parameters we have chosen. Region II separates regimes of instability due to different mode couplings (Ferriz–Mas and Schüssler, 1993). Its position and form depend on the values of the parameters, especially on q and on $\Omega_{e0} - \Omega$.

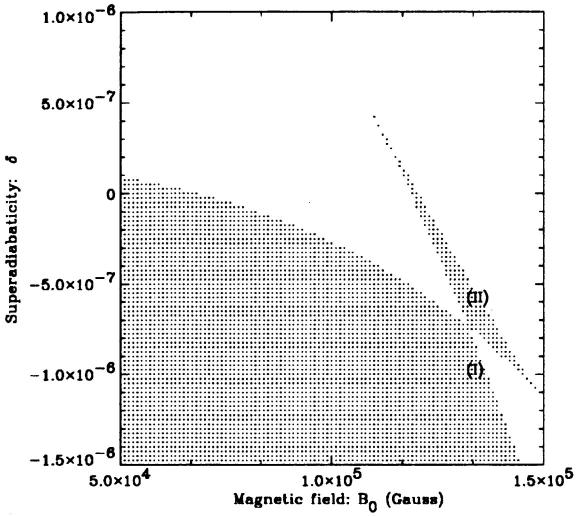

Fig. 1. Stability diagram on the (B_0, δ)–plane for the modes $m = 1$. The regions of stability are marked with dots. The values of the parameters are typical for the bottom of the solar convection zone: $r_0 = 5 \cdot 10^5\,\mathrm{km}$, $g_0 = 519\,\mathrm{ms}^{-2}$, $q = 0.06$, $f = 0.114$, $\sigma = -1.82$ and $\gamma = 5/3$. Here $\Omega = \Omega_{e0} = 2.7 \cdot 10^{-6}\,\mathrm{s}^{-1}$

5. Conclusions. Work in progress

An analytical stability criterion for the non-axisymmetric modes of a flux ring lying in the equatorial plane of a rotating star has been derived. Stratification, differential rotation and different rotation rates between the matter inside and outside the flux tube determine the stability. At this stage, we cannot decide on the physical relevance of the 'island-like' stability region in connection with the possibility of storing strong magnetic fields in the superadiabatically stratified lower part of the convection zone proper. To that end, the case of flux tube equilibria outside the equatorial plane has to be treated; this is the issue of an ongoing investigation.

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