NONLINEAR EVOLUTION OF THE RESISTIVE TEARING MODE

R. S. Steinolfson and G. Van Hoven
Department of Physics
University of California
Irvine, California 92717

ABSTRACT

Numerical solutions of the MHD equations are used to investigate the nonlinear behavior of the tearing instability. The mode evolves from a linearly growing excitation, followed by a period of greatly reduced nonlinear growth. Constant-Ψ solutions evolve much more slowly than comparable nonconstant-Ψ modes with orders of magnitude less conversion of the stored magnetic energy. The nonconstant-Ψ computations indicate a reduction by approximately 20% of the energy in the initial shear layer. For long-wavelength solutions, secondary-flow vortices, opposite in direction to the linear vortices, generate a new magnetic island centered at the initial x-point.

COMPUTATIONAL PROCEDURE

The nonlinear phase of the tearing mode (Furth et al., 1963) is studied in slab geometry using incompressible, constant-resistivity, MHD theory. The initially stationary plasma, with uniform thermodynamic properties, is embedded in a force-free, nondissipating magnetic field. A linear mode, at its maximum linear growth, provides the initial state for the nonlinear computation. We present results for a magnetic Reynolds number S (ratio of the resistive time to the hydromagnetic time) of 10^4 and values of the wavelength parameter $\alpha (2\pi a/\lambda$, where a is the shear scale and λ the disturbance wavelength) of 0.05, 0.13, and 0.50. [A larger parameter range and additional computational results are considered by Steinolfson and Van Hoven (1983b).] The mode with $\alpha = 0.5$ is a constant-Ψ solution in the linear regime, while the other two are nonconstant-Ψ , and the $\alpha = 0.13$ mode corresponds to maximum linear growth (Steinolfson and Van Hoven, 1983a).

The linear theory predicts a chain of x-points and islands in the magnetic field lying along the tearing surface (x-axis in our geometry) at y = 0. We isolate one wavelength of this initial disturbance and do

M. R. Kundu and G. D. Holman (eds.), Unstable Current Systems and Plasma Instabilities in Astrophysics, 273–276.

not allow it to interact with adjacent wavelengths. Because of the symmetries involved, our computation only extends from the center of one island (x = 0) to the adjacent x-point (x_{max}) and from the tearing surface (y = 0) to a relatively large distance y_{max}. The distance y_{max} is large enough that the perturbation is essentially negligible and decaying exponentially with y. Symmetry boundary conditions are applied at the remaining three boundaries. An expanding grid is used in the y-direction, with minimum spacing near y = 0, in order to resolve the tearing layer. The nonlinear equations are solved numerically using a fully-implicit, alternating-direction procedure.

NUMERICAL RESULTS

The evolution of two of the modes, as measured by reconnected flux and nonlinear growth, is shown in Fig. 1. The dashed curves represent continued linear growth. The long wavelength mode (α = 0.05) evolves almost identically to the α = 0.13 solution, in terms of these quantities, with somewhat (a few percent) more flux reconnection at the final time. Although the two nonconstant-Ψ modes display comparable nonlinear behavior, they are in sharp contrast to the considerably smaller reconnected flux for the constant-Ψ mode.

The total magnetic energy, per unit distance perpendicular to the tearing plane, removed from the magnetic fields is tabulated in the first row of part A of Table I. By contrast, the energy initially in the shear layer for the α = 0.05 solution is 3.6 x 10^{17} ergs/cm_z (scales inversely with α), which, for this case, means that the shear-layer energy has been reduced by 20%. The second and third rows in part A show the percent of the total energy that was removed from the x- and z-components, respectively. Longer wavelength modes remove more energy from the z-component, while none is removed from the z-component for the constant-Ψ solution (energy actually transfers into this component). The available energy in part A is distributed among the various components as shown (on a percentage basis) in part B. Note that more energy goes into heating as the disturbance wavelength increases, with less into the y-component of the magnetic field.

For all three of the modes, a secondary flow vortex, oriented oppositely in direction to the initial, linear vortex, forms near the x-point, as illustrated in Figure 2(a), which has a nonlinear y-scale. The velocity is parallel to the flux function contours on the left while some of the magnetic field lines near the tearing surface are shown on the right. The dashed flux-function contours indicate a clockwise vortex (the distorted linear vortex), and solid curves represent a counterclockwise vortex. The two modes not shown in this figure continue evolving for the duration of the calculation with the qualitative spatial behavior in Fig. 2(a). However, for the long wavelength mode, the secondary flows become strong enough to alter the basic magnetic topology and cause the formation of an additional magnetic island centered at the linear x-point [Fig. 2(b)].

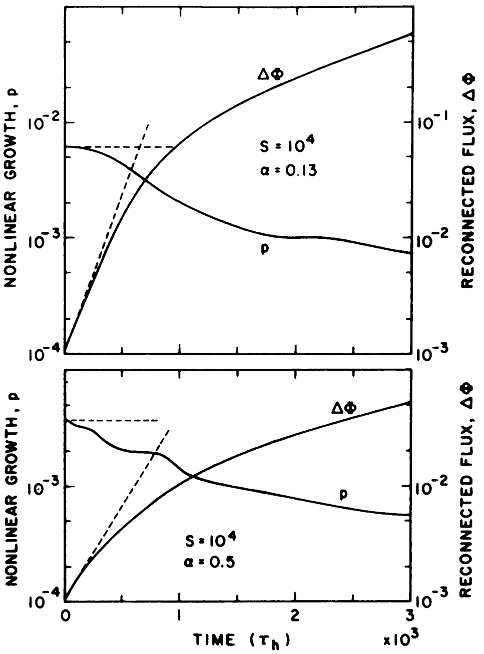

Figure 1. Nonlinear Evolution

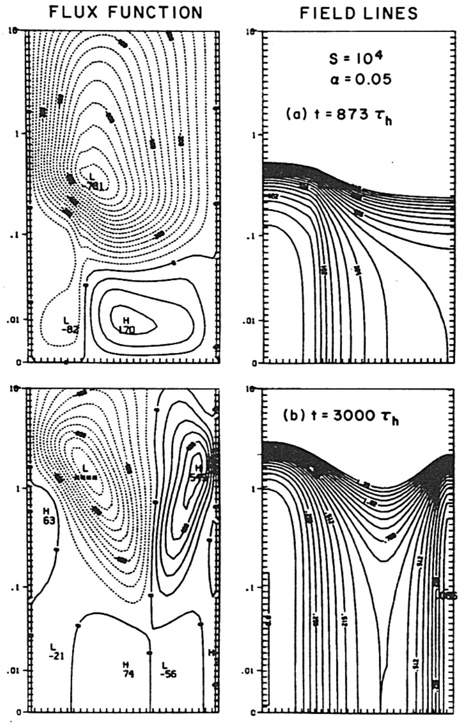

Figure 2. Formation of a new magnetic island

α	0.05	0.13	0.5
A. energy source:			
total	7.6×10^{16}	1.7×10^{16}	2.4×10^{13}
magnetic (x)	89.7	97.0	100.0
magnetic (z)	10.3	3.0	0.0
B. energy budget:			
magnetic (y)	8.0	16.2	49.2
magnetic (z)	0.0	0.0	2.6
kinetic (x)	4×10^{-2}	3×10^{-2}	8×10^{-2}
kinetic (y)	6×10^{-4}	5×10^{-4}	3×10^{-4}
kinetic (z)	3×10^{-3}	3×10^{-4}	5×10^{-5}
thermal	92.0	83.8	48.2

Table I. Energy balance. Total magnetic energy ($ergs/cm_z$) computed for reference B = 37.3 G, a = 10^7 cm; remaining energies given in percent.

CONCLUSION

A primary result in these computations is that the nonlinear evolution generally differs from one region of parameter space to another, and hence, a typical characterization of the evolution in the nonlinear regime is not possible. Some general statements that do apply to all solutions are: (1) The nonlinear spatial distributions of the physical variables differ substantially from the linear behavior; (2) Once nonlinear effects become important, the growth slows considerably from the linear rate; (3) Neither the linear growth rate nor the nonlinear growth is a good predictor of the nonlinear performance of a particular mode in terms of magnetic energy conversion; and (4) More of the stored magnetic energy is converted to thermal energy as disturbance wavelength increases (> 90% at $\alpha = 0.05$).

ACKNOWLEDGEMENTS

This work was supported by the Solar Terrestrial Theory Program of NASA under Grant NAGW-93. Acknowledgement is also made to the Institute of Geophysics and Planetary Physics at the Los Alamos National Laboratory, which provided funding under their grant program, and to the National Center for Atmospheric Research, which is sponsored by NSF, for the use of their computer facilities.

REFERENCES

Furth, H. P., Killeen, J., and Rosenbluth, M. N.: 1963, Phys. Fluids 6, pp. 459-484.
Steinolfson, R. S., and Van Hoven, G.: 1983a, Phys. Fluids 26, pp. 117-123.
Steinolfson, R. S., and Van Hoven, G.: 1983b, submitted to Phys. Fluids.

https://doi.org/10.1017/S0074180900075707 Published online by Cambridge University Press