THE BOUNDARY LAYER IN CATACLYSMIC VARIABLES

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ABSTRACT. Recent development in the theory of the boundary layer is considered. The model for the formation of a corona above the WD is discussed in some detail.

The interaction of the boundary layer with the stellar interior and possible mixing between the hydrogen-rich accreted matter and the core material is discussed in view of recent numerical simulations of the boundary layer.

INTRODUCTION

In the classical picture of geometrically thin accretion disks, the disks material rotates with Keplerian velocities. As the disk approaches the star, the rotational velocity in the disk must adjust to the inner boundary condition on the surface of the White Dwarf. The angular velocity in the disk increases as r decreases. However, to adjust to the WD surface the angular velocity must eventually decrease with decreasing r. Usually the angular velocity of the WD is significantly smaller than the Keplerian one. The region between the point where the angular velocity deviates from the Keplerian one and the surface of the WD is defined as the boundary layer (hereafter BL). It is assumed that this region is very small in radial extent and this is the source for the name.

The simplest energy release estimates indicate that about $\frac{1}{2}\ L_{acc}$ is released in the disk and about $\frac{1}{2}\ L_{acc}$ is released in the BL. In spite of the fact that half the accreation luminosity is released in the BL only few investigations on the structure of the BL have been carried out so far. As a rough estimate we have

$$L(BL) = 1/2\ L_{acc} = GM_{wd}\dot{M}/R_{wd}$$
$$= 4.3\times10^{51}(M_{wd}/M_0)(\dot{M}/M_0/yr)/(R_{wd}/cm)\ erg/sec$$

and for $\dot{M}=10^{-9}M_0/yr$ we find $L(BL)=8.5\times10^{33}$ erg/sec.

Paper presented at the IAU Colloquium No. 93 on 'Cataclysmic Variables. Recent Multi-Frequency Observations and Theoretical Developments', held at Dr. Remeis-Sternwarte Bamberg, F.R.G., 16–19 June, 1986.

Astrophysics and Space Science **130** (1987) 303–314.

Hence, the luminosity of the boundary layer is expected to be very high. The obvious question is of course, what is the emitted spectra of this radiation?

1) Pringle 1977 and Pringle and Savonije 1979

The first treatment of the BL was carried out by Pringle (1977) who applied the following assumptions:

a) The BL is in hydrostatic equilibrium
b) The BL emits Black Body radiation
c) The BL is optically thick.

On the basis of these assumptions Pringle concluded that the temperature in the BL should be of the order of $(2-5)\times10^5$K and emit predominately soft X-ray in the range (0.1-1.0KeV photons).

In 1979 Pringle and Savonije discussed the case of optically thin BL and hypothesised that the matter in the BL osses its angular omentum via a succession of strong shocks. They conclude that for the BL to emit hard X ray one has to assume:

a) The BL is optically thin
b) Dissipation takes place via strong shocks.

The basic idea was that strong shear and turbulence should create strong shocks. The temperature of a strong shock can be easily estimated to be

$$T_{shock} < T_{shock\ max} = (3/16)\mu m_p GM_{wd}/(kR_{wd})$$
$$= 2.3\times10^8 (M_{wd}/M_0)/(R_{wd}/9\times10^8 cm)\ K$$

The radiation from a strong shock should extend up to 20 KeV. Pringle and Savonije conclude also that:
When the cooling time << the adiabatic expansion time in the BL the shocked gas ollapses onto the surface of the WD. In the opposite case namely, the cooling time >> the adiabatic expansion time dissipation takes place via strong shocks. The two cases depend on the accretion rate. The first one is realized for $\dot{M}>>10^{16}$ gm/sec and vice versa.

The basic problem in this model is that no geometry of the shock is specified. It is based on global energy considerations and the emperature is derived by assuming almost omplete dissipation. It is difficult to see how strong shocks can be formed in a highly sheared layer (and remain stable).

2) Tylenda 1981.

Tylenda 1981 proposed a self-consistent model of the BL based on the following assumptions:

a) The BL is stationary (hydrostatics).
b) The BL is small in the radial and vertical directions.
c) Vr and Vz are negligble relative to the angular velocity everywhere.
d) The BL is optically thin and isothermal in the Z direction.
e) Energy dissipation by means of viscosity.
f) The Reynolds number is assumed to be 10^3.

Assuming steady state, Tylenda equates the viscous energy production in the boundary layer to the local bremsstrahlung losses. The results are

$$T(BL) = 5.7\times10^7 K(Re/10^3)^{-2}(M/M_0)^{5/2}(R_{wd}/6\times10^8 cm)^{-3/2}(\dot{M}/2\times10^{16} gm/sec)^{-1}$$

The temperature refers to radiation at 4.9 KeV. Re is the Reynold number and is given by

$$\nu = \Omega_k R^2 \delta / Re \qquad Re \sim 10^3$$

Here ν is the viscosity and δ is the width of the boundary layer. The width of the BL is determined by the assumed Reynolds number. The only consistent solution for the run of the temperature in the boundary layer under these conditions is one in which the temperature starts with zero at the inner part of the disc and reaches a finite high value at the surface of the WD. Tylenda finds

$$T_{surface} = 10 m_p R^2 \Omega^2_k \delta / 36k$$

The fact that the temperature in the BL ncreases towards the star is a consequence of the disc model. We will return to this point in the next section.

3) Regev 1983

https://doi.org/10.1017/S0252921100105019 Published online by Cambridge University Press

Regev 1983 tried to find a consistent solution to the boundary layer and assumed:

a) Steady state.
b) Axial symmetry
c) Viscous boundary layer.
d) Optically thick layer (high accretion rates).
e) The variables are not necessarily constant in the vertical direction

Regev expanded the solution in a small parameter. The obvious small parameter in the problem is the ratio of the speed of sound to the circular velocity which is equivalent to assuming a geometrically thin disc (height/radius << 1).

$$e = V_s / (\Omega_k R_{wd}) \ll 1.$$

A straightforward expansion in e is not consistent with the inner boundary condition because the leading term of the disc does not vanish. Such a situation is frequently found in boundary layers. Consequently Regev applied a matched asymptotic expansion technique to fit the two sides. In this way a fully consistent solution can be found.

Regev finds several results the most interesting of which is that the luminosity of the disc is given by:

$$L_{acc} = (3/2 - C) GM\dot{M}/R$$

where C is an integration constant and should be found numerically by matching the two solutions. Shakura and Sunayev 1971 and Pringle 1981 assumed that

$$\partial \Omega_0 / \partial r = 0 \qquad \text{for} \quad r = R/9\text{x}10^8 = 1$$

$$\text{where } \Omega_0 = \Omega / (GM/R^3)^{1/2} = 1$$

Hence theses authors derived C=1. This value for C is however inconsistent with the outer solution for the angular velocity. Moreover, this value for C leads to a vanishing temperature at the inner part of the disc (cf. Tylenda's treatment). In a numerical example Regev shows that C can be different from 1. In his particular case C = 0.61 for $\dot{M} = 2.4\text{x}10^{17}$ gm/sec. We should stress that Regev finds the temperature to reach a certain high value at the inner part of the disc and tends to zero at the surface of the WD. It should be emphasised that the setting of the boundary condition in the BL affects the entire disc structure. Obviously, the effect is Largest near the BL and decays away towards the outskirts of the disc. Accurate models for the disc radiation distribution are required in order to distinguish between such fine details.

Finally, Regev notes that not for every set of parameters there is a solution to the BL (under the above assumptions of course).

4) Ferland, Langer, MacDonald, Pepper, Shaviv and Truran 1982.

The above authors compared the expected X ray luminosity with the observed one and found that in most cases the soft X ray luminosity is much lower than the expected one. In view of this basic finding they conclude that: a) The BL structure is not predicted by the theory, or b) The dissipation law is different, or c) L(BL) was overestimated, or d) The optical luminosity of a typical old nova is produced partly by reprocessed UV from the WD, or e) The BL is larger than predicted.

In spite of the fact that the authors may have wrong estimates for the relevant X ray luminosity the basic facts are qualitatively correct.

5) Jensen 1984

Jensen (1984) evaluated the X ray luminosity and compared it not to the optical one (a la Ferland et al.) but to the total accretion luminosity. Jensen also found deficiency in X ray luminosity and concludes that it cannot be due to excess of optical radiation.

6) King and Shaviv 1984

King and Shaviv 1984 were motivated by the observations of SS-Cygni which can be summarized as follows:

1) The object is bright,very variable and emits hard X rays in quiescence. The hard X ray decrease at outburst while the soft X ray increase.
2) The hard X-ray decrease without an increase in intrinsic absorption.
3) The hard X ray can be fitted to thermal bremsstrahlung with temperature T correlated to the

https://doi.org/10.1017/S0252921100105019 Published online by Cambridge University Press

total X ray luminosity as:

$$L_x \sim T^a \quad \text{with} \quad a \sim 3/2.$$

King and Shaviv discuss the structure of the BL in terms of three time scales: The time scale for viscous heating given as

$$t_{heat} = 3\rho V_s^2/\Gamma$$

where $\Gamma = \nu\rho V_\phi^2/b^2$ $\qquad \nu = \alpha V_s b,$

the time scale for hydrodynamic expansion

$$t_{exp} = b/V_s$$

and the time scale for radiative cooling

$$t_{radiative} = 3\rho V_s^2/\rho^2\Lambda$$

where $\rho^2\Lambda$ is the volume energy loss rate (erg/sec cm^3).

The three time scales are functions of the density, the height of the BL b and temperature. In fig. 1 we have the regions in the b-T plane (the heating plane) in which the three time scale are important. The heating plane is divided into three regions depending on the ratio between the time scales. For low b we find that the time scale for heating is the shortest and hence the optically thin BL is unstable. As a consequence, any mass element in this region will heat up and expand until the heating time equals the radiation losses time scale. At this point the temperature of the element is essentially the virial temperature of the WD. The expansion and heating leads to the creation of a hot corona around the WD. The extent of the corona in the vertical direction is of the order of

$$H_v \sim V_s R_{wd}/V_\phi \sim 0.6 x R_{wd}$$

while the radial extent is approximately

$$H_r \sim (V_s/V_\phi)^2 R_{wd} \sim 0.3 x R_{wd}.$$

When the density in the BL is very high, the radiative cooling time is always the shortest and hence the BL is stable.

Three processes control the behaviour of the corona: The radiative cooling as before, the hydrodynamic time

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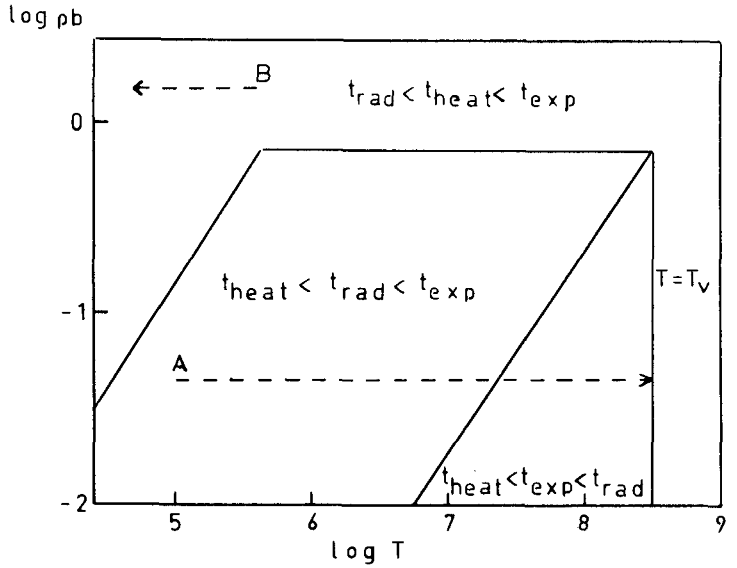

Fig. 1.

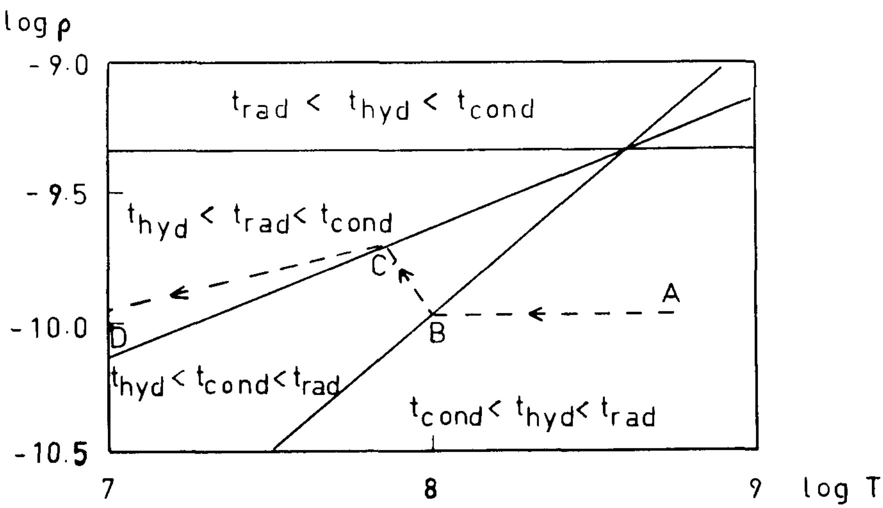

Fig. 2.

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scale is now given by:

$$t_{hydrodynamic} = H_r/V_s$$

and the conductive time scale

$$t_{conductive} = 5/2 \rho V_s^2 H_r^2 / k_0 T^{7/2}.$$

The relations between the different time scales are plotted on the cooling plane (fig. 2). At high temperatures and low densities typical to the corona, the conductive cooling time is the shortest. Hence, the corona will cool at first at essentially constant density. The radial scale height is proportional to the temperature and hence the total luminosity in the X will be roughly

$$L_x \sim H_r \rho^2 {}^2 T^{1/2} \sim T^{1/2}.$$

As the element cools it enters into the region where the hydrodynamic time scale is the shortest. The behaviour in this region is typical to the solar corona. The corona cools and the density decreases and hence

$$L_x \sim H_r \rho^2 T^{1/2} \sim T^a \qquad -1/2 < a < 3/2.$$

Finally the element reaches the region where the hydrodynamic time scale is the shortest but the radiative cooling is more important than conduction. In this region we find

$$L_x \sim T^{3/2+2e} \qquad \text{where} \quad e > 0 \text{ and small.}$$

Note that the X ray luminosity-temperature relation depends critically on the behaviour of the density. The density must decrease with time if $L_x \sim T^{3/2}$. For constant T we have:

$$\partial \rho / \partial t = -1/g\, \partial^2 P / \partial t \partial z = 2/3g\, \partial (\Lambda \rho^2) / \partial z < 0$$

because the density decreases with z. The corona collapses onto the WD. The condensation and falling of blobs decrease the density and the total emitting mass as a function of time on one hand and tir the surface of the WD on the other. It may well be possible that part of the energy of the falling blobs is released inside the surface of the WD and hence the luminosiy in the X ray range is decreased correspondingly. The energy so absorbed by the WD will subsequently be emitted in the EUV region and hence escape detection (cf. Ferland et al 1982, Jensen 1984)

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7) King Watson and Heise 1985

King Watson and Heise 1985 observed SS-Cygni with EXOSAT and confirmed the predictions of King and Shaviv.

8) Numerical simulations

The model of King and Shaviv is very rudimentary and demands further theoretical support. The problem of the BL is however very complicated and is not easily amenable to analytical studies. Recently two numerical works have carried out. The first one by Robertson and Frank 1986 and the second by Kley and Hensler 1986. These are 2D viscous flow calculations carried out under two limits. In the first the energy released by the dissipation is allowed to escape from the system and thus mimics an optically thin case. In the second, no energy losses are allowed. Both groups find that in the optically thin case the BL expands over a region of the order predicted by King and Shaviv. The flow continues from the equator to the pole.

Before we leave the numerical calculations we note that here is a problem of what to assume for the viscosity in the BL and to what extent the shear layer turbulence is isotropic etc. The calculations were carried out under certain assumptions. While the general picture will probably not change with a change in the viscosity law, the details will.

ELEMENT MIXING

An important question concerning the thermonuclear runaways (TNR) on the surface of WD is the possible mixing between the fresh hydrogen rich material accreted onto the star and the core material. Observational evidence indicates that many nova eject matter rich in C N O Ne elements and sometimes even elements with higher Z. The particular isotopic ratios found in few cases and the known maximum temperatures found in TNR lead to the conclusion that these isotopes were somehow dredged from the core of the WD. Several possible mechanisms for such mixing have been proposed and here we discuss only the element mixing by means of shear turbulence in the outer surface layers of the WD in or just below the BL. Could the stirring in the outer layers lead to mixing? Is the accretion restricted to the equatorial region? In the numerical calculations of Robertson and Frank and Kley and hensler the inner boundary condition is a rigid wall with no slip condition. Hence they cannot provide an answer o this question. On the other hand there is always a small layer that surrounds the core.

The first to analyze this question was Durisen 1977. Durisen calculated the long term evolution of accreting rotating WD. Since Durisen was interested only in the long term effects he assumed rapid Z readjustment and mixing over Z so as to yield constant angular momentum on cylinders. In the absence of circulation, an axisymmetric barytropic equilibrium state must have J constant over cylinders. If this condition is not satisfied, a rapid z-readjustment takes place on a dynamic time scale. Furthermore, Durisen assumed the existence of only molecular viscosity. The results were fast expansion of the envelope of the star and the concentration of high J material near z=0, the equatorial plane. Durisen hypothesised that instabilities in the region with high shear will lead to the establishment of a region in which the Richardson criteria is marginally satisfied.

The validity of the Richardson criterion was proven only for cylindrical stars (Sung 1974). Moreover the criterion is sufficient but not necessary. If the dominant unstable wave length is of the order of the radius of the star then clearly the validity of the condition to the case under consideration is not clear at all.

The problem is further complicated due to thermally driven circulation which hitherto was ignored. The dynamics demands that J be constant over cylinders. However circulation will try to build a small deviation from this state. Any deviation from this state causes secular and dynamical instability (Goldreich and Schubert 1967, Fricke 1968). Hence the long term evolution may depend on these small deviations. Kippenhahn and Mollenhoff (1974) suggest that the deviations from J=constant over cylinders may lead to oscillating nonequilibrium meridional circulation (on a dynamic time scale).

Kippenhahn and Thomas (1978) bserved the non-spherical shape of nova ejecta and concluded that the TNR are nonspherical. They hypothesize the formation of accretion belts and that the new fresh material does not move towards the poles of he WD. Two basic assumptions are included in the analysis.

The first one concerns the meridional circulation time scale. The velocity of the meridional circulation is given by

$$V_{circ} = CL/GMR\rho$$

where C contains thermodynamic derivatives which are all of the order of unity. Kippenhahn and Thomas assume a density of 100gm/cc and obtain as a typical time scale for the circulation 2×10^5 years. Such a high density is typical to the burning zone where the TNR takes place. The density in the outer layer is significantly lower, of the order of 10^{-7}gm/cc and hence the circulation time is drastically shorter.

The second assumption is that the angular momentum and composition have the same spatial distribution, namely

$$X = fX_0 \qquad \text{and} \qquad J = fJ_0$$

where f depends on the radial distance and atitude. In other words

$$J/X = \text{constant}.$$

Kippenhahn and Thomas conclude that meridional circulation by rotation of the accretion belt and energy dissipation is insufficient for mixing the belt towards the pole.

MacDonald (1983) followed Kippenhahn and Thomas (1978) and perturbed the assumed equilibrium (with the strong assumption of J/X=const.). He derived a questionable and not understood dispersion relation and concluded that mixing inward towards the interior occurs on a thermal time scale.

Regev and Shaviv (1986) examine this question and preliminary result indicate that the assumtion is not valid.

REFERENCES

Durisen R.H. 1977, Astrophys. J. 213,145
Fricke K.J. 1968, Zs. f. Ap. 68,317
Ferland G.J., Langer S.H., MacDonald J., Pepper G.H.,

Shaviv G. and Truran J. 1982, Astrophys. J. 262, L53
Goldreich P. and Schubert G. 1967, Astrophys. J. 150,571
Jensen
King A.R. and Shaviv G. 1984, Nature 308,519
King A.R., Watson and Heise 1985,Mon Not Roy Ast Soc
Kippenhahn R. and Thomas H. C. Asytron & Astrophys 63,265
Kippenhahn R. and Mollenhoff C. 1974, Astrophys & Space Sci 31,117
Kley W. and Hensler G. 1986,preprint
MacDonald J. 1983,Astrophys. J. 273, 289
Pringle J.E. and Savonije G.J. 1979,Mon Not Roy Ast Soc 187,777
Regev O. 1983, Astron. & Asrophys. 126,146
Regev O. and Shaviv G., 1986 in preparation
Robertson J.A. and Frank J. 1986, preprint
Sung C-H. 1974, Astrophys. J. 33, 99
Tylenda R. 1981, Acta Astron. 31,267