There was an astronomer whose spectra
Were of hot supernova ejecta
But her lack of detection
Of semi-convection
Made de Loore and Chiosi reject her.

SESSION 6

EVOLUTION WITH MASS LOSS: SINGLE STARS

Chairman: A. COWLEY
Introductory Speaker: C. DE LOORE

1. C. CHIOSI, E. NASI and G. BERTELLI: Theoretical evolution of massive stars with mass loss by stellar wind.
2. D.S.P. DEARBORN and J.B. BLAKE: Critical rates of mass loss.
3. S.A. LAMB: Supergiant mass loss and the Cassiopeia A progenitor.
4. S.R. SREENIVASAN and W.J.F. WILSON: The role of rotation in the evolution of massive stars losing mass.
5. H.J. FALK and R. MITALAS: Evolution of a 30 M_o star with mass loss.
6. T.J. MAZUREK: Mass conservation and rapid mass loss on the main sequence.

EVOLUTION OF SINGLE STARS WITH MASS LOSS

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1. EVIDENCE FOR MASS LOSS

A. Mass loss has been observed by means of UV, optical, IR and radio observations.
B. There exists also indirect evidence that stars lose matter during their evolution.

1.1. Mass determination of binaries

For stars on or near the ZAMS the masses agree well with the mass luminosity relation (see for instance Stothers, 1972). The effective temperatures and luminosities correspond with models of these stars for luminosity class V. For some evolved systems the masses and luminosities are such that mass exchange and mass loss might have occurred, since one of the members does not fit the mass luminosity law (Examples : Ly Aur (Andersen et al., 1974), HD 47129 (Hutchings and Cowley, 1976), HDE 228766 (Massey and Conti, 1977), BD+40°4220 (Bohannan and Conti, 1976)). For this last star the mass loss could have been as large as 60 $M_\odot$ in 3.10^6 years, corresponding with a mass loss rate of ~ 2.10^{-5} $M_\odot yr^{-1}$ (Conti, 1978).

1.2. Overluminosity of supergiants and Of-stars

The luminosities of early type supergiants and of Of-stars seem to be considerably higher than expected from the observed masses, derived from evolutionary tracks or the mass luminosity relation (Hutchings, 1976, Conti, 1976). Also the optical companions of massive X-ray binaries are overluminous; this was ascribed to mass loss by a strong stellar wind during the hydrogen burning stage by de Loore, De Grève, Lamers (1977), de Loore, De Grève, Vanbeveren (1978), Ziolkowski (1977). This has important repercussions on the evolution and the evolutionary status of massive X-ray binaries and WR stars.
More evidence is furnished by the mechanism for the production of the X-rays; these may either be produced by the accretion of matter expelled by a stellar wind acting on the hot companion (Lamers, van den Heuvel,

P. S. Conti and C. W. H. de Loore (eds.), Mass Loss and Evolution of O-Type Stars, 313–336.

Patterson, 1976) or by a stellar wind enhanced by Roche overflow from the hot companion of the neutron star.
The surface composition of supergiants can show products of nuclear burning; it might be that these products were brought from the interior to the surface by convection or that the outer layers were expelled, so that the inner regions which have undergone nuclear burning are revealed. Early type supergiants do not have convective envelopes hence in the stars of this group showing evolved surface abundances probably considerable mass loss occurred (Walborn, 1976; Dearborn and Eggleton, 1977). Another possibility to bring processed material to the surface, meridional mixing may be ruled out, since at no time during the evolution the envelope convection penetrates through the hydrogen burning shell (Lamb, Iben, Howard, 1976; Lamb, 1978).

1.3. Features of the HRD which cannot be explained by usual evolutionary computations

a) an uninterrupted distribution of stars with spectral types O through A0 for log $L/L_o < 5.3$
b) a relative lack of stars with spectral types later than B3 for log $L/L_o > 5.3$
c) the absence of stars of spectral type M for log $L/L_o > 5.3$.

If instead of using a conservative assumption upon the mass, mass loss is taken into account, most of these characteristics may be explained : the strange features as overluminosity (or undermassiveness) disappear and a new mass luminosity relationship may be derived, the X-ray production in binaries can be explained at least qualitatively, the position in the HRD becomes clear, certainly for the points b and c, i.e. the thinning out of more luminous stars with spectral type later than B3 and the absence of stars of spectral type M for log $L/L_o > 5.3$. Different assumptions for the mass loss may be adopted. The effect of these as-

<table>
<tr><th></th><th colspan="2">opacities</th></tr>
<tr><th>assumptions</th><th>Cox Stewart</th><th>Carson</th></tr>
<tr><td>no mass loss</td><td>no</td><td>no</td></tr>
<tr><td>mass loss only important during late type supergiant phase</td><td>no</td><td>no</td></tr>
<tr><td>sudden mass loss at some critical effective temperature</td><td></td><td>no</td></tr>
<tr><td rowspan="2">mass loss occurs continuously in all parts of the HRD</td><td colspan="2">can account for the presence of OBN and WN stars and the absence of very luminous M supergiants</td></tr>
<tr><td>no satisfactory explanation of WN stars with large H-deficiencies</td><td>effective temperature for the ZAMS too low to account for the observation of blue supergiants of the lowest luminosity</td></tr>
</table>

Table 1. Survey of various assumptions concerning mass loss and their influences on the features of the HRD. "No" means that the observed features of the HRD are not in agreement with the assumption.

sumptions is shown in Table 1. It turns out that only mass loss occurring continuously in all parts of the HRD explains these characteristics, and neither mass loss occurring only during the late type supergiant stage, nor sudden mass loss starting at some critical effective temperature.

2. EVOLUTION OF SINGLE STARS WITH MASS LOSS

Although evidence for mass loss in stars of different spectral types was discovered many years ago (Morton, 1967; Hutchings, 1976), with mass loss rates between 10^{-7} and 10^{-4} $M_{\odot}yr^{-1}$ for luminous stars, mechanisms for the explanation of this phenomenon have been developed recently, or are still being developed. It is clear that mass loss rates of this order will affect the evolution of the stars. Evolution of stars decreasing in mass was computed by Massevich (1958) in order to explain statistics for stars of the galaxy and the assumption that the mean molecular weight changes along the main sequence. She assumed that the star loses mass continuously owing to corpuscular radiation and calculated sequences of equilibrium models with decreasing mass. A mass loss rate, $\dot{M} = -kLR/M$ was proposed by McCrea in 1962. Tanaka (1966) calculated the evolution of two massive stars (~ 15 $M_{\odot}$ and ~ 47 $M_{\odot}$) from ZAMS to core H exhaustion. Hartwick (1967) used the McCrea relation with k such that for main sequence stars $\dot{M} = 10^{-6}$ $M_{\odot}yr^{-1}$. Chiosi and Nasi used the same relation with various k for the calculation of the evolution of stars with initial masses of 20 and 40 $M_{\odot}$. The tracks are shown in Figure 1. The most striking features are the following :

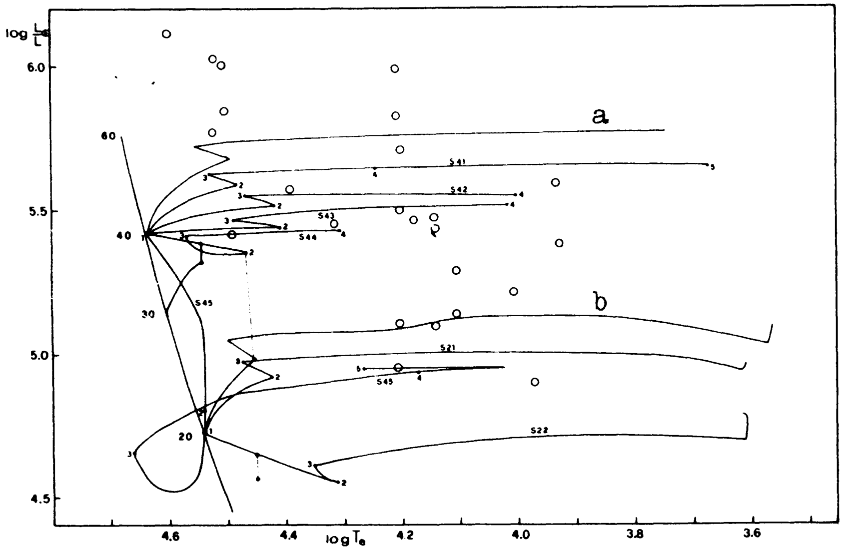

Figure 1. Evolutionary sequences with mass loss for initial masses of 20 and 40 $M_{\odot}$. The tracks a and b are tracks without mass loss (Chiosi and Nasi (1974).

1. for moderate $\dot{M}$ the evolutionary tracks are similar to those at constant mass but with decreasing luminosity for higher $\dot{M}$.

2. for very high $\dot{M}$ the evolutionary tracks show similarities with those of the mass losing primaries in massive close binaries with Roche lobe overflow. Dearborn and Eggleton (1977) paid especially attention to the problem of CNO abundances in stars that lost matter. According to their computations a 32 $M_\odot$ star, losing mass at a rate of ~ $1.5\ 10^{-6}$ $M_\odot yr^{-1}$ can be converted into an OBN star (Walborn, 1976) : the burning products of the CNO cycle show up at the surface. This is particularly interesting in connection with the problem of the WR abundances. The status of evolutionary computations with mass loss is given in Table 2.

Table 2. Survey of evolutionary computations including mass loss.

Massevich 1958	mass loss by corpuscular radiation
McCrea 1962	mass loss rate $-\frac{dM}{dt} = kLR/M$ (L,R,M in solar units, k in $M_\odot yr^{-1}$)
Tanaka	evolution of 46.8 $M_\odot$ 15.6 $M_\odot$
Hartwick	evolution of 15 $M_\odot$
Chiosi & Nasi 1974	evolution of 20 and 40 $M_\odot$
de Loore, De Grève, Lamers 1977	20-50 $M_\odot$
de Loore, De Grève, Vanbeveren 1978	50-100 $M_\odot$
Dearborn & Eggleton 1977	
Chiosi & Nasi, Sreenivasan 1978	20-100 $M_\odot$
Sreenivasan, Wilson 1978	15 $M_\odot$
de Loore, De Grève, Vanbeveren (Dec.1978)	20-100 $M_\odot$
Stothers & Chao wen Chiu	
Czerny, Michal (Poland)	40-180 $M_\odot$
Falk, Mitalas	30 $M_\odot$

3. COMPUTATIONS OF EVOLUTIONARY TRACKS WITH MASS LOSS

3.1. Mass loss rates

A considerable number of observations on mass loss exists but the number of stars with accurately determined rates is less extended (Snow, T., these proceedings). For the determination of the theoretical mass loss rates we calibrated the values of Hutchings (1976) and of Barlow and Cohen (1977) by means of 7 standard stars. Rather large errors (up to 1 magnitude) may be expected since discrepancies up to a factor of 10 might exist for the standard stars, and since extrapolation is necessary (standard stars of spectral type O9f to B1a). Barlow and Cohen (1977) derive an expression for the mass loss rate (from the IR excess of 34 OBA supergiants and 10 Of and Oe stars, together with a velocity law derived from the observations of P Cygni). For our computations we started with the equations proposed by Barlow and Cohen :

$$\dot{M} = \alpha (L/L_o)^{\beta}$$

$\alpha = 6.8\ 10^{-13}\ M_o yr$ $\beta = 1.1 \pm 0.06$ for the O-stars
$\alpha = 5\ 10^{-13}\ M_o yr$ $\beta = 1.2 \pm 0.08$ for the B and A

These values are about a factor of 2 smaller than those quoted for supergiants in the literature. Also the values of Sterken (1977), derived from the Hα profile of B1a and B2a supergiants agree within a factor of 2. β is approximately 1, hence we assumed the mass loss nearly proportional to the luminosity, and expressed the mass loss rates in terms of a parameter N defined as

$$N = -\dot{M}\ c^2/L$$

The N values derived from the mass loss rates of Barlow and Cohen are shown in Figure 2. In the figure are also given the values of ζ Pup and τ Sco and the 6 supergiants of Sterken (underlined). The lower limit for mass loss for O and B stars (Snow and Morton, 1976) is M_{bol} = -6. In the late B and A stars Rosendhal (1973) found a lower limit at M_{bol}= -7.8. From the figure it may be seen that M is of the order of 100±50 throughout the whole spectral range. The figure does not show variations

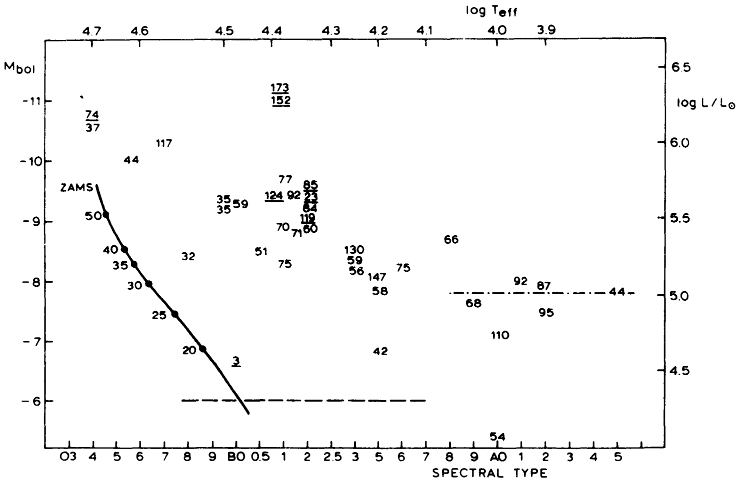

Figure 2. Observed mass loss rates converted into N-values. The lower limits for significant mass loss observed in the far UV (dashed line) and in the visual (dash-dotted line) are indicated also.

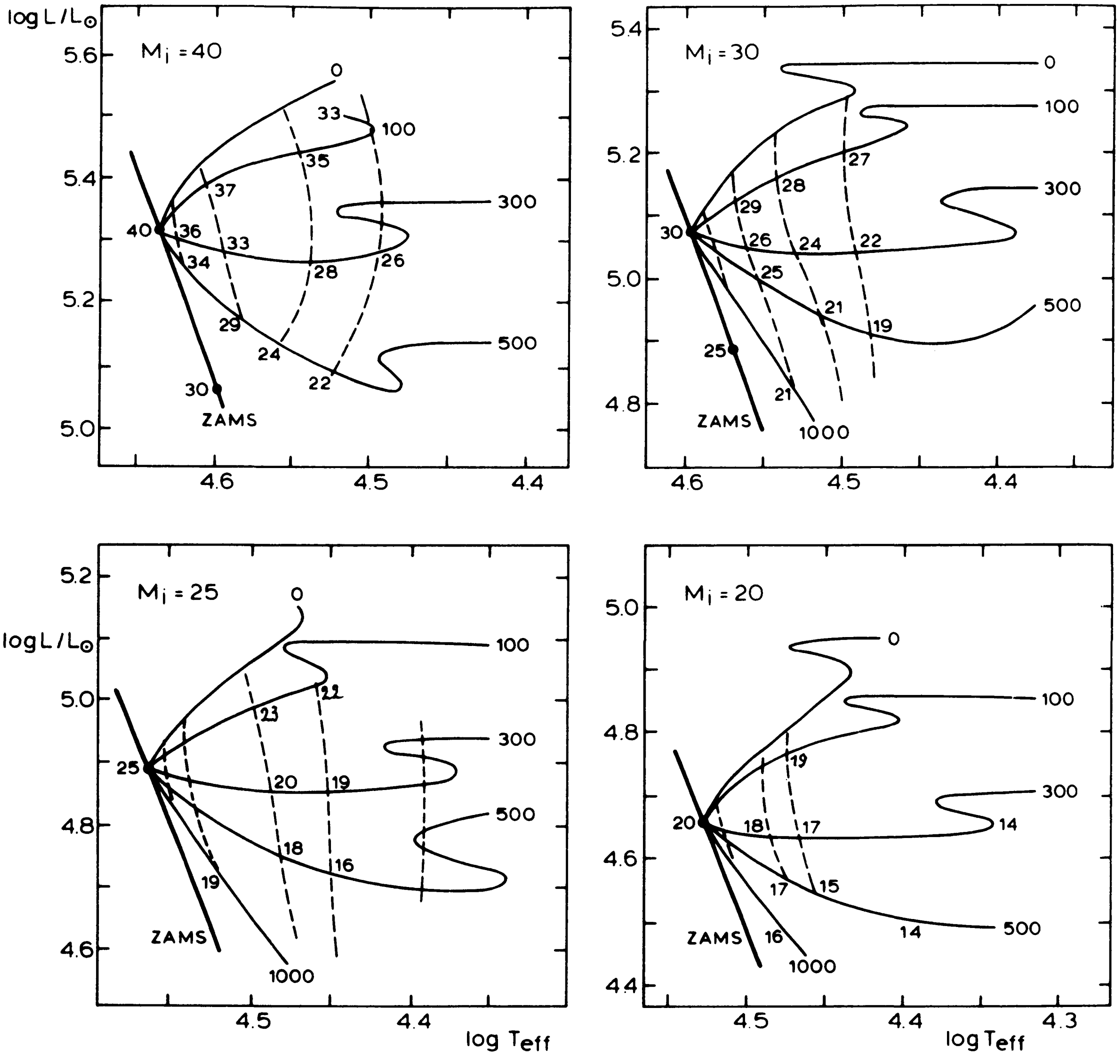

Figure 3. Evolutionary tracks in the HR-diagram of mass losing stars with initial masses of 40, 30, 25 and 20 $M_\odot$. The evolutionary tracks, in full line, are labeled with the N-values. The dashed curves are isochrones, given for intervals of one million years. Near the isochrones the stellar masses are indicated.

of N with luminosity or spectral type and this was the reason why we computed our tracks with constant N. We used N-values ranging between 100 and 300 (cfr. Fig.2), keeping in mind that the Barlow and Cohen values are about a factor 2 smaller than those of other observers.

3.2. Evolutionary tracks

For the computation of evolutionary tracks for massive single stars we adapted our code for the evolution of primaries of close binaries. Sets of atmospheres were calculated for various luminosities, effective temperatures, masses and mass loss rates in domains containing the pre-estimated point on the presumed track in the HRD. Entropy losses were taken into account. An initial chemical composition of X=0.7, Z=0.03 was used. Evolutionary tracks for masses between 20 and 50 M_0, for N=0, 100, 300, 500 and 1000 are shown in Figure 3. The tracks for different values of N (100-1000) for 20 M_0 and 40 M_0 may be compared with those of Chiosi and Nasi (1974); the agreement is rather good. Since they used a changing N instead of a constant value this indicates that the general conclusions concerning the supergiant stage are independent of the precise behavious of N. Striking features are :

1. the luminosity drops are more pronounced for larger N-values; for N=1000 the evolutionary tracks move downwards along the ZAMS (this was also noted by Chiosi and Nasi)

2. for increasing N-values the luminosity at the first turn point steadily decreases together with the effective temperature; hence the hydrogen burning phase covers a wider strip in the HRD.

After hydrogen exhaustion in the core the star follows a track similar to the conservative case (i.e. at nearly constant luminosity). If we compare the values of the luminosities in these regions with the luminosities of single stars at the end of the main sequence without stellar wind we find that the stellar wind remnants are overluminous by a factor 1.3 (N=100) to 5 (N=300)(Figure 3). Another consequence of the mass decrease is an increase of the hydrogen burning phase of about 30% (N=300) to 10 à 15% (N=100) compared with evolution at constant mass. Beyond the turn point the influence of the mass outflow on the evolution is only marginal due to the small time scale involved with core contraction prior to helium ignition (10^4 years). Stars with initial masses between 30 and 50 M_0 lose about 50% of their mass during the core hydrogen burning phase (N=300) or ~ 20% (N=100). For stars in the range 60-100 M_0 the luminosity is higher (of the order of 10^{-5} $M_0 yr^{-1}$), so that more hydrogen rich layers are expelled and helium rich layers appear at the surface already during hydrogen burning. The evolutionary tracks for this mass range (de Loore, De Grève, Vanbeveren, 1978) are shown in Figure 4. The figure shows the decreasing masses and the hydrogen abundance at the surface. These massive stars attain their turning point during core hydrogen burning and turn to the left. The most massive ones (80 and 100 M_0) even cross the ZAMS before evolving towards the red giant region. The time between red and blue main sequence points

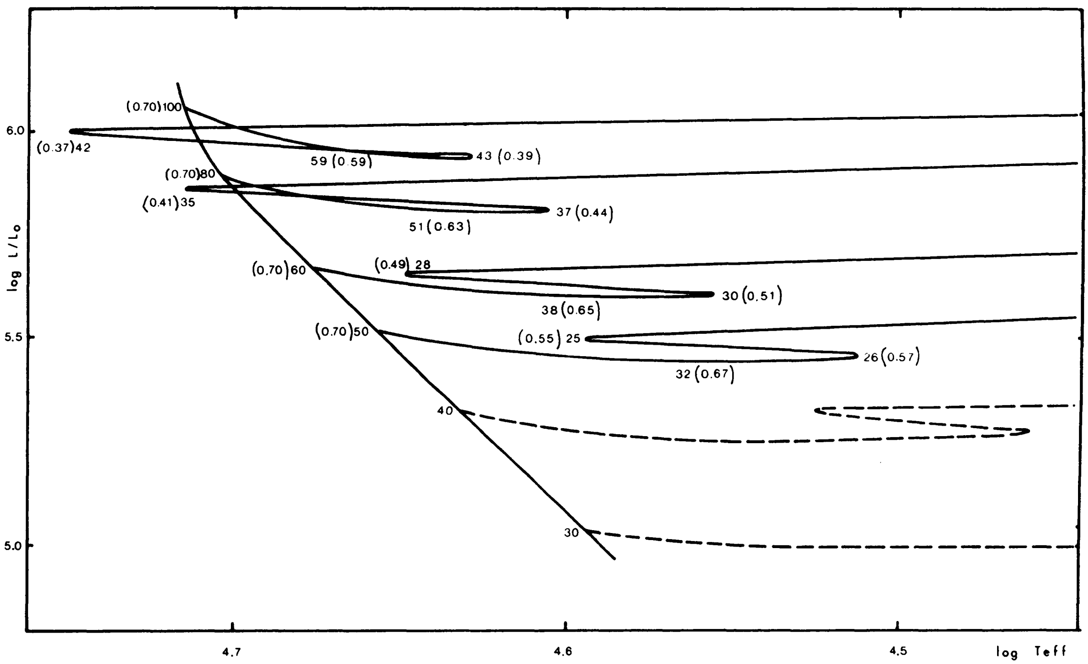

Figure 4. Evolutionary tracks for mass losing stars of 100, 80, 60 and 50 $M_⊙$ (N=300). The numbers at the tracks are the remaining masses in $M_⊙$. Between brackets is indicated the hydrogen content of the atmosphere.

is some 10^4 years. Figure 5 shows the masses at the end of the core hydrogen burning as a function of the initial masses. For the three N-values used for the computations a linear relation between final mass and initial mass may be obtained. The limiting value for the mass corresponding with vanishing mass loss is ~ 13 $M_⊙$ (where UV observations point to ~ 15 $M_⊙$ (Snow and Morton, 1976)).

Chiosi, Nasi and Sreenivasan (1978) have calculated evolutionary tracks with mass loss using the mass loss rates predicted by Castor, Abbott and Klein (1975).

$$\dot{M} = \frac{L}{cv_{th}} \cdot \frac{\alpha}{T} \left|\frac{1-\alpha}{1-\Gamma}\right|^{\frac{1-\alpha}{\alpha}} (K\Gamma)^{1/\alpha}$$

with L, M stellar luminosity and mass, c the light velocity, v_{th} the thermal velocity of random motion, Γ the ratio of the luminosity to the Eddington luminosity.

($\Gamma = \frac{\sigma_e L}{4\pi G M_c}$ with σ_e the mass scattering coefficient for free electrons, K and α were used as adjustable parameters). The α-value of 0.76 used by Chiosi et al. seems to correspond with our case N=100, and the value of α=0.9 is slightly smaller than our N=300 case. The agreement between

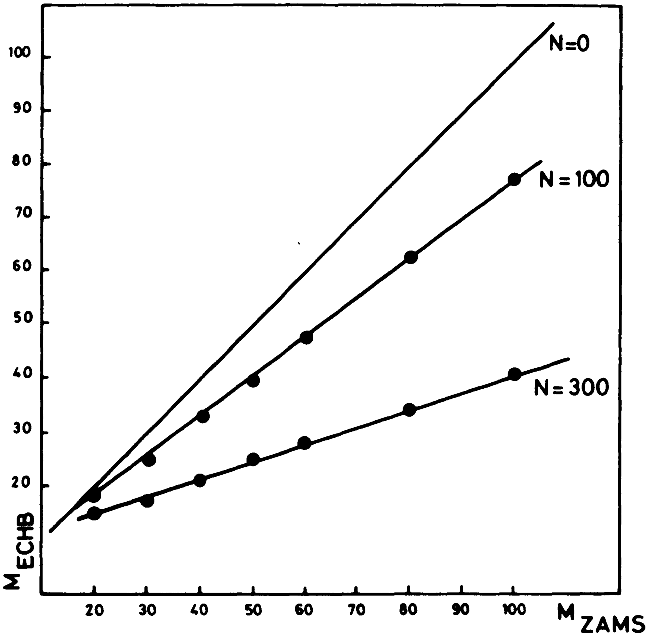

Figure 5. Mass at the end of core hydrogen burning as a function of the initial mass for three N-values.

the two sets of computations (de Loore et al. and Chiosi et al. (1978)) for the hydrogen burning stage is very good. Chiosi et al. have treated semi-convection in a very extended way, while in the computations of de Loore et al. only convection, but not semi-convection, is treated properly. However this has little effect on the evolution as the layers involved are removed. Chiosi et al. (1978) have also incorporated mass loss during the red giant phases when the evolutionary tracks cross the acoustic flux dominated region. As already calculated before (de Loore, 1970; Castellani et al. 1971) the acoustic flux is a sharply peaked function of T_{eff}, so that the switch from predominantly radiation pressure driven mass loss to acoustic flux driven mass loss occurs quite suddenly. The mass loss rate is given by

$$\dot{M} = \varepsilon \frac{L_{ac}}{v_e v_c}$$

with v_e the escape velocity of the star, v_s the sound speed, L_{ac} the acoustic flux and ε is the efficiency parameter (Fusi-Pecci and Renzini, 1975a,b, 1976). Chiosi et al. adopted a value of 10^{-4} for ε. Evolutionary tracks were computed for initial masses of 20 - 100 $M_\odot$, lowered by stellar wind losses (α=0.9) with further mass loss due to acoustic fluxes when log T_{eff} equals 3.84, 3.83 and 3.795 respectively. The mass loss rates involved have values of 5 to 6 10^{-3} $M_\odot yr^{-1}$, 5 10^{-3} $M_\odot yr^{-1}$ and 2.5 10^{-4} $M_\odot yr^{-1}$ respectively. This is shown in Fig. 6 and 8.

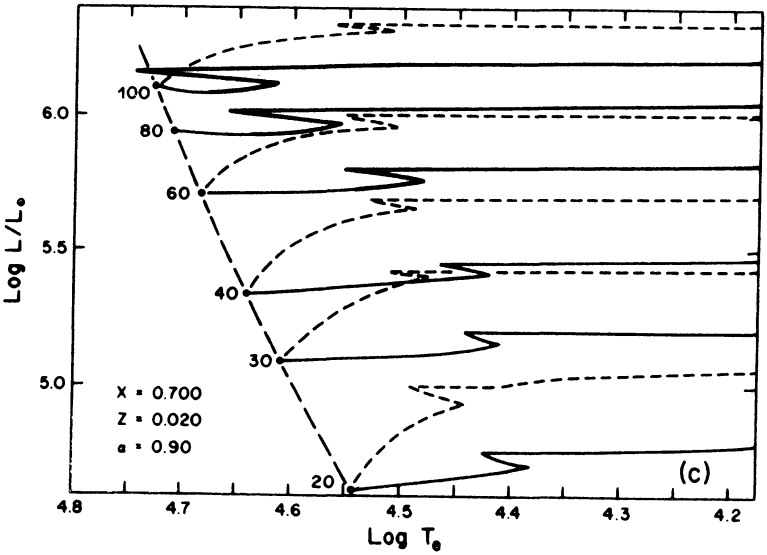

Figure 6. Theoretical HR diagram of the 20, 30, 40, 60, 80, 100 $M_{\odot}$ stars with mass loss by radiation pressure (α=0.76) during the main sequence and shell H-burning phases (Chiosi, Nasi, Sreenivasan, 1978).

The same set of calculations was repeated for α=0.76 (shown in Figure 7). The tracks show the following characteristics :

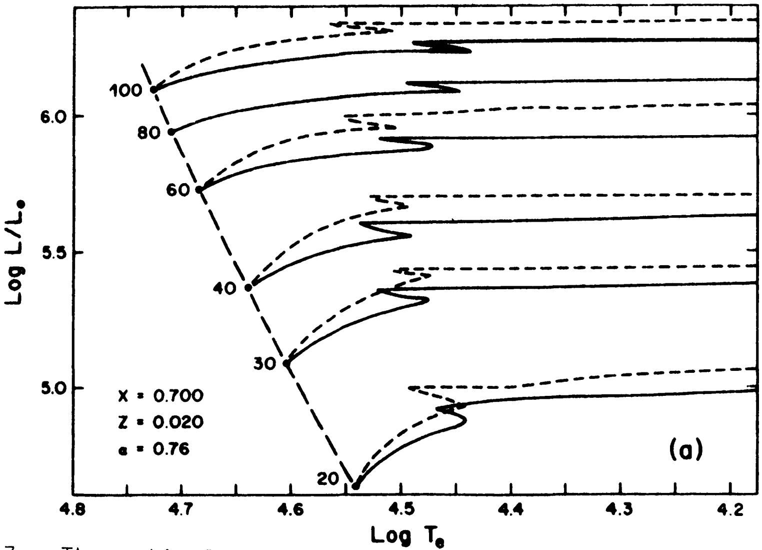

Figure 7. Theoretical HR diagram of the 20, 30, 40, 60, 80, 100 $M_{\odot}$ stars with mass loss by radiation pressure (α=0.90) during the main sequence and shell H-burning phases (Chiosi, Nasi, Sreenivasan, 1978).

1) as long as layers with the initial chemical composition are expelled the stars move towards lower effective temperatures. The acoustic flux increases hence the mass loss rate becomes higher. This stage ends when layers that have undergone internal mixing or CNO processes appear at the surface. This stage does not occur in higher masses since CNO-processed layers reached already the surface before

2) two competing effects occur now, leading to loops in the HRD : on the one hand gravitational core contraction and consequent He-burning causes an expansion of the envelope and a decrease of the effective temperature; the star tends to move to the right. On the other hand He-enriched layers will tend to shift the star to move to the left. The result is that the star describes loops (Figure 8).

3) as a consequence of the mass loss and the H-burning shell moving outwards, the outer layers are eaten from two sides. Consequently the relative He-core mass increases. When this value exceeds values of 0.6-0.7 the star moves towards the left, and again the high temperature mass loss mechanism takes over.

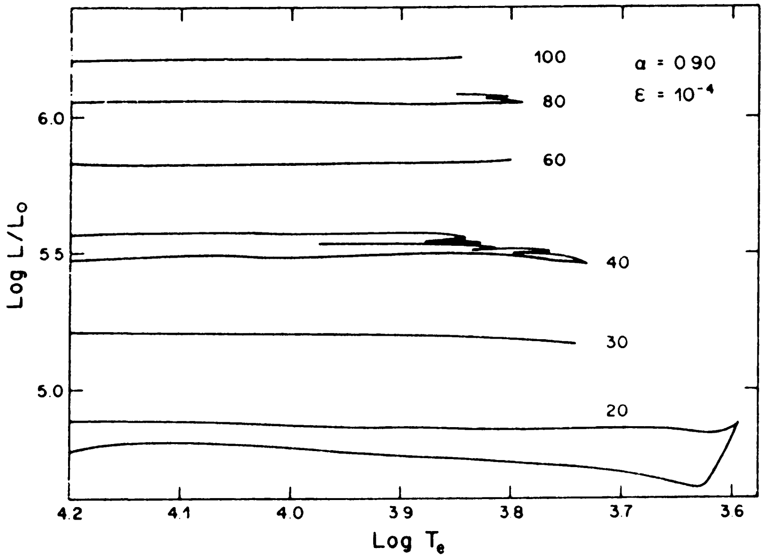

Figure 8. Theoretical HR diagram for the set α=0.90 during the phase of acoustic flux driven wind (ε=10^{-4}) and early stages of central He-burning. Each sequence is labelled by the initial mass in solar units (Chiosi, Nasi, Sreenivasan, 1978).

Recently Stothers and Chao-wen Chiu (1978) carried out evolutionary computations including mass loss for different assumptions

1) mass loss occurs in all regions of the HRD, according to (McCrea, 1962)

$$\dot{M} = -kLR/M$$

(for k a value of 10^{-11} $M_o yr^{-1}$ is used).

2) mass loss only occurs during the late type supergiant stage (log T_{eff} < 3.85)

3) sudden mass loss occurs at a critical effective temperature (T_{eff} ~ 5000K)(Bisnovatyi-Kogan and Nadezhin, 1972) for $M > 20\ M_\odot$.

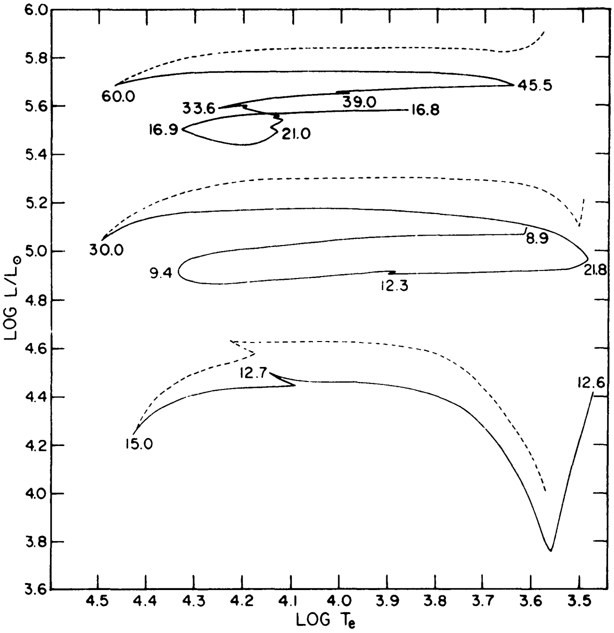

Figure 9. HR diagram showing the evolutionary tracks for initial masses of 60, 30 and 15 $M_\odot$. The dashed lines represent conservative tracks, the solid lines represent the case with mass loss occurring continuously in all parts of the diagram with $\dot{M}$ given by the McCrea expression (Stothers and Chao-wen Chiu, 1978).

Evolutionary tracks are shown in Figure 9. The results are similar to those of Tanaka (1966), Chiosi and Nasi (1974), Dearborn and Eggleton (1977), de Loore et al. (1977), Sreenivasan and Wilson (1978), Chiosi et al. (1978), de Loore et al. (1978).
The evolution of a 15 $M_\odot$ star is shown in Figure 10, with the assumption that mass loss is only important among late type supergiants. The star executes a long blue loop and ends as a star of 3.6 $M_\odot$. The hydrogen envelope has X=0.36. For very high masses the evolution is similar to the case of sudden mass loss : the star describes loops in the HRD (see Figure 10, 2 upper tracks) with rapid leftward motion and motions towards the right on a nuclear time scale. Much of the time is spent in an unstable area, bordered by log T_{eff} = 3.63 and log T_{eff} = 3.73, forming a yellow supergiant region. In contrast to this, a star losing mass continuously spends most of the time as a blue supergiant.

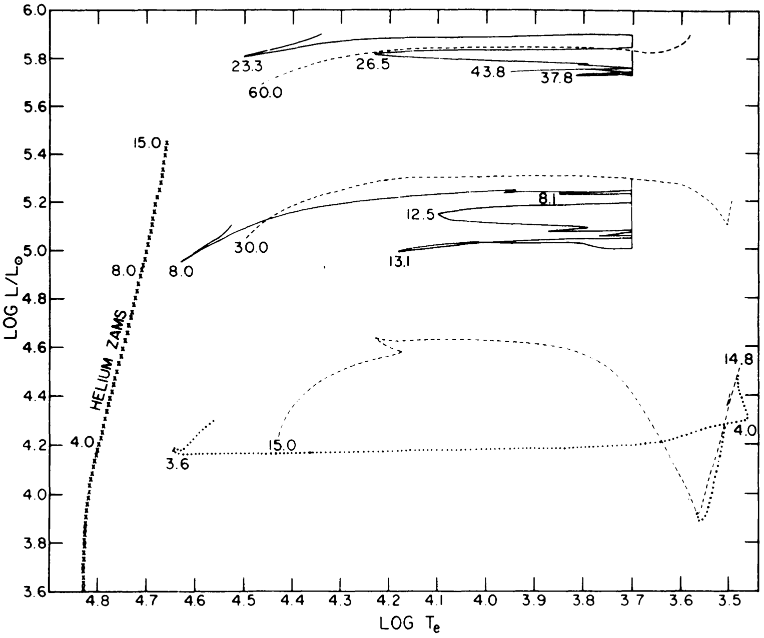

Figure 10. HR diagram showing the evolutionary tracks up to the stage of helium exhaustion for the conservative case (dashed curves), for the case of mass loss during the late supergiant stage (dotted curves) and for the case of sudden mass loss in the region of yellow supergiants of very high luminosity (Stothers and Chao-wen Chiu, 1978).

5. COMPARISON WITH OBSERVATIONS

5.1. Mass-luminosity relation

a) Masses of stars in the core hydrogen burning phase

A careful compilation of masses, luminosities and temperatures of a large number of OB-stars was carried out by Snow and Morton (1976); these data can be used for a comparison with the results of our calculations, and since the computations of Chiosi et al. (1978) and Stothers and Chao-wen Chiu (1978) are comparable, also with those. In Figure 11 are shown equal mass curves (EMC). These EMC are not identical with evolutionary tracks as is the case for conservative evolution. The EMC are in the case N=300 much steeper than the EMC for N=0. During shell burning the evolution is so fast that although the mass loss rates are considerable the stars lose only marginal fractions of their mass; for this stage the EMC may be considered as evolutionary tracks. In Figure 11 are shown also the stars of Snow and Morton's list, of classes V, IV and III with masses, derived by comparison with Stothers' conservative tracks. These values are compared with the EMC derived from our calculations for N=300. The agreement is very good. At the end of core H-burning the EMC are again very close to the conservative evolution tracks for the same mass. Consequently here again the mass estimates for both

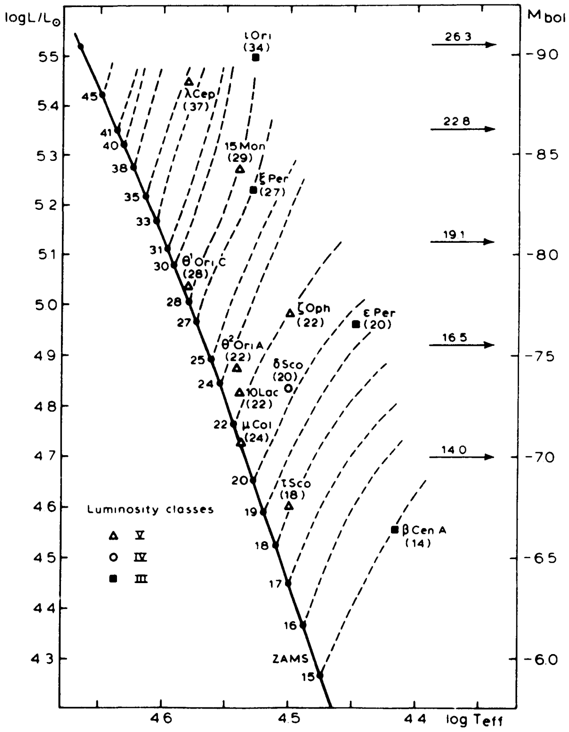

Figure 11. The equal mass curves for hydrogen core burning (dotted lines) for the case N=300 are compared with masses of stars of luminosity classes III, IV and V derived from the observations by using conservative evolutionary tracks (Stothers, 1972). The arrows indicate the luminosity of the equal mass curves during hydrogen shell burning.

cases are similar. The largest differences are found near the start of the contraction phase. Hence mass estimates of stars in the core hydrogen burning phase based on conservative tracks will differ only slightly from estimates based on tracks with mass loss, even if the mass loss is large (N=300).

b) OBA-Supergiants

During hydrogen shell burning the luminosity of the star remains nearly unchanged as well for mass losing tracks as for the conservative case. This enables us to establish a mass-luminosity relation for supergiants not dependent on the spectral type. No masses of supergiants are known accurately enough; only 6 supergiants could be found for which a reasonable accurate mass can be derived (ζ Ori, ε Ori, η CMa, o^2CMa, HD 7583, HD 33579). The masses and luminosities of these stars, with 1σ error bars are plotted in Figure 12. The uncertainty in the masses is too large to draw any firm conclusion about the exact value of N.

c) The lack of very luminous stars with $M_{bol} < -9$ ($\log L/L_{\odot} > 5.5$) and log T < 4.25 cannot be explained by a luminosity decrease during H shell

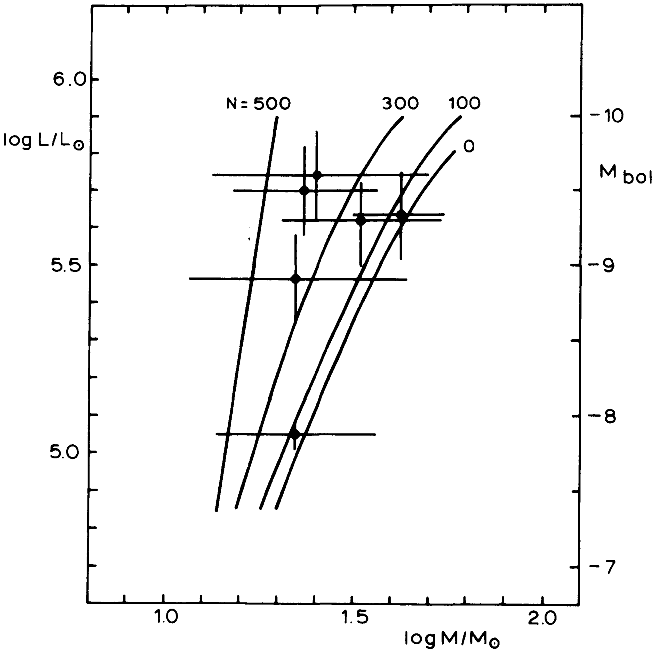

Figure 12. The observed masses and luminosities of six supergiants (with 1σ error bars) compared with the predicted mass-luminosity relation for different mass loss rates (N=0, 100, 300, 500).

burning since this phase is too short to allow a significant mass decrease, nor can this be explained by a speeding up of the evolution on the horizontal track (de Loore et al. 1977).

d) The assumption of a strong stellar wind has the consequence that in the main sequence band a variety of stellar remnants are produced that should be observed as "overluminous"(= undermassive), helium enriched or nitrogen rich. Among the population I stars of the galaxy, helium stars, OBN stars and Wolf Rayet stars show these characteristics. This is valid for Cox-Stewart opacities as well as for Carson opacities (Stothers and Chao-wen Chiu, 1978). Table 3 shows the changing atmospheric abundance and the N(H)/N(He) ratio for various evolutionary stages and different initial masses (N=300). Figure 13 shows again the evolutionary tracks for massive stars and the regions of the HRD where the WN 7 and 8 stars are located, the WR (WC) stars as well as the position of the optical companions of 5 massive X-ray binaries (Conti, 1978). As can be seen from the figure, single WN 7 and 8 stars might have evolved in a natural way from O stars with masses between 80 and say 120 $M_{\odot}$ with strong stellar winds. The formation of other WR stars, with more helium in the atmosphere cannot be explained by this mechanism. The evolutionary scenario of Conti (1976) implying that Of-stars would evolve into transition WR stars and further into WR stars is only partially confirmed. It is not easy to explain how such hydrogen deficient

X_{at}	N(H)/ N(He)	100 M_o			80 M_o		
		$t/10^6$	log L	log T_{eff}	$t/10^6$	log L	log T_{eff}
0.7	10.37	0	6.06	4.72	0	5.90	4.71
0.6	6.5	1.785	5.95	4.67	2.253	5.80	4.63
0.5	4.25	2.430	5.94	4.64	2.858	5.80	4.61
0.4	2.8	2.882	5.94	4.63	3.230	5.92	4.27
0.36	2.5	2.992	6.06	4.34			

X_{at}	N(H)/ N(He)	60 M_o			50 M_o		
		$t/10^6$	log L	log T_{eff}	$t/10^6$	log L	log T_{eff}
0.7	10.37	0	5.67	4.68	0	5.52	4.66
0.6	6.5	2.954	5.59	4.59	3.726	5.45	4.53
0.5	4.25	3.657	5.61	4.56			
0.4	2.8						
0.36	2.5						

Table 3. Atmospheric hydrogen abundance (X_{at}) and the ratio N(H)/N(He) for various evolutionary stages and different initial masses (N=300).

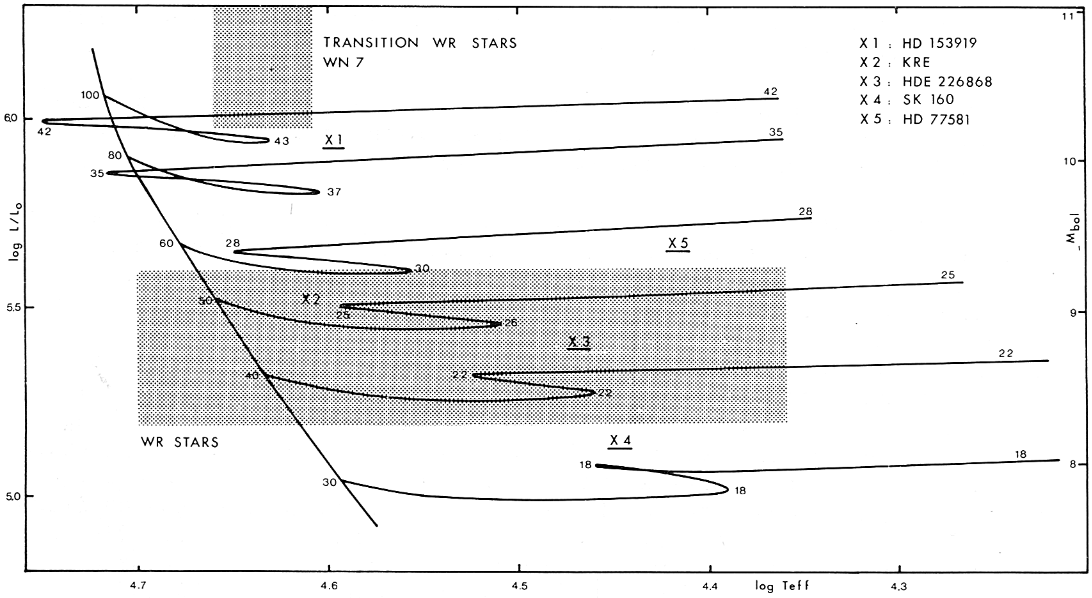

Figure 13. Evolutionary tracks for stars with initial masses between 30 and 100 M_o losing mass, and the position of WN7 stars, WR stars and companions of massive X-ray binaries.

stars could be formed by single star evolution : if the mass estimates for these stars are correct, they cannot have had as ancestors stars with masses larger than 60 M_o; for lower masses however the mass loss rates are not sufficiently high to allow the rich layers to show up at the surface. Two alternatives remain for these WR stars :

a) they are binaries (Kuhi, 1973) : for massive binaries it is perfectly possible to remove 70% of the stellar mass and to form pure He stars (Paczynski, 1967). We would therefore argue in favor of the picture that even the presumed "single" WR stars are in fact binaries composed of a pure He-star and a neutron star companion (van den Heuvel and de Loore, 1973; Massevich, Tutukov, Yungel'son, 1976; de Loore et al. 1975, 1977; van den Heuvel, 1976). Although it would be very difficult to detect the binary character of such systems, a clue could perhaps be found in the distance determinations : indeed as a consequence of the supernova explosion involved in the formation of the neutron star, such systems would have undergone an extra kick, and should on the average be farther away from the galactic plane than the other WR binaries composed of a luminous star and a helium star. Tentative observations carried out by Stenholm and Lundstrom (1977) seem to give some evidence to this idea. In this context, HD 197406 should be mentioned : this star is situated at z ~ 1 kpc from the galactic plane; it is probably a runaway star, a single line spectroscopic binary with a low mass companion, accelerated out of the galactic plane ~ 5.10^6 years ago (Moffat and Seggewiss, 1978).

b) Another solution of the problem is suggested by Chiosi et al. (1978). After the acoustic flux regime, when the relative mass of the core is 0.65-0.68 the star moves towards the left in the HRD. The lifetime spent in the region of large mass loss decreases with the initial mass loss. Average mass loss rates in the acoustic flux phase increase with luminosity. A consequence of the combination of these two effects is that the largest mass loss occurs during the lifetime spent in the yellow-red supergiant stage. Optimum mass loss occurs for stars with initial masses between 18 and 35 M_o. These stars could account for the box of WR stars. In this picture the occurrence of single WC stars would correspond with the fraction of the core He-burning phase after the removal of the H-rich envelope and before He-exhaustion.
However the large mass loss rates required are not in agreement with the values of Reimers (1977) for late supergiants. Moreover, the sense of the tracks, when rather large mass loss rates are applied, seem to go into the wrong sense : it is hard to understand that the luminosities will increase when extremely large mass loss rates are used. We have calculated a sudden mass loss, for a stellar wind remnant of 17.8 M_o, remnant of an initial 30 M_o star. A mass loss rate of 0.004 $M_o yr^{-1}$ was adopted when log T_{eff} attained a value of 3.85. Entropy losses were taken into account. Figure 14 shows the track. The luminosity drops and the star evolves towards the blue part of the HRD. As pointed out by Willis and Wilson (1978) the evolution is critically dependent on the mass loss rate. They argue that large mass loss rates for O type supergiants and Of stars could have been higher in previous stages. In the evolutionary computations with mass loss carried out with Carson opacities (Stothers and Chao-wen Chiu, 1978) the mass loss rates needed

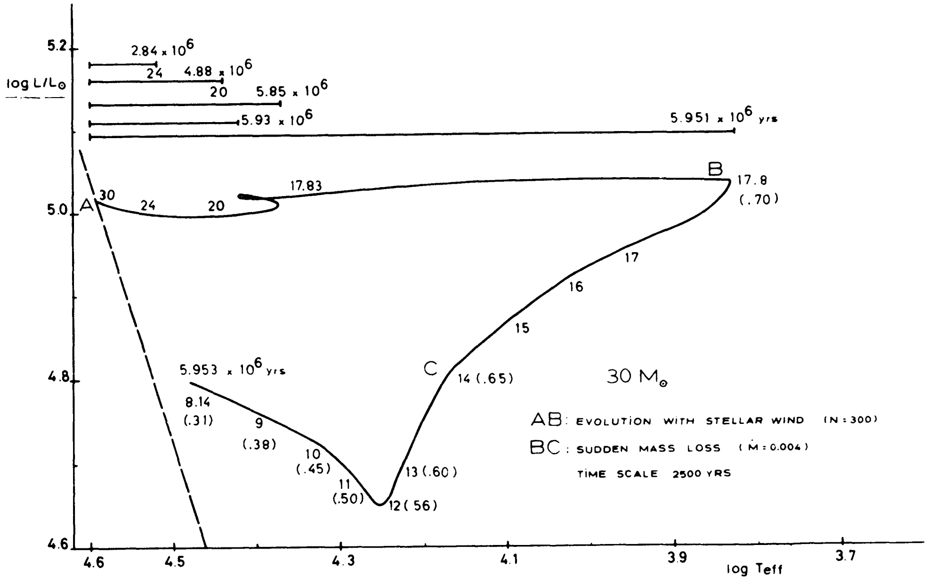

Figure 14. The evolutionary track for an initial mass of 30 M_0, first losing mass by stellar wind, leaving a remnant of 17.8 M_0. During the stage of red supergiant when log T_{eff} reached a value of 3.85 a sudden mass loss ($4.10^{-3} M_0 yr^{-1}$) was adopted, causing the star to move towards the left, with decreasing luminosity.

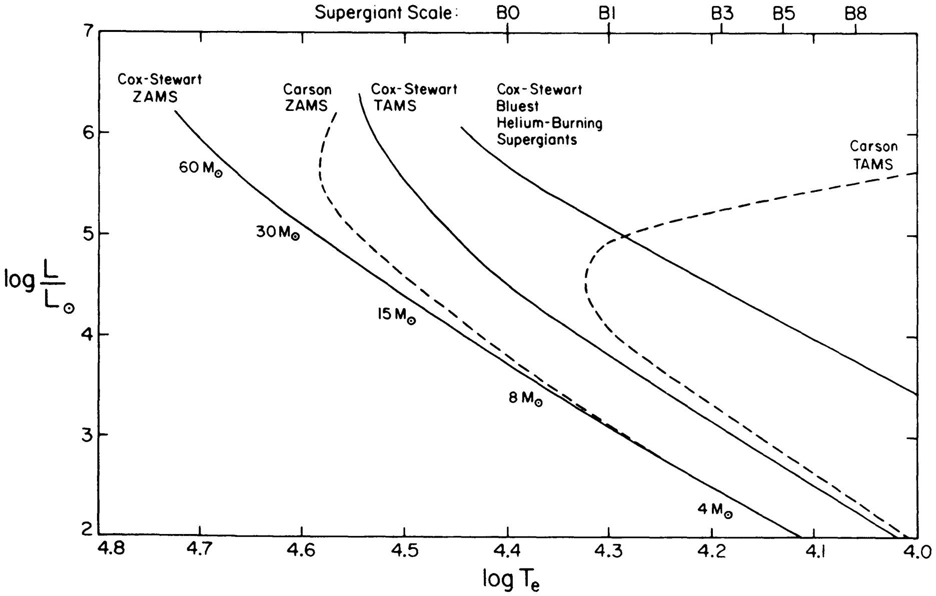

Figure 15. The ZAMS, the end of the hydrogen burning stage and the place of the bluest He-burning supergiants (Stothers, 1977).

for the explanation of the low number of very luminous blue supergiants of spectral type later than B3 are smaller. However they lead to effective temperatures for the ZAMS that seem to be rather cool (Figure 15) (log T_{eff} ~ 4.58). The observed spectroscopic and Zanstra effective temperatures of O stars seem to favor smaller opacities; the Carson opacities fail to account satisfactorily for the observations of the blue supergiants of lower luminosities.

5. EVOLUTIONARY COMPUTATIONS FOR LATE STAGES

Evolutionary computations for a 15 $M_{\odot}$ and 25 $M_{\odot}$ star were carried out by Lamb, Iben and Howard (1976) from main sequence through core carbon burning. The results of the processes of nuclear burning are dependent on the physical conditions in the star (temperature, density as function of radius, energy transport, and neutrino emission (pair, plasma, photo)). A very detailed treatment of convection is required; this was done by Lamb et al. (1976), not only for the later phases but also for previous burning stages. It turns out that a star of 15 $M_{\odot}$ evolves across the Hertzsprung gap burning carbon as a red supergiant, while a 25 $M_{\odot}$ star finishes its C-burning at higher effective temperatures, and does not reach the red-supergiant region. This explains not completely the minimum of M_{bol} ~ -9 for red supergiants; it would be very interesting to repeat the calculations, including mass loss.

6. INFLUENCE OF ROTATION

The influence of rotation on the evolution of stars losingmass was computed by Sreenivasan and Wilson (1978) for a 15 $M_{\odot}$ star. Rotation was considered as a factor leading to enhanced mass loss by assuming that gravity is weakened by the centrifugal force. Loss of angular momentum is carried through in the computations and conservation of energy and angular momentum are used to derive the spin down of such stars. Normally the effect of mass loss for an initial 15 $M_{\odot}$ star is marginal. Inclusion of rotation increases the mass loss rates to 3-4 10^{-7} $M_{\odot}yr^{-1}$ (ZAMS) and 4 10^{-9} $M_{\odot}yr^{-1}$ (red giant phase). The mass decreases to ~10 $M_{\odot}$.

7. INFLUENCE OF CONVECTION

One of the main features caused by mass loss is an increase of the main sequence lifetime. In that respect the treatment of convection, causing mixing and hence more efficient hydrogen burning is important. Very recently Roxburgh (1978) developed a new convection theory, retaining the kinetic energy flux and assuming the viscous dissipation to be small. The calculations show that with this new treatment the convective cores contain now 50 to 70% more mass than other models and the stars have slightly lower luminosities. The net effect is that the extension of the main sequence of stellar evolution is about 70% larger than in previous determination, hence the main sequence lifetime is larger by ~ 70%.

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The author is indebted to E.P.J. van den Heuvel, C. Chiosi, P. Conti, J. Ziolkowski and J.P. De Grève for discussions and constructive remarks. Drs. Chiosi, Nasi, Sreenivasan, Stothers and Chao-wen Chiu are acknowledged for the permission to reproduce figures and for sending preprints.

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DISCUSSION FOLLOWING DE LOORE

Marlborough: Do the evolutionary calculations including mass loss, calculated by Stothers using Carson's opacities agree with evolutionary calculations including mass loss by other workers such as Chiosi et al.?

de Loore: They show the same overall characteristics as our tracks and those of Chiosi et al. in the sense that models with both sets of opacities can explain some special features of the HRD. However, there are differences in the interpretation : the Carson opacities used by Stothers and Chao-wen-Chiu are so dominant that other parameters as thermodynamic functions and even the mass losses are of minor importance. The influence of the Carson opacities is so overwhelming that all other factors are not significant, and the stars expand to their largest possible configuration.

Garmany: I want to ask a question about the physics behind the increased length of time a star spends on the main sequence in the case of mass loss compared to the conservative case. Is the reason for this simply the decreased mass of the star, or is it more complicated?

de Loore: It is just the decreased mass of the star.

Garmany: I ask this because I have seen the times for the cases of different rates of mass loss, and there seems to be a very big difference between the conservative case and the cases of different mass loss rates.

Massey: In the slide you showed of real stars on your HR diagram, you said that the numbers listed were the "observed" masses - since at least some (if not all) of those stars are single, could you clarify what you meant by "observed"?

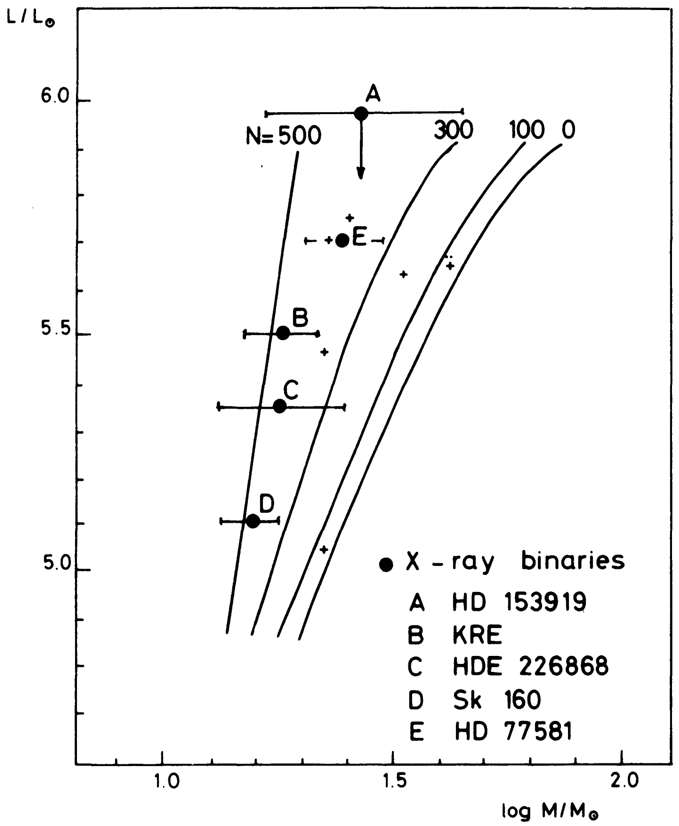
de Loore: The stars are selected from the list of Snow and Morton, and from spectral type and luminosity the masses are derived.

van den Heuvel: The masses in your diagram must be semi-empirical ones (i.e. not purely theoretical ones, derived from evolutionary tracks, because then you would be using a circular argument) obtained from a fine-analysis of the spectrum, for stars for which the distance is known. For such stars we have L, T_{eff} and $g = \frac{GM}{R^2}$ which allows one to determine M.

Hutchings: Please clarify what supergiant masses you used to determine N. (The X-ray primaries are all undermassive by ~ 50%, and the mass of Cen X-3 is certainly not well known at present).

de Loore: The supergiant masses were derived from effective temperatures and g-values (or radii) from ζ Ori, ε Ori, η CMa, HD 7583 and HD 33579. The optical companions of X-ray binaries were not used for this purpose.

Conti: I would like to show another numerical determination of the parameter N that Bert discussed in his paper. Stars are placed on a L,M diagram thus determining a value for N. In my paper on the companions to five X-ray binaries (Astron.Astrophys. 63, 225, 1978) I was able to determine L in four cases by appeal to the X-ray eclipse duration to give an estimate of R, and the spectrum to give an estimate of T_{eff}. The combination of these parameters gives the luminosity. The masses are found very accurately in those three cases where a pulsating neutron star is present, and somewhat less accurately in the other two by appeals to mass ratios. Placing these five stars on the L,M diagram is illustrated in the accompanying figure. The arrow for HD 153919 indicates that I may have overestimated R, hence L, if the stellar wind does indeed contribute to the X-ray eclipse duration (e.g., de Loore et al. 1978). Taken at face value, this figure indicates that for these binary objects, the value of N is near to 400 and larger on the average than shown by the single stars discussed by Bert in his paper. If this effect is real, it indicates that binaries



may well loose mass by a stellar wind at a more rapid rate than single stars. This has been also suggested by Hutchings using other kinds of data. Note that the mass loss suffered by those four stars with neutron star companions cannot have transferred to the collapsed object, it must have been removed from the system.

Ziolkowski: Would you like to comment on the point we discussed privately?

de Loore: The idea of Dr. Ziolkowski is that N-values of the order 100 are better in agreement with the observations than higher values. However the observed overluminosity can only be explained by rather large mass loss rates during the hydrogen burning stage.

Bisiacchi: From the observations of the number of O-type supergiants to dwarfs, we infer that only evolutionary models with large mass loss rates are able to reach, while burning hydrogen, the low gravity region of the log g, log T_{eff} plane where the supergiants lie. Our data agree with the tracks by Chiosi with α = 0.90, which apparently corresponds to your tracks for N = 300.