PRESIDENTIAL ADDRESS

THE LAWS OF MOTION*

By G. J. WHITROW

IN presenting his ideas about motion on the third and fourth days of his *Discorsi* of 1638, Galileo said that it was his purpose to "set forth a very new science dealing with a very ancient subject". Despite the remarkable advances that were made in the following centuries, three hundred and fifty years later one of the greatest nineteenth-century physicists, Heinrich Hertz, complained, in the classic introductory chapter to his book *The Principles of Mechanics*, that in expounding the foundations of dynamics in the traditional manner to his students he often felt tempted to apologize and to hurry on as quickly as possible to discuss particular examples which speak for themselves. My purpose in this Address is not a philosophical critique of the type Hertz developed as a preliminary to his reformulation of classical dynamics, but rather to produce what perhaps might be described as a chapter of "applied history of science" in an attempt to show how the *history* of dynamics can be used to illuminate our understanding of the foundations of the subject. If I were to quote a text to introduce this Address, I would choose the following sentence from Jouguet's *Lectures de Mécanique*:

"Il est fort instructif, pour qui veut comprendre la nature des principes et des lois de la Mécanique, d'en suivre l'histoire."[1]

At the dawn of Greek thought we find that the problem of motion was regarded as part of the problem of change in general. To explain change Empedocles introduced the four basic elements that could combine in different proportions, but Leucippus and Democritus had a better idea: the permanent feature in the world was the small invisible atom, and change was due to the chance motions of a large number of such atoms in empty space. An essential feature of their theory seems to have been the hypothesis that atoms are self-moving, that is to say by their nature they are endowed with motion.

This hypothesis was rejected by Plato, who considered that it was atheistic materialism to regard matter as self-moving. He believed matter to be essentially inert and only living organisms and "intelligences" were self-moving. In place of the atomists' concept of blind chance, as the guiding spirit he introduced the Demiurge—the Greek word δημιουργός. latinized as *demiurgus*, literally meaning "a skilled worker", but we may translate it as "Reason"—that produces order out of chaos according to

* Delivered at the Summer Meeting of the Society at Leicester, 3 July 1970.

[1] E. Jouguet, *Lectures de Mécanique* (Paris, 1908), vii.

THE BRITISH JOURNAL FOR THE HISTORY OF SCIENCE VOL. 5 NO. 19 (1971)

a preconceived plan. In Plato's philosophy, the objects that we perceive are imperfect imitations of truly real objects that can be perceived only by the mind. The Platonic idea of two worlds was one of the most important contributions to thought ever made. It survives today in the distinction we make between the world of our empirical experience and the world of our theoretical constructs. There was, unfortunately, a fundamental restriction imposed by Plato on his world of real objects that greatly delayed the successful development of dynamics. In his view, the essential feature distinguishing the world of reality and the world of appearance was that change is confined to the latter. Real objects are eternal and not subject to change—they are by nature *timeless*—and change occurs only in the visible world because this world is merely a moving image of ultimate reality which is static. It required the genius of that latter-day Platonist Galileo to extend the Platonic world of abstract theory to cover dynamics in the same way as geometry.

Plato's influence was also instrumental in delaying the successful development of dynamics in another respect. He drew a sharp distinction between circular and rectilinear motion, and regarded the former as superior, or more perfect, than the latter. He retained the sphere of Parmenides as the appropriate geometrical figure for the shape of the world but he made it rotate. He argued that the sphere is not only the most uniform of all solid figures but it is the only one which by rotating on its axis can move within its own limits without change of place (cf. Euclid, *Elements*, Book XI, Definition 14—where the sphere is defined not in terms of distance from the centre but in terms of the rotation of a semicircle about its diameter). This axial rotation, according to Plato, represented the motion of Reason, and was sharply distinguished from rectilinear motion. Motion that involves change of place was imperfect and not akin to reason. Moreover, in a finite universe, such motion, unlike circular motion, cannot continue for ever without interruption.

Plato's pupil Aristotle was the first to make a *detailed* inquiry into the concept of motion. On the whole, his views seem to have made little impact on later thinkers in antiquity but were of the greatest importance in the Middle Ages from about the fourteenth century onwards. It is no exaggeration to say that modern dynamics emerged from the criticism of his ideas. More empirically minded than Plato, for him the general object of scientific enquiry was not the transcendental world of abstract thought but the physical world about us. Paradoxically, this led to the greatest stumbling-block he placed in the path of knowledge—a stumbling-block that continued to exert its influence long after the seventeenth century and no doubt still has some effect today. For he rejected root and branch the idea that the order of nature is mathematical in character. Now, whether God is or is not a geometer is not the point at issue. The point is this: How can we best devise a powerful science of motion? Without

https://doi.org/10.1017/S0007087400011201 Published online by Cambridge University Press

mathematics this is impossible. Unfortunately, Aristotle was a logician, and logicians have always had a baleful influence on science. He defined mathematics as the science of changeless things and of the unchanging aspects of changing things, and physics as the science of things that change. Consequently, he concluded that physics, by definition, cannot be applied mathematics. Thus, in their different ways, both Plato and Aristotle erected barriers in the path of knowledge that severely hampered the study of motion.

Nevertheless, Aristotle must be recognized as one of the great pioneers of the study of motion. In his *Physica*, *De Caelo*, *De Motu Animalium*, etc., we find a detailed discussion, from the standpoint of his cosmology, of what we may regard as the common-sense origins of the subject. One of these is the idea that motion is the opposite of rest, and that a moving body is essentially different from a body that is stationary. I need hardly remind you what an obstacle this was to prove for the development of dynamics.

Another difficulty under which Aristotle laboured was the inadequacy of his kinematic concepts, let alone the vagueness of his dynamical ideas such as "force". Aristotle had no idea of velocity as an instantaneous magnitude in its own right, different from both distance and time. Indeed, even the idea of velocity as a simple ratio of space and time was foreign to Greek thought, which restricted the concept of ratio to quantities of the same kind (or, as we now say, of the same physical dimensions). *A fortiori*, the idea of velocity as the instantaneous limit of such a ratio was quite beyond the resources of Greek thought. In place of velocity, Aristotle usually referred to the time required to describe a given distance. He distinguished between uniform and non-uniform motion, and he had two definitions of the quicker of two bodies: as that which traverses the same distance in the shorter time, or as that which traverses the greater distance in the same time. These alternatives led to great confusion before the modern concept of acceleration could be defined: Which was to be taken as the independent variable, distance or time?

In the seminal five hundred years (*c.* 1100-1600 A.D.) preceding the scientific revolution of the seventeenth century the central problem of the methodology of science was the role of mathematics and its application to the analysis of experimental results. Not only as regards the use of mathematics, but even as regards the experimental method, Aristotle's influence tended in the course of time to become a hindrance rather than a help. In the medieval universities physics became the *logical* interpretation of Aristotle's treatises. It was not the triumph of mathematics over empirical investigation but of mathematics over "logic" that was necessary for the advance of dynamics and other branches of physics.

The first requirement was to develop an adequate kinematic formulation of the concept of motion. This was basically the great achievement—

https://doi.org/10.1017/S0007087400011201 Published online by Cambridge University Press

at least as regards the initial stages—of the Merton school of the early fourteenth century and of Nicole Oresme in the latter part of that century.

The oldest kinematic treatise of the Latin West known to us is the *Liber de Motu* of Gerard of Brussels (first half of the thirteenth century). He did not define velocity as a ratio of unlike quantities but assumed that the speed of a motion can be assigned some number or quantity that is neither distance nor time. His mathematics was very poor, but it influenced the Merton school in the following century, stimulating them to study the kinematics of non-uniform, or accelerated, motion. In groping their way towards this concept, which was extremely difficult to formulate before the invention of the differential calculus, the schoolmen of the fourteenth century were greatly handicapped by their lack of algebraic symbolism.

Before the kinematic concept of acceleration could be correctly formulated, two other ideas were necessary:

(i) the idea of time as the *independent*, and of space as the *dependent*, variable;

(ii) the idea of instantaneous velocity.

The general mathematical notion of the variable was gradually developed by the post-1277 critics of Aristotle. Aristotle, as I have already said, made a rigid distinction between mathematics and physics. This was gradually abandoned. In particular, motion had been regarded by Aristotle as a *quality* (distinct from rest) and *not a quantity*. Duns Scotus considered the idea of the variability of qualities, i.e. their quantitative variation, but the pioneer figure in the *mathematical* development of the idea of the variable was Thomas of Bradwardine in his *Tractatus de Proportionibus* (1328). As his modern editor remarks: "Bradwardine used mathematics for the systematic and general expression of theory, Galileo used it for the systematic generalization of experimental observation".[2] Bradwardine's work is noteworthy for introducing into mathematics functions more complex than simple linear proportionality.

The question of the extent to which the fourteenth-century scholastic investigations of kinematics may have influenced Galileo is a contentious one. Mach's claim, that before Galileo could propound his theory of falling bodies he had to invent the very idea of acceleration, was certainly too extreme. On the other hand, the claim, stemming from Duhem, that scholastic ideas could have exerted a significant influence on Galileo has recently been attacked by Leon Rosenfeld. For representing a magnitude of any kind by a linear segment, the schoolmen had Aristotle's authority; the innovation Oresme introduced consisted in disposing the segments which represented the intensions (dependent variable) orthogonally to the line which represented the extensions (independent variable).

[2] H. Lamar Crosby, Jr., *Thomas of Bradwardine His Tractatus de Proportionibus: its Significance or the Development of Mathematical Physics* (Madison, Wisconsin, 1955), 17.

https://doi.org/10.1017/S0007087400011201 Published online by Cambridge University Press

Rosenfeld discusses the significance of Oresme's application of his method to uniformly accelerated motion and the speculation that Galileo could have been helped by Oresme's work when he studied falling bodies. Rosenfeld writes:

"Oresme's solution proceeds smoothly enough: the extension being the time and the intension the linearly increasing velocity, he realizes that the space is given by the area of the resulting triangular configuration and he thus finds for it the correct quadratic law. It does not occur to him, however, that this type of motion could have anything to do with the fall of material bodies. On the other hand, if Galileo, when he had found the law of spaces by experiment, had known, or remembered, Oresme's formal result, he would not, in his attempt to derive by mathematical reasoning the corresponding law of velocities, have made the false start of assuming the velocity to be proportional to space instead of time. Too much is made of the similarity between Galileo's ultimate formulation of the law of accelerated motion in the *Discorsi* and that given by Oresme: both state that the accelerated body will travel through the same space in a given time as it would in uniform motion with half the final velocity. This just corresponds to the statement that the area of a right triangle is the same as that of a rectangle of the same base and half the height—a simple thumb-rule familiar, no doubt, to every surveyor, but which our medievalists pompously call the 'Merton Theorem' as it happened to be a popular subject of verbal exercise in Merton College."[3]

There is, in my view, much sense in this criticism, but it overlooks or underrates the achievements of Galileo's medieval predecessors in (i) recognizing that time is the independent variable, (ii) introducing the idea of instantaneous velocity. As Clagett has written apropos of the second point, "It is this concept which differentiates decisively the Merton analysis of acceleration from all preceding treatments of the problem."[4] Of course, for the ultimate treatment we require the calculus developed by Newton and Leibniz, but they built on the gropings of their predecessors. We should note that the fourteenth-century terms *fluens* and *fluxus* were destined to be used by Newton when he spoke of a variable as a "fluent" and its rate of change as a "fluxion".

In addition to the ideas of instantaneous velocity and acceleration, the development of modern dynamics would have been impossible without a fundamental change in the actual concept of motion: the abandonment of the idea that it is absolute and the consequent development of the idea that it is relational in nature. This development began in the fourteenth century. Duns Scotus maintained that motion was a *forma fluens*, a continual flow that cannot be divided into successive states (a Bergsonian conception), whereas Gregory of Rimini argued that it was a *fluxus formae*, or "flux of form", a continuous series of distinguishable states. Gregory said that during motion the moving body acquired from instant to instant a series of distinct attributes of place. In this view he seems to have been influenced

[3] L. Rosenfeld, *Nature*, ccxxii (1969), 197.

[4] Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wisconsin; London, Oxford University Press, 1959), 261.

by William of Ockham, who denied that motion was due to the real existence of some flowing form in the moving body. Instead, it was sufficient to regard the moving body as having at different instants different spatial relationships to some other body. This idea, *that motion is a relation and not a quality*, was also adopted by Nicholas of Autrecourt (*c.* 1350). His definition of motion has been expressed by Weinberg in the form: "*x* is moved" means "*x* is at *a* at time *t* and separated from *b*; *x* is at *b* at time *t*' and separated from *a*".

The idea that motion is a relation rather than a quality is a necessary presupposition of the law of inertia. A second crucial step towards this law was the development of the idea of the relativity of the perception of motion. However, the need for this did *not* first arise in the context of uniform *rectilinear* motion but of uniform *rotational* motion, in the writings of Jean Buridan and Nicole Oresme. They discussed the idea that the Earth rotates on its polar axis, and both stressed the doctrine of the relativity of the perception of motion. Both illustrated the doctrine by positing observers on ships moving relatively to each other (this is linear motion, of course), noting that neither observer would detect his own motion (assuming a calm sea) but would describe the motion of the other ship in terms of his own ship being at rest. Both held that because of the relativity of the detection of motion all astronomical phenomena could just as well be accounted for by assuming terrestrial diurnal motion as celestial.

Both Buridan and Oresme advanced the argument of simplicity (Ockham's razor), the Earth's daily rotation being, in a sense, "simpler" than the daily rotation of the heavens. More important is the fact that Oresme specifically introduced the idea of a *closed mechanical system*: due to the relativity of the perception of motion, the observer describes all motions as if they were a part of his system only. This idea was used by Oresme to explain the apparent rectilinear motion of objects falling to the Earth. The analogy was again with a man on the ship. As Oresme showed, it would appear to this observer on the ship that his hand was descending in rectilinear motion if he made it slide down the mast:

> "The arrow is trajected upwards and (simultaneously) with this trajection it is moved eastward very swiftly with the air through which it passes and with all the lower mass of the universe . . . , it all being moved with diurnal movement. For this reason, the arrow returns to the place on earth from which it left. This appears possible by analogy: if a person were on a ship moving towards the east very swiftly without his being aware of the movement and he drew his hand downward, describing a straight line against the mast of the ship, it would seem to him that his hand moved with rectilinear movement only."[5]

Incidentally, Gassendi in a series of experiments in the harbour of Marseilles showed that a stone or bullet dropped from a masthead will

[5] *Ibid.*, 587.

https://doi.org/10.1017/S0007087400011201 Published online by Cambridge University Press

fall at the foot and not lag behind. He was apparently the first to *publish* such an experiment, in 1641.

Oresme also appears to have suggested that the basic reason for the phenomena of the closed mechanical system is that all objects therein share the same horizontal velocity, e.g. all objects aboard a ship. The idea of the *rotation* of the Earth was discussed by many writers on astronomy before Buridan and Oresme, including, according to Alberuni, Brahmagupta, followers of Âryabhaṭa, and also some later Islamic writers. They considered, too, the familiar counter-arguments, e.g. birds not being able to return to their nests if they flew west above a rotating Earth. Nevertheless, Oresme's discussion is more significant because of the idea he seems to have grasped of a closed mechanical system.

Oresme in another passage rejected the diurnal motion of the Earth on scriptural grounds. An important step beyond Oresme was taken by Copernicus, who attributed not one but three motions to the Earth (rotation, revolution and precession) and was even more committed to the idea of the relativity of our perception of motion. In attributing to the Earth the circular motion that was thought natural to the heavens and at the same time being unable to deny that rectilinear motion (fall) was natural on and near the Earth, he was forced to abandon the rigid distinction between celestial and terrestrial physics which had for so long been the fundamental feature of the Aristotelian cosmos. He had to modify Aristotelian physics so as to allow rectilinear motion and circular motion to co-exist in the same body, so that the Earth might rotate, while objects on it moved relative to it in straight lines.

The argument that bodies on Earth participate in its motion in the same way as bodies on a ship participate in its motion was advanced and sharpened by Giordano Bruno. In his *Cena de la Ceneri* (1584) he refuted the Aristotelian idea that a stone thrown up will not fall to its former place if the Earth rotates. He imagined two men, one on a moving boat and the other on the bank, dropping from the same line a stone on to the boat. He argued that the stone dropped by the second will fall behind that dropped by the first, since the stone dropped by the man on the boat will already have a certain horizontal *virtus impressa*, so that to the man on the boat it will appear to fall straight down. Marshall Clagett has commented on Bruno's ideas: "With Bruno the concept of the closed mechanical system had emerged more clearly than with Oresme and Buridan, but that we are dealing with basically the same argument cannot be doubted."[6]

The concept was illustrated by Galileo in a beautiful passage in the Second Day of his *Dialogue concerning the Two Chief World Systems* (1632):

"Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small

[6] *Ibid.*, 666.

https://doi.org/10.1017/S0007087400011201 Published online by Cambridge University Press

flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to a friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt when the ship is standing still everything must happen this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still . . . . The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also. That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted."[7]

Nevertheless, the idea that relativity of motion applies essentially to uniform *linear* motion was not clear to Galileo, whose thought was still dominated by circular motion. His Venetian predecessor G. B. Benedetti (1530-90), who foreshadowed the inertial concept more clearly than any of Galileo's other predecessors, drew attention to the fact that when a body is freed from a sling it moves automatically along the tangent to its circle of revolution. (Galileo mentions the same phenomenon.) Nevertheless, and this is an excellent example of the difficulties that confronted the founders of dynamics, Benedetti got thoroughly confused over the distinctions that we now know must be made between conservation of linear momentum and conservation of angular momentum. Noting that a piece of a spinning wheel, if suddenly detached, will fly along the tangent, he *rejected* the correct view of Buridan, Albert of Saxony, and Nicholas of Cusa that a wheel set spinning horizontally about its centre can rotate indefinitely. (Air resistance, etc., must be neglected, of course.)

The principle of inertia, contrary to the idea often suggested in textbooks, was not formulated by Galileo nor first propounded by Newton. Instead, we first find it emerging, in the early 1640's, in the writings of Gassendi and Descartes. (The medieval concept of "impetus"—even if it foreshadowed the concept of linear momentum—was actually a stumbling-block, since it buttressed the belief that motion is utterly distinct from the rest.) Gassendi freed himself from "the spell of circularity" that haunted Galileo. Galileo thought of a perfectly smooth ball on the *idealized* smooth surface of the Earth, and argued that it would move uniformly for ever because it would always be in the same relation to the centre. *Galileo was unable, however, to make the abstraction required of abolishing gravity*, because he believed that gravity was innate in a body itself.

[7] Galileo Galileo, *Dialogue concerning the two Chief World Systems—Ptolemaic and Copernican*, trs. Stillman Drake (Berkeley and Los Angeles, 1953), 186-187.

Gassendi, on the other hand, because he had the idea of gravity as an attraction could make it. He abolished, mentally, the attraction of other bodies and let the body move in a void, in which, of course, there was no centre. Hence, he could replace uniform horizontal motion by uniform rectilinear motion *in any direction*. Nevertheless, he still fell short of the law of inertia because he regarded rest as essentially distinct from motion, and this traditional polarity had to be abolished before the law could be properly formulated.

This essential step was taken by Descartes, who also appears to have been the first to introduce the term "laws of motion". The step taken by Descartes involved an entirely new concept of motion. The essential point was that uniform rectilinear motion was no longer regarded as a process requiring an agent but as a state, just as rest is a state. As Koyré says,

> "Motion and rest are . . . placed by this word on the same level of being, and no longer on different ones, as they still were for Kepler, who compared them to darkness and light, *tenebrae et lux* . . . It is only motion as *state* that does not need a cause or mover. Now, not all motion is such a *state*, but only that which proceeds uniformly and in a right-line, *in directum*, that is, in the same direction and with the same speed."[8]

Incidentally, it is significant that the introduction of the law of inertia came at a time when the old cosmology of a closed universe was being replaced by that of a universe in infinite space in which a straight line can continue for ever.

Descartes introduced his idea of motion as a state in his unpublished *Le Monde* of 1630. A body in curvilinear or accelerated motion changes its *state* every instant as it changes its direction and speed. Nevertheless, it is at every instant *in statu movendi uniformiter in directum*, but in practice the actual motion of a body never continues like this. It is only its "inclination", or *conatus*, to move in this way.

The law was also stated by Descartes in his *Principia Philosophiae* of 1644 in the following manner:

"1. Any particular thing, in so far as it is simple and undivided, remains always to the best of its ability in the same state, nor is ever changed [from this state] unless by external causes.
2. If [a body] is at rest we do not believe it is ever set in motion unless impelled thereto by some [external] cause. Nor is there any more reason if it is moved, why we should think that it would ever of its own accord, and unimpeded by anything else, interrupt this motion.
3. Any particular part of matter, regarded by itself, is never inclined to prosecute motion in any curved lines but only in straight lines."[9]

How did Descartes arrive at this law which was destined to be the means whereby the theory of motion would at last be placed on a firm

[8] A. Koyré, *Newtonian Studies* (London, 1965), 66-67.
[9] J. W. Herivel, *The Background to Newton's Principia* (Oxford, 1965), 44-45.

foundation? I believe that he was influenced by his conception of the nature of time. He believed that time is composed of time-atoms and that the universe could exist for only one of them were it not for the continual intervention of God. In his view, since a self-conserving being requires nothing but itself to exist, self-conservation must be the unique prerogative of God. He therefore argued that a material body has the property of spatial extension but no inherent capacity for temporal endurance, and that God by his continual action recreates the body at each successive instant. (He regarded continual creation and conservation as synonymous.) A body that endures can either be in rest or in motion, and a particular state of motion of a body can either endure or last only an instant. If the body is in accelerated or curvilinear motion, then its state of motion changes from instant to instant, but nevertheless at each instant the body is in a state of uniform motion in a given direction. Unless something external to the body intervenes to affect its tendency to move uniformly in this direction, this state of motion, like the body itself, will be conserved from instant to instant and hence the body will persevere in a state of uniform motion in the same straight line for all time.

Newton owed his First Law of Motion to Descartes, although he never admitted this, and to Galileo the law of independent motions and composition of motions, i.e. the parallelogram of velocities. As for the formal enunciation of the basic principles of dynamics, so far from completing it, he began it, as Truesdell has repeatedly emphasized. The First Law is far from precisely stated in the *Principia*. In Motte's translation it runs: "Every body perseveres in its state of rest or uniform motion in a right line unless it is compelled to change that state by forces impressed thereon." What does Newton mean by "body"? Certainly, it would seem, not a rigid body, because such a body under no forces can precess like a top. The law as stated only applies to that abstract entity, the Newtonian particle. But, if we examine Newton's comments immediately after his statement of the law, we find him first talking about projectiles, in so far as they are not retarded by the air or impelled downwards by the force of gravity, and then, more surprisingly, he actually goes on to talk about a spinning top, as if there were still some confusion in his mind between rectilinear inertia and rotational inertia. Moreover, he uses the term *vis inertiae*, in Definition III, suggesting still the idea of a motor impetus, or "force", that has to be overcome before a body can change its state of motion. Finally, Newton's law of inertia needs to be supplemented by Corollary V to his Laws of Motion, where he states what we now call the Newtonian-Galilean Principle of Relativity in the form: "The motions of bodies enclosed in a given space are the same relatively to each other whether the space is at rest or moving uniformly in a straight line without any circular motion." In his comments on this Corollary, Newton refers to the experiment of a ship—where all the motions happen after the same manner, whether the ship is at rest or is carried uniformly forwards in a straight line.

Turning to the Second Law, we find that Newton does not state it in the form that is now customary but refers to the *change*, not the rate of change, of motion, i.e. of what we now call linear momentum. He says: "The change of motion is proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed." In fact, Newton's Second Law is not a law connecting force (in the modern sense) and acceleration, but a law relating impulse and change of momentum traceable to the laws of impact which had been studied not by Galileo but by Newton's contemporaries. The action of a continuous force, such as gravity, was visualized as the limiting case of a succession of discrete jerks when the number in a given interval of time becomes infinite.

The Third Law, concerning the equality of action and reaction, is somewhat obscurely discussed by Newton. He cites the case of a single body and refers to the gravitational attractions between one part and the rest. If these were not equal and opposite then there would be a net force on the body and it would be accelerated without the intervention of an external force impressed upon it, in contradiction with the First Law of Motion. So far, the Third Law would seem to be a consequence of the First. But we also require this law when we consider the interactions between two (or more) distinct bodies. The First Law can tell us nothing about these, and so Newton falls back on empirical considerations, i.e. the experiments in impact of bodies by Wallis, Wren and Huygens. Incidentally, he makes no mention of the principle of conservation of linear momentum.

Following Newton, a long time elapsed before the formulation of the equations $F_x = M\ddot{x}$, etc., connecting force, mass and acceleration. These equations were first stated in their full generality by Euler in a paper in 1752 entitled "Discovery of a new principle in mechanics".[10] Euler recognized that in these differential equations the mass M may be either finite or infinitesimal and that they apply to every part of a dynamical system. As the greatest living authority on Euler's applied mathematics, Clifford Truesdell, has commented:

"The modern student may find it hard to understand how sixty years of experience with special cases had to follow before this simple conclusion, which he is taught to accept unquestioningly in a first course in physics, was seen. It is unlikely, however, that he has a better grasp of mechanical principles than Newton or Euler had. No-one doubted the correctness of 'Newton's second law', at least as a rule for problem-solving, but what no-one saw, until it was shown, was that among all the various mechanical principles then used it was this one

[10] L. Euler, "Découverte d'un nouveau principe de mécanique", *Mem. Acad. Sci. Berlin*, vi (1752), 185-217.

which was *general: It applies to every part of every system*, and more than this, *it suffices to get all the equations* determining the motion of many systems."[11]

The value of the new principle was evident in the paper in which it appeared, for in it Euler succeeded in obtaining the general equations of motion of a rigid body about its centre of mass known today as Euler's dynamical equations for a rigid body acted on by an external couple. In this paper, too, first appeared what we now call the angular velocity vector and the six components of what we now call the inertia tensor. (The idea of "moment of inertia" had been known to Huygens, although not to Galileo when rolling balls down an inclined plane, but it was first named by Euler.) Euler showed that inertia, in the sense of a body's intrinsic resistance to change of motion, was determined by the inertia tensor. Thus, as Truesdell remarks,

> "the distinction of mass from weight, begun by Newton, was completed, and at the same time inertia and mass, confused in the older works, were separated. The position of 'Newton's second law' was fixed as appropriate only to infinitely small bodies or to the centres of mass of finite bodies."[12]

Nevertheless, Euler was wrong in asserting in his memoir of 1752 that his new principle "contains by itself all the principles that can lead to the knowledge of the motion of all bodies, of what nature soever they be". Another quarter of a century passed before Euler found, near the end of his life, that to obtain the equations of motion of a rigid body directly and easily *two* principles were required. This result he arrived at in 1776 when he stated that the following laws are applicable to every part of every body, whether rigid or deformable:

Law 1. The total force acting on a body is equal to the rate of change of its total linear momentum.
Law 2. The total torque, or couple, acting on the body is equal to the rate of change of its angular momentum, provided that both the torque and the angular momentum are taken with respect to the same fixed point or the centre of mass.

Truesdell has suggested that these laws should be called *Euler's Laws of Mechanics*. The fact that there are *two* is associated with the theorem that every automorphism of Euclidean space may be decomposed into a translation and a rotation.

The Newtonian-Eulerian science of dynamics, nowadays often called *vectorial dynamics*, is based on the ideas of "force" and momentum as primary concepts. Newton's contemporary, Leibniz, was the originator of an alternative development called *analytical dynamics*. The relation of this to early classical dynamics is rather like that of analytical geometry to

[11] C. Truesdell, *Essays in the History of Mechanics*, Berlin (Heidelberg and New York, 1968), 116.

[12] *Ibid.*, 118.

https://doi.org/10.1017/S0007087400011201 Published online by Cambridge University Press

Euclid's geometry. Each problem in early classical dynamics had to be solved in its own way, whereas the motive-force behind the development of analytical dynamics was the search for general principles applicable to a wide class of problems. The oldest of such principles—the conservation of *vis viva*—goes back to Leibniz, although it was related to the previous rejection by Simon Stevin and others of artificial perpetual motion. In the nineteenth century this train of thought culminated in the law of the conservation of energy, still called by Helmholtz the conservation of "force". It is well known that the starting-point for Leibniz's reflections was his rejection of Descartes's idea of the conservation of motion, not to be confused with the later principle of the conservation of momentum, for Descartes's idea was a scalar and not a vector. The principle of the conservation of energy in dynamics is confined to material systems acted on by conservative forces. The extension of the principles to cover all natural processes was not effected until about 1848 by Mayer and Helmholtz, who approached the problem from the standpoint of physiology, and by Joule, from the standpoint of physics.

Other conservation laws applying to closed mechanical systems are the laws of conservation of linear momentum and angular momentum. These were formulated in the course of the eighteenth century.[13] Another great generalization that was first developed at about the same time was the Principle of Least Action, by Euler and Maupertuis. Unlike Maupertuis, Euler realized that both the actual and the varied motion considered in connection with this principle must satisfy the law of conservation of *vis viva*. Others who introduced key ideas into analytical dynamics were Laplace and Lagrange. Laplace introduced the potential function, and Lagrange his famous equations involving the energy function expressed in terms of generalized co-ordinates. Lagrange's *Mécanique Analytique*, which first appeared in 1788 on the eve of the French Revolution, came almost exactly one hundred years after Newton's *Principia*. It laid the foundations of modern analytical dynamics and introduced the idea of invariance under transformation of co-ordinates.

Lagrange's work stands in marked contrast to Newton's and Euler's.[14] Just as Laplace is alleged to have claimed, when quizzed by Napoleon, that in his celestial mechanics he had no need to appeal to the hypothesis of the Deity, so Lagrange in the preface to his great treatise wrote: "The reader will find no figures in this work. The methods which I set forth do not require either constructions or mechanical reasonings; but only algebraic operations, subject to a regular and uniform procedure." Here

[13] *Ibid.*, 239 *et seq.*

[14] Truesdell's playing down of Lagrange as compared with Euler, cf. his *Essays in the History of Mechanics*, 133-135, reflects his own interests. Euler's work in mechanics is of great value for the classical applied mathematician interested in fluid mechanics, etc., but Lagrange's work is of more value to the theoretical physicist interested in relativity, quantum theory, particle physics, etc.

I have time only to make the briefest of comments on Lagrange's achievement, but I must say something about his debt to d'Alembert, co-editor with Diderot of the famous *Encyclopédie*.

In his *Traité de Dynamique*, first published in 1743, d'Alembert tried to recast Newtonian dynamics in the general spirit of Cartesian rationalism with its quest for certainty and consequent desire to take mathematics, without appeal to experiment, as the pattern of the sciences. D'Alembert's ambition was to rid Newtonian dynamics of the dubious concept of "force", which he regarded as "obscure and metaphysical". The attack on forces was begun by Malebranche because they appeared to him to imply a kind of "soul" or active principle in inert matter. Controversy raged for years over the correct definition of "force" (*vis viva* or momentum), but for d'Alembert this was "a dispute about words too undignified to occupy the philosophers any longer".[15] (That was, of course, long before the days of our linguistic philosophers!) However, his use of the method of virtual velocities in dynamics introduced quantities as unobservable as the forces he wished to discard. The principle of virtual velocities, nowadays replaced by the principle of virtual work, was first explicitly formulated by Johannes Bernoulli about 1717 and originated in statics.

D'Alembert's treatise, which is one of the most difficult to read in the history of the subject, was based on the law of inertia, the parallelogram addition-law of velocities, a law similar to that of conservation of momentum, and his famous Principle—which in effect reduces dynamical problems to quasi-statical problems. In his formulation it differs somewhat from that stated in modern textbooks. The great merit of d'Alembert's treatise was that it gave a *general* principle for the solution of a large class of dynamical problems. As the editor of its most recent edition has remarked, "It may not have reduced Mechanics to a 'simple game' as d'Alembert claimed, but it did at least provide a simple method."[16]

D'Alembert's Principle is the fundamental principle of analytical dynamics. It makes possible the use of moving reference systems and can be regarded as a forerunner not only of Lagrange's theory but also of Einstein's ideas concerning relativity. It depends on d'Alembert's simple but brilliant idea of the "force of inertia", defined as the negative of the product of mass and acceleration. The Principle states that any system of forces acting on an arbitrary mechanical system is in equilibrium if we add to the impressed forces the forces of inertia. Consequently, the total virtual work of all these forces vanishes for small reversible displacements satisfying the given kinematical conditions or constraints.

Professor Lanczos has argued that those scientists who claim that analytical dynamics is nothing but a mathematically different formulation

[15] J. d'Alembert, *Traité de Dynamique* (2nd ed., Paris, 1758, ed. T. L. Hankins, New York and London, 1968), "Discours Preliminaire", xxii.

[16] *Ibid.*, Introduction, pp. xxxiii-xxxiv.

of Newton's laws of motion must assume that from these laws we can deduce the principle of virtual work, which applies to dynamics by virtue of d'Alembert's Principle. As Lanczos says, Newton's third law of motion is not wide enough and the Principle must be regarded as an addition to his laws of dynamics.[17]

Newton's point of view as regards his laws of motion was different from ours. In analysing the motion of a single body under a continuing dynamical effect, he visualized a sequence of blows and, as I have already mentioned, he regarded the continuous case as the limiting form of this. Taking his laws in reverse order, the Third Law was a way of isolating an individual body from the rest of a system, the Second Law was concerned with the alteration in motion when a blow was applied, and the First Law was concerned with what happens when no blow is applied. From Newton's point of view, the First Law was required as an addition to the Second Law and was not just a special case of it. Thus to determine motion, two vectors had to be added, one from the First Law and one from the Second.

Nowadays, we look on "Newton's Laws" (so-called) quite differently. First of all we begin with the concept of *inertial frames of reference*, the term being first introduced in the nineteenth century by James Thomson, brother of William Thomson better known as Lord Kelvin. The function of the First Law is to define the frames of reference in terms of which the Second Law is valid. (As far as Newton was concerned, there was no need for this, because he had recourse to his concept of absolute space. In the Third Book of the *Principia* he tries, in a rather underhand way, to "con" the reader into imagining that he is *not* invoking a hypothesis, but in fact he *postulates* that the centre of the mass of the solar system is to be regarded as what he calls "the centre of the world", so as to have an identifiable point in this space.)

The modern foundation for dynamics is the Principle of Relativity. The term is due to Henri Poincaré, who introduced it early this century just before Einstein's great paper of 1905, in the title of which there is no mention of it. The Newtonian-Galilean Principle of Relativity asserts that the laws of mechanics are expressible in an invariant form with reference to all inertial frames. As far as dynamics is concerned, this principle is a special case of Einstein's Principle of Special Relativity with the velocity-of-light parameter c taken to be formally infinite. (Einstein's Principle, of course, also covers other branches of physics besides dynamics, but has to be generalized to cover gravitation.) Einstein's Principle of Special Relativity leads to important consequences for the dynamics of high-energy particles and rapidly moving bodies, the most celebrated being the replacement of the independent classical laws of conservation of mass and

[17] C. Lanczos, *The Variational Principles of Mechanics* (Toronto, 1949), 77 (footnote).

of energy by a single law that covers both, encapsulated in the magic formula $E = Mc^2$.

Due primarily to the influence of Einstein, the modern point of view stresses the significance of the invariants of motion under laws of transformation from one frame of reference, or observer, to another and also of the laws of conservation. In recent years, particularly in connection with theoretical developments in particle physics, conservation laws have been associated with principles of symmetry. This association was due originally to one of the two great contributors to the development of dynamics in the first half of the nineteenth century, C. G. J. Jacobi. (The other, Sir William Rowan Hamilton, made contributions of the greatest importance to the canonical formulation of analytical dynamics, although the full power and significance of his work only became apparent this century with the rise of quantum and particle physics.)

Jacobi showed that the canonical formalism of Lagrange-invariance under displacements of time, position and angle gives rise, respectively, to the conservation of energy, linear and angular momentum.[18] In other words, *for a closed mechanical system*, the homogeneity of time leads to the conservation of energy, the homogeneity of space to the conservation of linear momentum, and the isotropy of space to the conservation of angular momentum. Hence, the conservation laws of dynamics are found to be interdependent with certain basic cosmological assumptions.

Jacobi's remarkable discovery seems to have excited little interest in his day and to have remained virtually unknown, and certainly unappreciated, until about fifteen years ago when attention was drawn to it by theoretical workers in particle physics. Meanwhile, owing to the growth of interest in cosmology, stemming from Eistein's paper of 1917 on the cosmological consequences of general relativity and the discovery of the expansion of the universe by Hubble in 1929, attempts have been made to relate the laws of motion to the properties of various world-models. A pioneer in this field was the late E. A. Milne, who followed Ernst Mach in regarding inertia not as an intrinsic property of each particle of matter but rather as a relation between each particle and the whole system of masses of the universe. In the context of a particular world-model which he devised for studying the expansion of the universe, Milne proved that with suitable conventions of measurement the motion of a "free particle" (test-particle) is uniform.[19] This suggested that inertial frames of reference may be defined by the free motions in the smoothed-out universe, i.e. the background of the universe obtained by smoothing-out all local irregularities in the distribution of matter. In 1953, D. W. Sciama sought to explain inertial mass as an inductive effect of distant

[18] C. G. J. Jacobi, *Vorlesungen über Dynamik*, ed. A. Clebsch (Berlin, 1866), Lectures 21, 23 and 24.

[19] E. A. Milne, *Kinematic Relativity* (Oxford, 1948), 75.

matter in the universe and found that by using a somewhat different world-model from Milne's he could derive Newtonian equations of motion for two special cases—rectilinear motion and uniform rotation.[20]

The hypothesis that there is some causal relationship between the inertial properties of matter and the basic structure of the universe is nowadays called "Mach's Principle". This term was originally coined by Einstein when formulating his general theory of relativity about 1915, but he used it to denote his postulate of the interdependence of the geometry of space-time and the distribution of matter and energy. But Mach's Principle in the sense in which the term is now used was not shown by Einstein to be a feature of his theory, and whether this is so is still an open question.

One consequence of Mach's Principle is that a rotating mass should drag along the inertial system with it relative to the other masses in the universe. In 1918 H. Thirring, using an approximate form of Einstein's theory valid only for weak gravitational fields, found that a slowly rotating mass-shell, producing a weak gravitational field, drags along the inertial frames within it. This still left open the question of what would happen in the case of a shell whose mass is not small compared with that of the rest of the universe. Would such a shell have a *large* effect on an inertial system within it? In 1959 R. H. Dicke argued that for Mach's Principle to be satisfied this must be so, and since then he has developed a modification of general relativity (the scalar-tensor theory) to make sure that this result is obtained. However, in 1965 D. R. Brill and J. M. Cohen used general relativity to prove that as the mass of the shell becomes larger and larger the induced rate of rotation of the inertial frames within it comes closer and closer to the rotation of the shell.[21]

The general validity of Mach's Principle is, however, still in dispute, but it is noteworthy that many investigations made in connection with it concern rotation. The peculiar character of rotational motion has been recognized since Newton's famous bucket experiment, from which he deduced the absolute nature of uniform rotation in contrast to the relative nature of uniform rectilinear motion. This conclusion is generally accepted. In other words, if we inhabited a planet such as Venus surrounded by clouds, we believe that with the aid of a Foucault pendulum we could decide whether or not to regard our world as rotating, and that if the clouds were to evaporate we should find that the results of our experiments would agree with our observations of the fixed stars.

More than forty years ago the late P. W. Bridgman suggested that a physical basis for this difference between rotation and translation might be found in the enormously different translational and rotational displacements attainable in practice. Whereas we may regard 2π as a

[20] D. W. Sciama, *Monthly Notices of the Royal Astronomical Society*, cxiii (1953), 34-42.
[21] D. R. Brill and J. M. Cohen, *Phys. Rev.*, cxliii (1966), 1011-1015.

https://doi.org/10.1017/S0007087400011201 Published online by Cambridge University Press

natural unit of rotation, Bridgman proposed tentatively that the diameter of the Galaxy might be the corresponding natural cosmic unit of translation. Instead, a more significant cosmic unit is likely to be a length of the order of 10^{10} light-years (derived from the current value of Hubble's constant), which may be about the radius of the universe. Measured in these respective units the angular displacements and velocities attainable in practice are far greater than the linear. Bridgman suggested that the real state of affairs may be as follows: "phenomena in any system are affected by motion with respect to the entire universe, whether that motion is one of translation or rotation, and the magnitude of the effect is connected with the velocity of motion by a factor which is of the general order of unity when velocity is measured in cosmic units".[22] The linear displacements attainable in practice are, however, so small that their effect has not yet been detected experimentally, but angular displacements are large and their effect is easily demonstrated. In Bridgman's view, "the special theory of relativity is no different in character from any other physical law; it is only approximate, and some day our measurements may become refined enough to detect its limitations".

These ideas and speculations indicate that there is still a wide field waiting to be explored concerning cosmology and the laws of motion. Even if, in the end, the consequences prove to be less revolutionary than was Galileo's work in dynamics, we too may be witnessing the birth of "a very new science dealing with a very ancient subject".

[22] P. W. Bridgman, *The Logic of Modern Physics* (New York, 1927, reprinted 1960), 183.