

Mira Variables: Theory versus Observation

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Abstract. The identification of the mode of pulsation of Mira variables remains a major problem. Recent angular diameter measurements of solar neighbourhood Miras suggest large radii consistent with first overtone pulsation. On the other hand, the pulsation velocity amplitudes of Mira variables point to fundamental mode pulsation. There is some marginal evidence from multiple modes in Miras that the prime pulsation mode is the first overtone, while distinct $(K, \log P)$ sequences in the Magellanic Clouds point to the Miras being fundamental mode pulsators. The above confusing evidence is presented and discussed. Finally, the $(K, \log P)$ relations in the SMC and LMC are compared in order to determine if there is any metallicity dependence.

1. Introduction

Reliable identification of the pulsation mode of the Mira variables is proving to be very difficult (Willson 1982; Wood 1990b). The basic reason for this difficulty is that Miras do not have sharp edges like our Sun, they just fade gradually as angular diameter increases. For example, in a theoretical model for a Mira atmosphere, the radius changes by a factor of ~ 2 as Rosseland optical depth changes from 0.01 to 1.0 (Bessell et al. 1989). The definition of radius is not simple in this situation, but modellers usually adopt a radius defined at Rosseland optical depth 1.0 (or $\frac{2}{3}$) (this radius will be referred to herein as the photospheric radius). In spite of the fact that the ratio of fundamental mode to first overtone period in a Mira is typically ~ 2.5 -3.5 (Fox & Wood 1982), observational estimates of Mira radii are so uncertain that a reliable identification of either fundamental mode or first overtone mode as the true pulsation mode has not yet been made.

In the last few years, some new observational and theoretical results relevant to the determination of the Mira pulsation mode have become available. These results are discussed in this review.

2. Angular diameter measurements of Mira variables

The angular diameter of a star such as a Mira variable varies greatly depending on the wavelength of observation (Bessell et al. 1989; Scholz & Takeda 1987). This is because of the large variation of opacity with wavelength caused by molecular species such as TiO. In deep bands, where the opacity is high, the

diameter (defined at monochromatic optical depth 1.0) can be at least twice as large as the stellar photospheric radius (Rosseland optical depth unity) (Bessell et al. 1989). However, in regions of the spectral continuum longward of $1 \mu\text{m}$ (and possibly shortward of $0.4 \mu\text{m}$), the radius at monochromatic optical depth unity appears to coincide well with the photospheric radius. Angular diameter measurements at these wavelengths should provide the most reliable estimates of Mira photospheric radii as there will not be a large correction factor for conversion of the monochromatic observed radius into the photospheric radius. Direct angular diameter measurements for Mira variables have been accumulating for about 20 years using speckle and lunar occultation techniques. Most recently, Tuthill et al. (1994) have used a non-redundant masking technique to measure the angular diameter of R Leo at wavelengths shortward of $1 \mu\text{m}$, and Haniff & Tuthill (1995) have applied the technique to a number of other Miras (see Haniff 1995, this volume). Because of the wavelength of these observations, the observed monochromatic radii need some quite uncertain correction factors for estimation of the photospheric radius.

Some angular diameter measures collected from the literature are given in Table 1. The table gives the monochromatic, uniform disk angular diameter ϕ at the wavelength of observation (λ). Adopting a limb-darkened rather than a uniform disk leads to similar angular diameters (Di Giacomo et al. 1991; Ridgway et al. 1992). No correction to photospheric angular diameter has been made, but since many of the observations were made in the H and K bands or around $0.4 \mu\text{m}$, these corrections should be small in such cases (i.e., the smaller of the measured angular diameters should be approximately equal to the photospheric angular diameter). The distance estimates required to derive radii were obtained by assuming the Miras lay on the (K, logP) relation for LMC Miras derived by Feast et al. (1989) and by using mean K magnitudes from the photometry in Catchpole et al. (1979). Finally, the period-mass-radius (PMR) relations for fundamental mode and first overtone pulsation (Fox & Wood 1982; Wood 1990a) were used to derive radii (R_0 and R_1 , respectively, in R_\odot) the stars would have in each mode for a mass of $1 M_\odot$.

Table 1. Observed Mira diameters

Star	P(d)	d(pc)	$\lambda(\mu\text{m})$	$\phi(\text{mas})$	R_{obs}	R_0	R_1	Reference
R Leo	310	113	2.16	30-35	352-411	224	405	Di Giacomo et al. 1991
			0.83, 0.90	44	517			Tuthill et al. 1994
			0.54-0.74	30-54	352-634			Labeyrie et al. 1977
o Ceti	332	113	<1	30-120	352-1410	232	424	Bonneau et al. 1982
			2.2	36.1	424			Ridgway et al. 1992
U Ari	371	743	1.62	6.11	472	246	456	Ridgway et al. 1979
S Psc	405	1240	2.23	3.84	495	257	484	Ridgway et al. 1979

It is clear from this table that the large observed radii of Mira variables provide strong evidence that these stars are first overtone pulsators. A similar conclusion was reached by Tuthill et al. (1994) and Haniff & Tuthill (1995). If the Miras are actually pulsating in the fundamental mode, as some of the evidence presented below suggests, then the radii need to be smaller by almost a factor of 2. In this case, some combination of the following factors must apply.

- (1) The distances to the solar neighbourhood Miras have been overestimated.

Wood (1990b) presented arguments that the $(K, \log P)$ relation for Miras should be metal dependent, with the result that the distances derived above should be reduced by 10-20%. By itself, this effect is much smaller than required to reduce the radii to fundamental mode sizes.

(2) The radii were systematically measured at maximum expansion of the star. However, many of the observations were made near maximum light when the star is expanding through mean radius (Bessell, Scholz & Wood 1995). The total variation in photospheric radius (maximum to minimum) in model Miras is 30-50% in the fundamental mode, much less in the first overtone, so that even if all observations were made at maximum expansion, the error in the mean radius estimate would be <25%.

(3) The PMR relations used, which were derived from hydrostatic models, do not apply to Miras pulsating at large amplitude. Very little theoretical work has been done on this problem but some preliminary results presented in Sections 3.1. and 3.2. suggest that the PMR relation for large amplitude pulsators is not greatly different from that derived from linear theory.

(4) The model atmospheres (Bessell et al. 1989; Bessell et al. 1995) which are used to relate the observed monochromatic radii to the photospheric radii are inadequate. There is a large amount of uncertainty in these models. Firstly, the density structure of the models was made up in a semi-empirical manner (Bessell et al. 1989) or derived from piston driven pulsating atmospheres (Bessell et al. 1995). Since schemes to produce Miras pulsating in the fundamental mode have recently been found (see Section 3.1., and Yaari & Tuchman 1995), better model structures will be possible in future. The models of Bessell et al. (1989) are much more extended than those of Bessell et al. (1995) due to the different density structures. But probably the largest uncertainty in the models lies in calculating the thermodynamic equilibrium and opacity in a molecular atmosphere with a velocity gradient throughout.

3. Nonlinear pulsation models

Producing nonlinear pulsation models for Mira variables presents significant difficulties. The fundamental mode in Mira models generally is very unstable and the amplitude grows enormously, leading to a series of thermal relaxation oscillations and mass loss unlike the variations seen in real Miras (Wood 1974; Tuchman, Sack & Barkat 1979). Attempts to produce first overtone pulsators with low masses and periods greater than ~ 300 days have also been generally unsuccessful as the fundamental mode tends to overwhelm the first overtone pulsation.

Yaari & Tuchman (1995) (see also Tuchman 1995, this volume) have reported recently some LMC Mira models that were pulsating stably in the fundamental mode. Two factors seem to have helped stabilise these models: firstly, the models are somewhat less luminous than expected for the LMC ($M_{\text{bol}}, \log P$) relation, and, secondly, the models were run for a long time starting from numerical noise so that a large envelope structural adjustment occurred before full amplitude was reached. Yaari & Tuchman found that their models contracted and ended up with a smaller period than they had initially - similar results were also found by Wood (1974) and Keeley (1970). In contrast, an example of a

fundamental mode pulsator which *expands* and lengthens its period is described below. This model had an abundance and luminosity appropriate for an LMC Mira. For comparison, a first overtone pulsator is also described.

3.1. Fundamental mode pulsation

The model discussed here is one of several for which stable fundamental mode pulsation has been produced. The nonlinear pulsations were performed with the code of Wood (1974), updated to include the new OPAL opacities (Iglesias & Rogers 1993) and molecular opacities as described in Chiosi, Wood & Capitanio (1993). When run with standard parameters, the models all showed the typical growth of the fundamental mode pulsation to amplitudes well beyond observed values, mass loss occurred from the surface and ultimately convergence failed (Wood 1974; Tuchman et al. 1979). Since Yaari & Tuchman (1995) had found that Mira models underwent a significant amount of envelope relaxation over hundreds of years of pulsation, a method of stabilising the violent initial pulsation was sought. Basically, more pulsation damping was needed, and this was provided by the artificial viscosity used to treat shock waves (Richtmyer & Morton 1967). The artificial viscosity parameter α^2 was increased to the large value of 15 for ~ 50 years while the pulsating envelope relaxed and was then reduced to a more normal value of 2.5 (together with the addition of Stellingwerf's 1975 velocity cutoff parameter $\alpha=0.1$).

Results for an LMC model with $M=M_{\odot}$, $L=4000L_{\odot}$, $Y=0.3$ and $Z=0.008$ are shown in Figures 1 and 2. In Figure 1, the large artificial viscosity was used up until time $t \sim 2.6 \times 10^4$ days and was then removed causing the amplitude of surface pulsation to increase enormously. Most envelope readjustment occurred in the stage of initial amplitude growth for $t < 6 \times 10^3$ days, and again when the excess artificial viscosity was removed around $t \sim 2.6 \times 10^4$ days.

Several points should be noted about the models. There is a very large extension of the surface layers caused by pulsation, and in the surface layers, motions with quasi-periods of ~ 3 -12 times the fundamental mode period occur (this is the cause of the variation with time of the mean radius of the outermost zone for $t > 4 \times 10^4$ days). Oscillations at $2 P_0$ are very common, these oscillations occurring near and above the photosphere (as can be in Figure 2). The total velocity amplitude near the photosphere is $\sim 40 \text{ km s}^{-1}$, while the surface layer velocities are small and quite irregular, as observed in optical Mira absorption lines (Joy 1954).

In order to see if the PMR relation derived from linear pulsation analysis of hydrostatic models gives a reasonable approximation to the PMR of pulsating Miras, the mean photospheric radius for the relaxed pulsating model was compared with the value predicted from the linear PMR relation. According to linear theory, for fundamental mode pulsation, P varies approximately as $R^{1.94} M^{-0.9}$ (Wood 1990a) so that $R = kP^{0.52} M^{0.46}$, where k is a constant and R is the photospheric radius in the hydrostatic model. For the present model, k was evaluated using the static radius ($191R_{\odot}$), period (209 days) and mass (M_{\odot}). Then the relaxed nonlinear period of 243 days used in this (linear) formula predicted a model photospheric radius of $206 R_{\odot}$ whereas the mean photospheric radius of the nonlinear model was actually $218 R_{\odot}$. This difference between predicted and actual radius is small, at least when it is seen that the linear theory

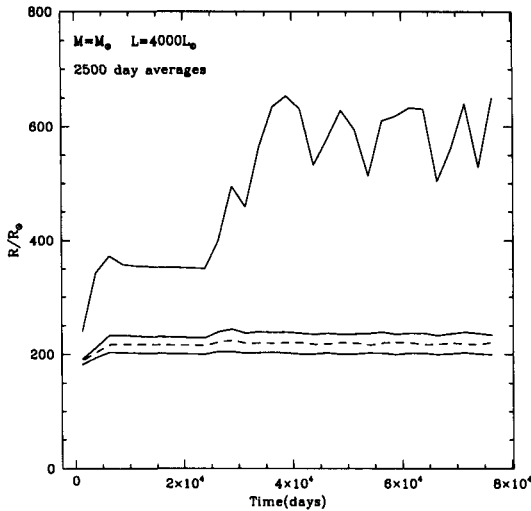


Figure 1. Radii of 3 mass points averaged over 2500 days (~ 10 pulsation cycles) plotted against time (solid lines). The point with the largest radius is the surface mass zone. The dashed line is the time-averaged photospheric radius.

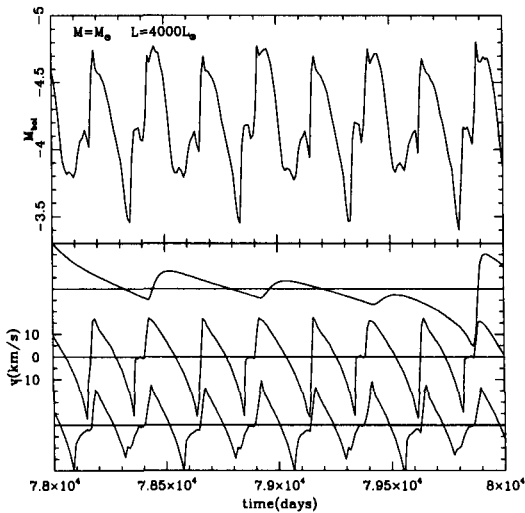


Figure 2. Bolometric luminosity (top panel) and velocity (bottom panel) of the 3 mass zones shown in Figure 1 plotted against time after the pulsation has reached a relaxed state.

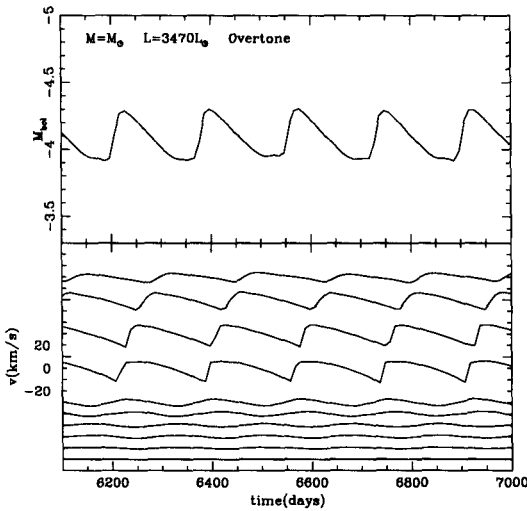


Figure 3. Bolometric luminosity (top panel) and velocity of 10 mass zones (bottom panel) plotted against time after the pulsation has reached a relaxed state. Rosseland optical depth unity lies near the mass zone whose velocity is given by the fourth line down in the bottom panel.

predicts a radius of $400 R_{\odot}$ when the nonlinear fundamental mode period of 243 days is used in the PRM for first overtone pulsation. The results indicate that structural adjustment of the envelope due to nonlinear pulsation does not radically alter the PMR relation. However, this statement needs verifying with a larger range of models, particularly models with longer periods.

3.2. Overtone pulsation

Production of first overtone pulsation models with masses of $M \sim M_{\odot}$ and periods $\gtrsim 300$ days is very difficult because, for these parameters, the fundamental mode always overwhelms the first overtone pulsation. Some overtone pulsators have been reported in the literature (Wood 1974, 1975; Tuchman 1991) but always with large mass ($\gtrsim 1.5 M_{\odot}$) and/or low luminosity and short periods. An example with $M = M_{\odot}$ and $L = 3470 L_{\odot}$ is shown in Figure 3. The nonlinear first overtone pulsation period of this model showed only small differences between the linear and nonlinear periods and static and mean pulsation photospheric radii of $<10\%$. Another first overtone model run for 160 years showed similar small changes in period and radius (this model had a stable fundamental mode). The PMR relation derived from linear pulsation theory should therefore be quite accurate when applied to full amplitude first overtone pulsators.

The important point to note in comparing Figures 2 and 3 is that the velocity amplitude of fundamental mode pulsation is about 40 km s^{-1} whereas it is only $\sim 25 \text{ km s}^{-1}$ for first overtone pulsation. The directly observed Mira pulsation velocity amplitudes are $\sim 24\text{--}28 \text{ km s}^{-1}$ (Hinkle, Scharlach & Hall 1984; Wood 1987). However, these velocities have not been corrected for geometric

and limb darkening effects which, for warmer stars, generally require a factor of 1.3 to 1.4 increase in observed velocity to get the true pulsation velocity. An estimate of the size of this correction factor for Miras can be obtained from the line profiles given for the Mira model atmospheres of Bessell et al. (1995). The line splitting shown for the fundamental mode pulsators in Bessell et al. is $\sim 20 \text{ km s}^{-1}$ near maximum light, slightly smaller than the observed line splitting ($\sim 24\text{--}28 \text{ km s}^{-1}$) observed in Miras (Hinkle 1978; Hinkle et al. 1984). The pulsation velocity amplitude in the fundamental mode models is $\sim 36 \text{ km s}^{-1}$, so that real Mira pulsation velocities of at least 36 km s^{-1} are predicted by the model atmosphere velocity correction factor. It should also be noted that the first overtone pulsator in Bessell et al. (1995) gave a line splitting of only $\sim 8 \text{ km s}^{-1}$ (for a true pulsation velocity amplitude of $\sim 17 \text{ km s}^{-1}$).

As noted before in the literature (Willson & Hill 1979; Bowen 1988; Wood 1990b; Bessell et al. 1995), the inability of low mass first overtone pulsators to give pulsation velocity amplitudes $\geq 25 \text{ km s}^{-1}$, and the agreement of theoretical fundamental mode pulsation velocities with observations provides one of the most powerful arguments for fundamental mode pulsation in the Miras. Some disagreement with this point of view has been expressed by Tuchman (1991).

4. Evidence for more than one pulsation mode

Possibly the most definitive method of determining the mode of pulsation of the Mira variables is the identification of multiple modes. Since $P_0/P_1 \sim 3$ while $P_1/P_2 \sim 1.2$ (Fox & Wood 1982), the period ratio of the dominant Mira mode to that of secondary modes should allow identification of the primary pulsation mode (under the assumption that the fundamental, first and second overtones are the modes most likely to be present).

4.1. Individual LPVs with multiple modes

Barthès & Tuchman (1994) have Fourier analysed AAVSO light curves of the two Miras S CMi and χ Cyg in order to look for secondary periods. They did find some formally significant secondary periods, compared them with the pulsation periods of models for Mira variables, and concluded that the best fit between observed and theoretical periods could be obtained assuming that the dominant pulsation mode was the first overtone (largely because of the existence of some observed secondary periods > 1600 days which could only be explained as due to the fundamental mode). However, the secondary periods found are only marginally significant and some secondary periods have no explanation in terms of radial pulsation modes. Better data is required to confirm these results.

One other characteristic feature of the light curves of Mira variables that is quite common is the existence of an oscillation at a period twice that of the primary period (Wood 1975). This can be explained not as a separate oscillation but as an oscillation induced in the outer layers with period twice that of the interior pulsation, as shown in Figure 2.

4.2. LMC sequences corresponding to different modes

In general, pulsation models of AGB stars show that as the star evolves to higher luminosities, lower order modes become unstable. In that case, one might

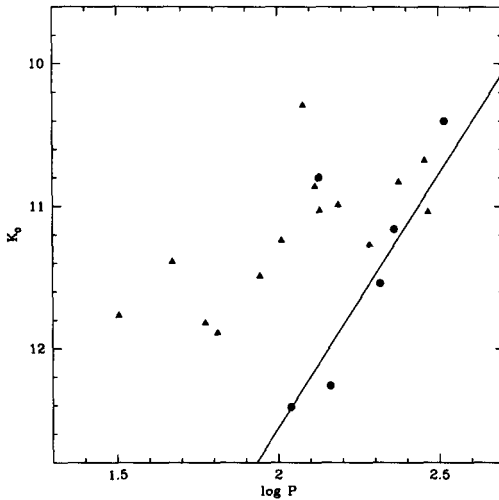


Figure 4. Small and large amplitude red variables in the $(K, \log P)$ plane. Periods are given in Sebo & Wood (1995a) for LPVs around NGC 1850 (circles) and Sebo & Wood (1995b) for variables around NGC 2058 and 2065 (triangles). The line is the sequence occupied by large amplitude (Mira) variables in the LMC (Feast et al. 1989; Hughes & Wood 1990).

expect to have groups of AGB stars pulsating in the fundamental mode, the first overtone, and so on. If these stars are in the LMC, then they will occupy separate period-luminosity sequences that will be readily seen in *apparent* magnitude since all the stars are at essentially the same distance. Sebo & Wood (1994, 1995a, 1995b) have recently found samples of red variables with both large and small amplitude ($\delta V \gtrsim 0.2$ mag.) in the LMC. A $(K, \log P)$ plot for all stars in the sample (Figure 4) shows that some stars lie on the well-known Mira sequence in the LMC but that there appears to be a second sequence (or sequences) at periods shorter by a factor of ~ 2.2 , with no stars at intermediate periods. This situation is just what would be expected if the Miras were fundamental mode pulsators while the second, shorter period sequence corresponded to first overtone pulsation.

5. Metal dependence of the Mira PL relation

As noted in section 2., there are good theoretical reasons to expect the Mira $(K, \log P)$ relation to depend on metal abundance (Wood 1990a). One test of the metallicity dependence of the Mira PL relation is to compare the $(K, \log P)$ relation for LMC and SMC Miras. This is done in Figure 5, from Wood, Moore & Bessell (1995), where the SMC LPVs are plotted along with the LMC relation of Feast et al. (1989) shifted by 0.4 magnitude to account for the different distance moduli of the SMC and LMC according to Cepheids (Caldwell & Laney 1991). It is clear that both the LPVs and Cepheids give the same distance modulus offset

between the LMC and SMC. The theoretical Cepheid calculations of Chiosi et al. (1993) predict very little dependence of the Cepheid PL relation on metallicity, at least for a change from SMC to LMC metallicity, so it seems that the SMC and LMC ($K, \log P$) relations for LPVs are similar. Note that if there was a factor of 2 difference in metal abundance between LMC and SMC LPVs (as for young stars in the Clouds - Russell & Bessell 1989), then the formula in Wood (1990a) would predict that the LMC stars were fainter by 0.13 magnitudes in K .

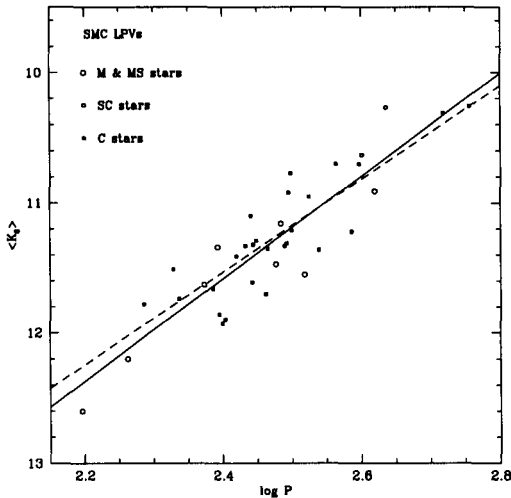


Figure 5. SMC LPVs in the ($K, \log P$) plane. The solid line is a least squares fit to the SMC data while the dashed line is the LMC relation of Feast et al. (1989) shifted by 0.4 magnitude to account for the different distance moduli of the SMC and LMC.

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Discussion

Kawaler: How does mass loss change the pulsation periods relative to the periods you compute for the models which aren't losing mass?

Wood: The effect of mass loss on pulsation has not been explicitly included in these calculations. At the surface, $P_{gas} = 0$ is assumed. For normal Miras, the mass loss rate is too low to be likely to affect the pulsation period.

Feast: (i) There is an additional piece of information regarding the mode of pulsation: The $2\mu\text{m}$ diameter of R Leo from an occultation agrees with overtone pulsation for Miras and all models agree that one gets to the photosphere at $2\mu\text{m}$. (ii) Your interesting results for low amplitude variables in the Magellanic Clouds are perhaps a parallel to the semi-regular variables in galactic globular clusters discussed by Patricia Whitelock.

Wood: (i) The Di Giacomo diameter at $2.16\mu\text{m}$ is certainly a strong piece of evidence for a large radius for R Leo. Additional measurements around $2\mu\text{m}$ should soon be forthcoming and will allow a check on this result. (ii) I agree.

Kovács: As far as the theoretical P-L relation is concerned: (i) should not one use the non-linear period rather than the linear one (see Y. Tuchman's talk)? (ii) is not the P-L relation for Miras rather a P-L-C (or some even more involved relation)?

Wood: (i) We don't have a theoretical P-L relation for Miras, only a P-M-R relation. The non-linear calculations I presented show that both P and R increase as a result of non-linear effects, but the P-M-R relation is not greatly altered. (ii) Theoretically, yes. Observationally, there is quite a tight P-L relation which indicates some additional constraint on P, M and R, a combination of stability and lifetime considerations, which we cannot currently compute.

Welch: Did I understand that the low-amplitude variables found near NGC 1850 and other clusters were being interpreted as having masses near $1M_{\odot}$? The turnoff mass of these clusters is closer to $5M_{\odot}$.

Wood: The LPVs are field variables, not showing any spatial connection with the clusters.

Haniff: Can observed radial velocities be used to infer the corresponding changes in "photospheric" radius as a function of pulsation phase? And if so, are these radius changes consistent with your theoretical modelling?

Wood: The observed velocities in the visible range always correspond to infall onto the star (absorption lines). Clearly, different mass zones are seen at different pulsation phases so that we can't integrate the velocity with time to derive motion of a mass point, or the photosphere. Similar remarks also apply to infrared velocities.

Maeder: Would you please comment on the driving mechanism of Miras' pulsations; in particular, is there any contribution from the molecules?

Wood: Driving is produced largely in the hydrogen ionization zone, and the helium ionization zones as well. There is essentially no driving produced in the outer, molecular layers. Details can be found in Fox & Wood (1982).

Kanbur: In order to get overtone models you might try giving it a 'high kick' so that it reaches its limit cycle from above. This works in the case of building ~10d Cepheid models.