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ARTICLE

Untimely destruction: pestilence, war, and accumulation in the long run†

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Abstract

This paper analyses the effects of disease and war on the accumulation of human and physical capital. We employ an overlapping generation framework in which young adults, motivated by old-age provision and possibly altruism, make decisions about investments in schooling and capital. A poverty trap exists for a wide range of constant war losses and premature adult mortality. If parents are altruistic and the sub-utility function for own consumption is more concave than that for their evaluation of their children's full income in adulthood, the only possible steady-state growth path involves full education. Otherwise, steady-state paths with incompletely educated children may exist. When mortality and destruction rates are stochastic, the initial boundary conditions and agents' beliefs have a strong influence on the paths generated by a sequence of shocks. Calibrating the model to Kenya, simulations for stochastic settings yield the finding that a trap exists and is always avoided, but the chances of a slow recovery are substantial.

Keywords: Capital accumulation and destruction; premature mortality; poverty traps; stochastic environments

1. Introduction

Dürer's woodcut, "The Four Horsemen of the Apocalypse", is a terrifying vision of the great scourges of humanity from time immemorial. This paper deals with three of them—pestilence, war, and death—and with their accompanying destruction of human and physical capital. Its particular concern is how these calamities affect the accumulation of both kinds of capital, with special reference to the existence of growth paths and poverty traps. Its treatment of these questions is necessarily stylized, simple, and, in contrast to Dürer's masterpiece, desiccated.

The distinction between human and physical capital is vital. Not only are they complementary in production but they are also subject to different, albeit not fully independent, hazard rates. The attendant risks are not, moreover, equally insurable. These considerations weigh heavily in the decision of how much to invest and in what form, with all the ensuing consequences for material prosperity in the long run.

A few examples of such calamities will convey some flavor of the historical dimensions of what is involved. The Black Death carried off about one-third of the entire European population

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between 1347 and 1352. The so-called "Spanish influenza" pandemic is estimated to have caused at least 50 million deaths globally between 1918 and 1920, with exceptionally high mortality among young adults. In recent times, the AIDS pandemic, far slower in its course like the disease itself, still threatens to rival that figure, despite the improved availability of antiretroviral therapies. Pestilence and war also ride together. Half a million died in an outbreak of smallpox in the Franco-Prussian War of 1870–1871 [Morgan (2002)]. For every British soldier killed in combat in the Crimean War (1854–1856), another six died of disease, and in the Boer War (1899–1902), the ratio was still one to three.

War losses in the 20th Century make for especially grim reading. Between 15 and 20 million people died in the First World War, the great majority of them young men. Almost two million French soldiers fell, including nearly 30% of the conscript classes of 1912–1915. Joining this companionship of death were over 2 million Germans, including almost two of every five boys born between 1892 and 1895 [Keegan (1998: 6–7)], almost a million members of the British Empire's armed forces, and many millions more in those of Imperial Austria, Russia, and Turkey. Its continuation, the Second World War, was conducted, in every respect, on a much vaster scale. Most estimates suggest that it resulted in at least 50 million deaths, directly and indirectly. Among them were 15 million or more Soviet soldiers and civilians, 6 million Poles (20% of the pre-war population), and at least 4 million Germans [Keegan (1990: 590–1)]. With these staggering human losses went the razing of German and Japanese cities and massive destruction in the western part of the Soviet Union as well as the states of Eastern Europe. The catalog of conflicts in the second half of the 20th Century is also unbearably long, with particularly appalling casualties in Southeast Asia and Rwanda.

Great epidemics and wars capture the headlines and grip the imagination, but the majority of those adults who die prematurely fall victim to low-level, "everyday" causes, especially in poor countries: notable killers are infectious diseases, accidents, violence, and childbirth. These are competing risks—one dies only once; but even in O.E.C.D. countries, their combined effect is not wholly negligible, and in many poorer ones, it is quite dismaying. According to the WHO (2007), those who had reached the age of 20 in the O.E.C.D. group could expect to live, on average, another 60 years or so, their counterparts in China and India another 50–55 years, and those in sub-Saharan Africa but 30–40. The odds that 20 years old in the O.E.C.D. group would not live to see his or her 40th birthday were 1 or 2 in a 100, for the 50th, 2.5–5 in a 100. These odds were a little worse for young Chinese, decidedly worse for young Indians, and for young Africans less favorable than one in five—in some countries where the AIDS epidemic was raging, indeed, scarcely better than even.

At the time of writing, the world is beset by the SARS-CoV-2 pandemic. The current global tally is about 630 million reported cases, scores of millions of them involving moderate or severe morbidity, and 6.6 million deaths, to say nothing of the ensuing short- and long-term burden of mental health problems and the disruption of children's education. The frequent emergence of new, viable variants threatens a continuation of the pandemic for years to come. In that event, it will cease to be a high-frequency shock like the Spanish influenza, becoming instead endemic. A salient feature of Covid-19 cases is the very high level of mortality among the old; the young, in contrast, rarely experience more than mild symptoms and very few die. This mortality profile is naturally accommodated in the theoretical structure developed in this paper, but we do not pursue it further.

Whether caused by great epidemics and wars, or by endemic communicable diseases and low-level conflicts, the resulting human and material losses have long-run as well as immediate economic consequences. Taking as given the technologies for producing output and human capital in the presence of these hazards, we address two questions, the answer to the first of which is the basis for investigating the second.

1. If mortality and destruction rates do not vary over time, are both secular, low-level stagnation and steady-state growth possible outcomes? If so, are both locally stable equilibria, thus establishing the existence of a poverty trap?
2. If mortality and destruction rates are stochastic, under what conditions would the economy fall into, and remain in, such a trap when it would otherwise grow, albeit not steadily?

The overlapping generation model (OLG) offers a natural framework within which to address these questions. In the variant adopted here, there are children, young (working) adults, and the old. Young adults decide how much schooling their children will receive and how much to put aside to yield a stock of physical capital in the next period. In doing so, they are bound by certain social norms, which govern the distribution of aggregate current consumption among the three generations. Untimely destruction can undo these plans. Children may die prematurely at some point in young adulthood, and war can wreak havoc on the newly formed capital stock: the lottery of such deaths and the destruction of physical capital involve two objects, not one. These losses, if they occur, will reduce the resources available to satisfy claims on consumption in old age in the following period. In making their decisions, parents may also be motivated by altruism toward their children, so that their premature deaths will be felt as a distinct loss quite independently of the ensuing reduction in old-age consumption under the prevailing social norms—and arguably all the more keenly if the children have been well educated. That is to say, providing more amply for old age through the accumulation of human and physical capital necessarily enlarges the children's opportunity set, and this consideration will normally promote accumulation.

The institutional form is assumed to be a very large extended family, in which the surviving young adults raise all surviving children. Given such pooling, the law of large numbers makes the level of consumption in old age—for those who survive to enjoy it—virtually certain when mortality and war loss rates are forecast unerringly. Yet even then, the idiosyncratic risk of dying earlier remains. War losses are uninsurable and operate much like cohort-specific mortality. When these rates are stochastic, they constitute unavoidable systemic risks, with consequent effects on investment in both forms of capital.

Our main insights are as follows. Balanced growth paths with endogenous physical and human capital may not exist [Uzawa (1961)], so we first establish conditions for the existence of two extreme steady states, namely, permanent "backwardness", wherein there is no schooling, and unbounded growth with a fully educated population, which we term "progress". Backwardness and progress can coexist as equilibria under various technologies and for a wide range of mortality and destruction rates, but neither need be an equilibrium.

Parents' altruism influences the set of equilibrium paths in two ways. First, if sufficiently strong, it can rule out backwardness. Yet a robust numerical example shows that a poverty trap can exist even with quite strong altruism. Second, progress is the only steady-state growth path if the sub-utility function for parents' own consumption is more concave than that for the evaluation of their children's full income. Under the converse of the latter condition, other steady-state paths with incompletely educated children may exist, some of them stationary, even if altruism is strong. These results stem from the fact that, with two objects entering the lottery, weaker concavity with respect to one also weakens diminishing returns to the realized pay-offs.

Turning to the second question, and using the answers to the first as foundation, we explore whether an economy can grow despite unforeseen outbreaks of war and epidemics. Such events, even if temporary and rare, may pitch a growing economy into backwardness. We establish that these risks depress investment in both physical and human capital, and only extreme destruction of physical capital could induce an increase in schooling. We also establish thresholds for human and physical capital above which an economy can withstand a particular configuration of shocks: large endowments confer robustness. We show with simulations that the duration of a specific sequence of adverse events is often decisive in determining whether an economy can regain growth. In other simulations, we explore the random development paths of economies in a common, stationary, stochastic environment, but with different starting capital stocks. We also apply the model to Kenya, whose AIDS epidemic is among the world's worst. Based on a calibration for the period 1920–1990, corresponding simulation analyses are undertaken for 1990–2070. A trap exists and is always avoided, but the chances of a slow recovery are substantial.

There is an extensive literature on the relationship between the health of populations and economic activity. Notable is the general empirical observation that good health has a positive and statistically significant effect on aggregate output [Barro and Sala-I-Martin (1995); Bloom and Canning (2000); Bloom et al. (2001); Bloom et al. (2019)]. Especially relevant for present purposes is a body of work on the macroeconomic effects of AIDS, in which there are varying points of emphasis. Corrigan et al. (2005a, 2005b) adopt a two-generation OLG framework in which the epidemic can affect schooling and the accumulation of physical capital, but expectations about future losses play no role. In two contrasting studies of South Africa, Young (2005) uses a Solovian model to estimate the epidemic's impact on living standards through its effects on schooling and fertility, with a constant savings rate. Bell et al. (2006b) apply a two-generation OLG model with pooling through extended families and a vital role for expectations, but no role for physical capital.

Closely related theoretical contributions include Chakraborty (2004), in whose OLG framework endogenous mortality is at center stage. Better health promotes growth by improving longevity, and investment in health emerges as a prerequisite for sustained growth. Individual investment in health is also the prime mechanism in Augier and Yaly (2013).[1] Young adults, who have only wage income, pay a fixed fraction thereof as taxes into a fund managed by the government. This fund provides all capital for the next period, with the gross returns going to the survivors. In Boucekkine and Laffargue's (2010) two-period framework with heterogeneous levels of human capital, a rise in mortality among adults in the first period reduces the proportion of young adults with low human capital in the second period because the mortality rate among children at the end of the first rises more sharply in poor families. The number of orphans in the first period increases, so that the proportion of young adults with low human capital in the second period will increase if orphans get little education. Bell and Gersbach (2013) analyze growth paths and poverty traps when epidemics take the form of two-period shocks to mortality, paying particular attention to their effects on inequality in nuclear family systems.

In an earlier study of the impact of mortality shocks on long-run development, Lagerlöf (2003) employs a model without physical capital wherein the escape from Malthusian stagnation arises due to a sequence of mild epidemic shocks. The probability of survival improves with the stock of human capital. A larger population reduces that probability, but allows for a faster transmission of human capital. A sequence of sufficiently favorable shocks can lead to a stock of human capital that renders the economy less susceptible to further shocks. Drawing on a unified growth-theory model à la Galor (2005), Aksan and Chakraborty (2014) demonstrate that an escape from Malthusian stagnation cannot occur due to a reduction in child mortality alone, but also requires lower adult morbidity; for otherwise the incentives to invest in human capital are insufficient. Other important contributions to the development-disease nexus are Chakraborty et al. (2010, 2016), wherein the authors present a dynamic law for the prevalence rate of transmissible diseases that depends on epidemiological parameters such as the number of contacts and the probability of being infected. In both papers, there are multiple steady states. The former paper focuses on the macroeconomic impact of Malaria and HIV in sub-Saharan Africa (SSA), while the latter more broadly investigates the overall disease burden in SSA and compares different types of policies to lift an economy out of a poverty trap. Building on this work, Gori et al. (2021) analyze optimal policies for mitigating the HIV/AIDS epidemic in SSA when public policies are the only means to reduce the spread of the virus.

A salient feature of many of these studies is the central importance of premature adult mortality. The current stock of physical capital, when it does appear, is not subject to similar, exogenous hazards. Voigtländer and Voth (2009, 2013) espouse a Malthusian explanation of the rise of growth in early modern Europe. Disease and war rode together, but "[war] destroyed human life quickly while not wreaking havoc on infrastructure on a scale comparable to modern wars". [Voigtländer and Voth (2013: 175)]. In contrast, the possibility of destruction on such a scale is an essential element of the present paper, in which there are no fixed factors like land. In this regard, exponential depreciation at a constant rate in Solovian models does not lend itself to the task of representing the shocks of war losses. To our knowledge, no other contribution addresses the possibilities of long-term stagnation and growth when both forms of premature destruction are salient features of the environment wherein agents make decisions about accumulation.

Accumulation in the broad sense also involves fertility decisions, which are a central feature of Young's (2005) analysis. He found that the HIV/AIDS epidemic was inducing lower fertility and hence, through the usual Solovian mechanism, higher per capita income in the longer run. He arrived at the same conclusion in a follow-up study of 27 countries in sub-Saharan Africa [Young (2007)]. This did not go uncontested. Kalemli-Ozcan and Turan (2011) and Kalemli-Ozcan (2012) found that Young's results are not robust with respect to specification. More importantly, indicators of the strength of the epidemic had a positive effect on fertility in between-country and between-region specifications, though the effect was ambiguous for within-country ones. Such a positive effect is a troubling development when the demographic transition is already tardy and slow. Gori et al. (2020) provide theoretical underpinnings for this effect when parents are faced with a pressing tradeoff between the quality and quantity of their offspring. Fertility is indeed a central element in decisions about accumulation. In treating it as exogenous, we plead that dealing with education and savings involves difficulties enough. More generally, agents are denied opportunities to change their behavior in ways that reduce individual mortality, nor are there public health measures, which require collective action and taxation.

The paper's theme is also broadly related to the existence and relevance of balanced growth paths. The classic problem examined by Uzawa (1961) is whether such paths exist in neoclassical growth models with capital accumulation, population growth, and labor- or capital-augmenting technological progress. Wan (1971) and Schlicht (2006), with clarifications by Jones and Srcimgeour (2008), completed Uzawa's argument that, with constant rates of population growth and technological progress, the existence of such paths requires either a unitary elasticity of substitution between capital and labor, or purely labor-augmenting technological progress. Grossman et al. (2017) present a class of production functions for which balanced growth in a neoclassical growth model with capital-augmenting technological progress is possible. This possibility arises when education is endogenous and capital is more complementary with schooling than with raw labor. In this connection, we explore a complementary balanced growth problem: Does balanced growth exist in an OLG framework with endogenous physical and human capital accumulation, with or without altruism? We establish conditions on the sub-utility functions with respect to altruism and own consumption that allow balanced growth, but without imposing very strong restrictions on the production technology.

The paper is organized as follows: Section 2 lays out the model's structure, Section 3 the family's decision problem under perfect foresight. Section 4 analyzes steady states, which necessarily involve unchanging mortality and destruction rates. It begins by establishing the conditions for stable backwardness. It then demonstrates that both these conditions and those under which steady-state growth is also an equilibrium can be satisfied simultaneously. Settings in which the destruction rates are stochastic are analyzed theoretically in Sections 5 and 6. Section 7 is devoted to a variety of illustrative numerical simulations. The application to Kenya is treated in Section 8. It is followed in Section 9 by a discussion of Europe's experience in the 19th century and the historical role of human capital. Section 10 draws together the chief conclusions.

2. The model

There are three overlapping generations: children, who split their time between schooling and work; young adults, who work full time; and the old, who are inactive. The timing of events within

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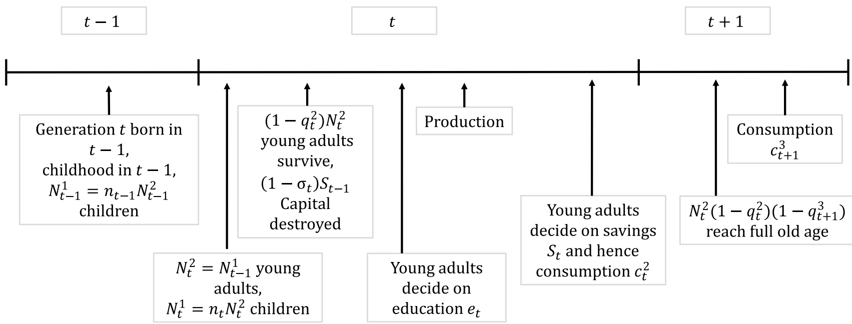

Figure 1. Sequence of events for the generation born in period $t-1$.

period t relates to those individuals born in period $t-1$, who become young adults at the start of period t. It is displayed in Figure 1. Those individuals who survive early old age in the following period $t+1$ therefore live for the three periods $t-1$ to $t+1$.

All individuals belong to numerous, identical, and very large extended families. The number of young adults in each family at the beginning of period t is N_t^2. They marry and have children at once. Each couple within the extended family has $2n_t$ children, all of whom survive into adulthood in the next period. Thus, n_t is the net reproduction rate (NRR). Death then claims some young adults and some of those who have just entered old age. The surviving young adults rear all children collectively and decide how to allocate the children's time between schooling and work, as well as the resulting aggregate output between consumption and savings. The numbers of young adults and their offspring who reach maturity are $N_t^2 = n_{t-1}N_{t-1}^2$ and $N_t^1 = n_t N_t^2$, respectively. Let q_t^a denote the mortality rate among age group $a(=2,3)$. Then, the numbers of young and old adults who make claims on output in period t are as follows:

$\left(1-q_t^2\right)N_t^2$ young adults survive to raise all children, and

$\left(1-q_t^3\right)N_t^3$ old adults live for three full periods, where $N_t^3 = \left(1-q_{t-1}^2\right)N_{t-1}^2$.

Two social rules govern consumption-sharing in the extended family:

1. When each surviving young adult consumes c_t^2, each child consumes βc_t^2 ($\beta < 1$).
2. All surviving old adults receive the share ρ of the family's current "full income", $\bar{Y}_t$, which is the level of output that would result if all children were to work full time.[2] Since the extended family is very large, each surviving old adult will consume

$$c_t^3 = \frac{\rho \bar{Y}_t}{\left(1-q_t^3\right)N_t^3}. \tag{1}$$

Output is produced under constant returns to scale by means of labor augmented by human capital (i.e., labor is measured in efficiency units) and physical capital, which is made of the same stuff as output. All individuals are endowed with one unit of time. The time a child spends in school in period t is denoted by $e_t \in [0,1]$. Each young adult possesses λ_t (≥ 1) efficiency units of labor, each child $\gamma \in (0,1)$ units. Each fully educated child ($e_t = 1$) requires $w \in (0,1)$ young adults as teachers, so that the direct cost of providing each child with schooling in the amount e_t is $w\lambda_t e_t$, measured in units of human capital. The total endowment of the surviving young adults' human capital is $\Lambda_t \equiv \left(1-q_t^2\right)N_t^2\lambda_t$; $\bar{L}_t \equiv \Lambda_t + \gamma N_t^1$ is the household's endowment of labor (measured in efficiency units) at time t. The amount of labor supplied to the production of the aggregate good is

$$L_t \equiv \left[\left(1-q_t^2 - wn_te_t\right)\lambda_t + n_t\gamma\left(1-e_t\right)\right]N_t^2.$$

The aggregate savings of the previous period, S_{t-1}, are also subject to losses early in t; what remains has a lifetime of one period. The capital stock available for current production is $K_t = \sigma_t S_{t-1}$, where $\sigma_t \in (0, 1]$ is the survival rate of savings. The distinction between aggregate output and full income is important, the former reflecting the choice of e_t. The level of aggregate output, Y_t, is

$$Y_t = F(L_t, \sigma_t S_{t-1}),$$

where the function F is assumed to be monotonically increasing and strictly concave in both arguments, continuously differentiable and homogeneous of degree 1, with both inputs necessary in production. Putting $e_t = 0$ to yield output when the whole endowment is engaged in production, we obtain full income:

$$\bar{Y}_t \equiv Y_t(e_t = 0) = F\left(\Lambda_t + \gamma N_t^1, \sigma_t S_{t-1}\right).$$

Current aggregate output finances the consumption of all three generations in keeping with the social rules and savings to provide the capital stock in the next period:

$$P_t c_t^2 + S_t + \rho \bar{Y}_t = Y_t, \tag{2}$$

where $P_t \equiv \left(1 - q_t^2 + \beta n_t\right) N_t^2$ is the price of one unit of a young adult's consumption in terms of output, the numéraire.

The formation of human capital involves the contributions of parents' human capital as well as formal education. The human capital attained by a child just before reaching adulthood is assumed to be given by

$$\lambda_{t+1} = z_t h(e_t)\lambda_t + 1. \tag{3}$$

The positive multiplier z_t represents the strength with which capacity is transmitted across generations; it may depend on the number of children each surviving young adult raises. The function $h(e_t)$ may be thought of as representing the educational technology, with a fixed pupil-teacher ratio of $1/w$. Let h be increasing and differentiable on $(0, 1)$, with $h(0) = 0$. The property $h(0) = 0$ implies that unschooled children attain, on reaching adulthood, only some basic level of human capital, which has been normalized to unity. In view of the fact that the early stages of education involve laying the foundations to develop skills, it is further assumed that $h'(e)$ is bounded as $e \to 0+$.

2.1. Preferences and choices

Young adults, who make all allocative decisions, have preferences over lotteries involving current consumption, consumption in old age and, if they are altruistic, the level of (net) full income accruing to each of the children in their care upon reaching adulthood in $t + 1$, denoted by $(1 - \rho)\bar{y}_{t+1} \equiv (1 - \rho)\bar{Y}_{t+1}/N_{t+1}^2$. The latter is the central measure of the size of the children's opportunity set on reaching young adulthood and hence exercises a heavy influence on the level of well-being they can attain. When choosing an allocation (c_t^2, e_t, S_t), young adults must forecast mortality and destruction rates in the coming period, higher levels of which weaken the effect of current investments in human and physical capital on $\bar{y}_{t+1}$. If these forecasts are unerring, those who survive through old age will obtain c_{t+1}^3, as given by (1), which the law of large numbers renders virtually non-stochastic. The stochastic element in the lotteries in question therefore arises only from the individual risks of failing to reach old age and, where altruism toward the children is concerned, that the latter will suffer the misfortune to die prematurely in young adulthood. If, in contrast, the outbreaks of war and disease in the future are viewed as stochastic events, there will be systemic risks, which are analyzed in Sections 5–8.

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3. Perfect foresight

The current levels of $\left(n_t, z_t, N_t^1, N_t^2, N_t^3, q_t^2, q_t^3, \lambda_t, K_t\right)$ and those in the next period are known with certainty. The surviving young adults' preferences are assumed to be additively separable in $\left(c_t^2, c_{t+1}^3, \bar{y}_{t+1}\right)$:

$$V_t = u\left(c_t^2\right) + \delta\left(1 - q_{t+1}^3\right) u\left(c_{t+1}^3\right) + \frac{b\left(1 - q_{t+1}^2\right)}{\left(1 - q_t^2\right)} n_t v\left[(1-\rho)\bar{y}_{t+1}\right], \tag{4}$$

where $1 - \delta \geq 0$ is the pure impatience rate and $b \geq 0$ is the taste parameter for altruism. The term $1/\left(1 - q_t^2\right)$ accounts for the children in the extended family whose parents have died.[3] The sub-utility functions u and v are assumed to be strictly concave. Although $\lim_{c \to 0} u'(c) \to \infty$, the same restriction need not be imposed on v, its argument being always positive. In view of the considerations they represent, there are good reasons to suppose that these functions are not the same; for own consumption in old age is a quite different object from the provision of opportunities for one's children. The intrusion of the term $(1-\rho)$ in v reflects the fact that the social norm governing the claims of those in old age is effectively a flat tax at the rate $(1-\rho)$ on the full income that results from their investment decisions when they were young. The yield of these investments accrues to the children on reaching adulthood, and they are bound by the same norm. In the absence of altruism $(b = 0)$, only the retained part $\rho \bar{Y}_{t+1}$ counts.

The adults' decision problem is

$$\max_{(c_t^2, e_t, S_t)} V_t \text{ s.t. (1), (2), (3), } c_t^2 \geq 0, e_t \in [0,1], S_t \geq 0. \tag{5}$$

Let $\left(c_t^{2,0}, e_t^0, S_t^0\right)$ solve (5), where it should be noted that the adults' decisions in period t are not influenced by their successors' in subsequent periods.

A preliminary step is to normalize the system by the size of the cohort N_t^2, exploiting the assumption that F is homogeneous of degree one. Let $l_t \equiv L_t/N_t^2$ and $s_t \equiv S_t/N_t^2$, so that (1) and (2) can be written respectively as

$$c_{t+1}^3 = \frac{\rho n_t}{\left(1 - q_{t+1}^3\right)\left(1 - q_t^2\right)} \cdot F\left[\left(1 - q_{t+1}^2\right)\lambda_{t+1}(e_t) + n_{t+1}\gamma, \frac{\sigma_{t+1} s_t}{n_t}\right] \tag{6}$$

and

$$\left[\left(1 - q_t^2\right) + \beta n_t\right] c_t^2 + s_t + \rho F\left[\left(1 - q_t^2\right)\lambda_t + n_t\gamma, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right] = F\left(l_t, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right). \tag{7}$$

Recalling that $L_t/N_t^2 = (1 - q_t^2 - w n_t e_t)\lambda_t + n_t\gamma(1 - e_t)$, normalized output is

$$y_t \equiv F\left[\left(1 - q_t^2 - w n_t e_t\right)\lambda_t + n_t\gamma(1 - e_t), \sigma_t s_{t-1}/n_{t-1}\right].$$

The analogous definition of normalized full income is $\bar{y}_t \equiv F\left(\bar{l}_t, \sigma_t s_{t-1}/n_{t-1}\right)$, where $\bar{l}_t \equiv \bar{L}_t/N_t^2$ denotes the normalized endowment of labor at time t. Closely associated with these normalizations is the ratio of human to physical capital at the start of period t, $\zeta_t \equiv \lambda_t/s_{t-1}$, which depends on investment decisions in the previous period.

Together with the constraints $c_t^2 \geq 0$, $e_t \in [0,1]$ and $s_t \geq 0$, the budget identity (7) defines the set of all feasible allocations $\left(c_t^2, e_t, s_t\right)$. Upon substitution for c_{t+1}^3 from (6) into (4), it is seen that V_t is likewise defined in the same space. In order to ensure that V_t is concave over the feasible set, some restrictions are needed. Although $\bar{y}_{t+1}$ is concave in Λ_{t+1} and S_t, Λ_{t+1} is concave in e_t if and only if h is concave in e_t.[4] Yet u and v are strictly concave, so that h can be weakly convex and

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still satisfy the requirement that V_t be concave. We therefore impose the following, less restrictive conditions:

$$u\left[\bar{y}_{t+1}\left(zh(e_t)\lambda_t+1,s_t\right)\right] \text{ and } v\left[(1-\rho)\bar{y}_{t+1}\left(zh(e_t)\lambda_t+1,s_t\right)\right]$$

are concave $\forall e_t \in [0,1]$ and $\forall s_t \geq 0$.

4. Steady states

In a steady state, the levels of inputs, output, and (dated) consumption change at a constant rate. Thus, fertility, mortality, and destruction rates $(n_t, \mathbf{q}_t, \sigma_t)$ must be constant, as well as the transmission parameter z_t; and since $\lambda_{t+1} = z_t h(e_t)\lambda_t + 1$, the level of education must be constant, too. In such environments, foresight will indeed be perfect.

If all per capita levels are growing at the same, positive rate, the economy is said to be on a steady-state growth path. In a slight abuse of terminology, a path along which all per capita levels are constant will be called stationary, even if the population is changing. Two notable steady states involve the extreme values of education. If all children go uneducated in period t, so that $\lambda_{t+1} = 1$, the state of "backwardness" is said to rule in period $t+1$. If, once reached, such a state persists, that stationary path implies the existence of a poverty trap. If, at the other extreme, all children born in period t are fully educated ($e_t = 1$), and all generations that follow them are likewise, this path will be called "progress".

Given stationary $(n_t, \mathbf{q}_t, \sigma_t)$, output per capita can increase only if there is some form of technical progress. If time t does not appear as an explicit argument of F, the only possible form of technical progress in the present framework is the labor-augmenting kind, which is expressed by an increase in the average level of human capital. This much will be assumed. The first question is whether backwardness can be equilibrium. The second question is whether backwardness is locally stable. If it is, there is a poverty trap.

A technical preliminary, on which the following analysis draws, is consigned to Appendix A. It yields the condition, for all interior solutions $e_t \in (0,1)$,

$$\left(1-q_{t+1}^2\right) zh'(e_t)\cdot F_1\left(\bar{l}_{t+1}, \frac{\sigma_{t+1}s_t}{n_t}\right) = (w+\gamma/\lambda_t)F_1\left(l_t, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right)\cdot \sigma_{t+1}F_2\left(\bar{l}_{t+1}, \frac{\sigma_{t+1}s_t}{n_t}\right), \quad (8)$$

where F_i denotes the partial derivative of F w.r.t. its ith argument. Multiplying both sides by λ_t, the l.h.s. is the product of the yield of a marginal increase in education in period t, in the form of the surviving children's human capital in period $t+1$, and the marginal yield of human capital, in the form of full income, in that period. The r.h.s. is the product of the cost of this investment in education in period t and the marginal yield of physical capital, adjusted by the corresponding survival rate σ_{t+1}. Since fixed shares of $\bar{y}_{t+1}$ are the arguments of $u\left(c_{t+1}^3\right)$ and v, altruism has no influence on the choice between the two forms of investment if $e_t^0 < 1$, and so does not appear in (8).

4.1. Backwardness

We seek to establish conditions that yield $e_t^0 = 0\ \forall t$. Along such a path,

$$\bar{y}_t\left(e_t^0=0\right) = y_t\left(e_t^0=0\right) = F\left(1-q_t^2+n_t\gamma, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right)\ \forall t,$$

since $\lambda_t = z_t h(0)\lambda_{t-1} + 1 = 1\ \forall t$. As defined above, the path is stationary, so the index t can be dropped. From the f.o.c. (see Appendix A), we have

$$u'\left(c^2\right) = \sigma\left(1-q^2+\beta n\right) F_2\left(1-q^2+n\gamma, \frac{\sigma s}{n}\right)\Omega(e_t^0=0),$$

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where $\nu_t \equiv b\frac{(1-q_{t+1}^2)n_t}{(1-q_t^2)}$ and

$$n_t\Omega_t \equiv \frac{\rho\delta n_t}{1-q_t^2}\cdot u'\left(\frac{n_t\rho\bar{y}_{t+1}}{\left(1-q_t^2\right)\left(1-q_{t+1}^3\right)}\right)+(1-\rho)\nu_t v'[(1-\rho)\bar{y}_{t+1}].$$

Since $l_t = 1-q^2+\gamma n$, the budget constraint (7) specializes to

$$\left(1-q^2+\beta n\right)c^2+s=(1-\rho)F\left(1-q^2+\gamma n, \sigma s/n\right),$$

and (6) to

$$c^3=\frac{n\rho}{\left(1-q^2\right)\left(1-q^3\right)}\cdot F\left(1-q^2+\gamma n, \sigma s/n\right).$$

Remark: $F(1-q^2+\gamma n, \sigma s/n)$ is the output per person entering young adulthood at the *start* of each period. Each young adult has n children, but only the fraction $\left(1-q^2\right)$ of such adults survive early adulthood thereafter.

Substituting for c^2 and c^3 in the first-order conditions, we obtain an equation in s, which features the constellation (n, q^2, q^3, σ), the preference parameters (b, δ) and the generational parameters (ρ, β, γ). The smallest positive value of s that satisfies this equation is denoted by $s^b = s^b\left(n, q^2, q^3, \sigma, \ldots\right)$.

The final step is to examine the counterpart of (8) when $e_t^0 = 0\ \forall t$. Rearranging terms, we obtain

$$(w+\gamma)\sigma F_2\left(1-q^2+\gamma n, \sigma s^b/n\right) \geq \left(1-q^2\right)zh'(0). \tag{9}$$

A marginal investment in a child's education will yield $zh'(0)$ units of human capital in the next period, with the fraction $1-q^2$ of all children later surviving early adulthood, and so contributing to output. The cost of this investment involves the sum of the opportunity and direct costs of education at the margin, measured in units of human capital. When $\lambda_t = 1$, this combined direct cost is $(\gamma + w)$ for each child. The latter is surely less than the basic endowment of unity, for a child is much less productive than an uneducated adult and w is the teacher-pupil ratio, with some allowance for an administrative overhead.

The alternative is to invest in physical capital. The marginal product thereof, F_2, is a pure number, since capital is made of the same stuff as output. When adjusted by the survival rate σ, it measures the marginal yield of investing in physical capital, the proportional claim on future full income being ρ for both forms of investment. Hence, σF_2 is the opportunity cost of a marginal investment in education.

The existence of a poverty trap in this model depends heavily on the steepness of h at $e=0$, as seen from the right-hand side of (9). Since we assume that $h'(0)$ is finite, it is possible that the marginal pay-off to investment in physical capital is larger than that of investment in human capital, even in the absence of schooling. If this be the case, parents would invest only in physical capital, and to such degree as merely to maintain the (normalized) stock of capital, keeping the economy in a perpetual poverty trap.

It is arguable that $h'(0)$ is at most $h(1)$ (footnote 4). Since $s^b < (1-\rho)F\left(1-q^2+\gamma n, \sigma s^b/n\right)$, condition (9) is not an exacting requirement, even though $\gamma + w < 1$. If (9) indeed holds as a strict inequality, then by continuity, it will be preserved when there are sufficiently small changes in $\mathbf{q}$ and σ. This establishes:

Proposition 1. *If condition (9) holds as a strict inequality, then there exists a locally stable, steady-state equilibrium in which children work full time and output per head is stationary.*

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4.2. Steady-state growth

The (asymptotic) rate of growth of λ_t and s_t at any fixed e, denoted by $g(e)$, is given by $g(e) = zh(e) - 1$. A growth path with $e_t = e$ is feasible if and only if $zh(e) > 1$. Each such path is effectively defined by the value of e and the initial values of s and λ, where c_t^2, c_t^3, s_t, y_t and $\bar{y}_t$ also grow at the asymptotic rate $g(e)$ and the potential contribution of child labor can be neglected for all sufficiently large t.

Since the first derivatives of F are homogeneous of degree zero,

$$u'\left(c_t^2\right) / \Omega_t = \sigma\left(1 - q^2 + \beta n\right) \cdot F_2\left[\left(1 - q^2\right) \zeta(e), \sigma/n\right], \tag{10}$$

where $\mathbf{q}_t$ and F_2 are constant, and $\forall e_t \in [0, 1]$

$$\Omega_t = \frac{\rho\delta}{1 - q^2} \cdot u'\left[\frac{n\rho\bar{y}_{t+1}}{\left(1 - q^2\right)\left(1 - q^3\right)}\right] + (1 - \rho)bv'\left[(1 - \rho)\bar{y}_{t+1}\right]. \tag{11}$$

It is seen that Ω_t cannot be changing at the same rate as $u'\left(c_t^2\right)$ unless either u and v are identical, or there is no altruism $(b = 0)$. In the latter case, $u'\left(c_{t+1}^3\right) / u'\left(c_t^2\right)$ must also be constant. That is to say, steady-state growth is possible only with some restrictions on preferences beyond those underpinning (4). The requirement that $u'\left(c_{t+1}^3\right) / u'\left(c_t^2\right)$ be constant motivates a standard assumption:

Assumption 1. $u(c_t) = c_t^{1-\xi}/(1 - \xi)$.

Independently of restrictions on preferences, the f.o.c. yield, along such a path,

$$\frac{F_1\left[\left(1 - q^2\right) \zeta(e), \sigma/n\right]}{F_2\left[\left(1 - q^2\right) \zeta(e), \sigma/n\right]} \cdot \left[\left(1 - q^2\right) zh'(e)\right] \geq \sigma w F_1\left[\left(1 - q^2 - wne\right) \zeta(e), \sigma/n\right], \ e \leq 1. \tag{12}$$

The ratio F_1/F_2 is the (constant) $|MRTS|$[5] in producing full income, F_1 on the r.h.s. is the marginal product of human capital, and both are evaluated at $\zeta(e)$ and σ/n.

4.2.1. No altruism

Since v plays no role, (10) may be written as

$$F_2\left[\left(1 - q^2\right) \zeta(e), \sigma/n\right] = \frac{\left(1 - q^2\right)\left[(1 + g(e))\left(c_t^3/c_t^2\right)\right]^{\xi}}{\delta\rho\sigma(1 - q^2 + \beta n)}. \tag{13}$$

Lemma 1. *Let e vary parametrically to yield steady-state growth paths. Then, ζ is increasing in e for all F that are:*

(i) sufficiently close to Cobb-Douglas in form, provided $\xi \leq 1$; or

(ii) members of the CES family whose absolute value of the elasticity of substitution, $|1/(\epsilon - 1)|$, is at most 1, provided $\xi + \epsilon \leq 1$.

Proof. See Appendix A. □

Remark: The condition $\xi \leq 1$ in part (i) can be weakened to include values exceeding, but sufficiently close to, 1. Regarding part (ii), if $\epsilon = -1$, the result holds for all $\xi \leq 2$.

Under what conditions are steady-state paths possible? Let e^p denote the smallest value of e satisfying $zh(e) = 1$, where $e^p > 0$ in virtue of $h(0) = 0$. If $e^p \geq 1$, there exists no steady-state growth path.

Suppose, therefore, that $e^p < 1$. Under the conditions of Lemma 1, $\zeta(1) > \zeta(e)\ \forall e < 1$. Hence, if the optimality condition (12) is violated at $e = 1$, then, by definition, progress is ruled out. If,

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with progress ruled out, the l.h.s. of (12) exceeds the r.h.s. at e^p, then by continuity, there exists at least one value of $e \in (e^p, 1)$ such that (12) holds as an equality. This establishes:

Proposition 2. *In the absence of altruism, there are three possibilities when e is parametric.*

(i) If $e^p \geq 1$, there exists no steady-state growth path.

If $e^p < 1$ and F satisfies the conditions in Lemma 1, then:

(ii) if (12) is violated at $e = 1$ and the l.h.s. exceeds the r.h.s. at e^p, there exists at least one steady-state growth path such that $e \in (e^p, 1)$;

(iii) if (12) holds at $e = 1$, the progressive state is a possible path, and if it holds as a strict inequality, then that path is also locally stable.

The direct costs of education, which arise from the teacher-pupil ratio w, exert a strong influence on which of these possibilities holds. If w is sufficiently close to zero, it follows from (12) that progress is the only possible outcome, which accords with intuition. In fact, the educational system is a fairly heavy user of its own output, so that the other outcomes cannot be ruled out.

In view of the role played by condition (12), we have established

Corollary 1. *The parametric growth paths defined by parts (ii) and (iii) will be sustained by families' optimal choices.*

4.2.2. Altruism

If v differs from u in the degree of concavity, it follows at once from (11) that Ω_t cannot be changing at a constant rate along a steady-state growth path.

Assumption 2. *Let v also be iso-elastic:* $v\big[(1-\rho)\bar{y}_{t+1}\big] = \big[(1-\rho)\bar{y}_{t+1}\big]^{1-\eta}/(1-\eta)$.

With v in play in this form, (10) becomes:

$$F_2\big[\left(1-q^2\right)\zeta(e), \sigma/n\big] =$$

$$\frac{\big[(1+g(e))c_t^3/c_t^2\big]^{\xi}}{\sigma\left(1-q^2+\beta n\right)}\left(\frac{\rho\delta}{1-q^2} + (1-\rho)^{1-\eta}b\big[c_t^3(1+g(e))\big]^{-\eta+\xi}\left(\frac{\rho n}{(1-q^3)\left(1-q^2\right)}\right)^{\eta}\right)^{-1},$$

so that Lemma 1 continues to hold.

This equation may also be expressed as

$$u'\left(c_t^2\right)/\Omega_t = \left[\frac{\rho\delta}{1-q^2}\cdot\left(\frac{n\rho}{\left(1-q^2\right)(1-q^3)}\right)^{-\xi} + (1-\rho)^{1-\eta}b\cdot\bar{y}_{t+1}^{(\xi-\eta)}\right]^{-1}\left(\frac{\bar{y}_{t+1}}{c_t^2}\right)^{\xi}$$
$$= \sigma\left(1-q^2+\beta n\right)\cdot F_2\big[\left(1-q^2\right)\zeta(e), \sigma/n\big]. \tag{14}$$

Given that the economy is on a steady-state growth path, the r.h.s. is constant, attaining its upper limit when $e = 1$, since ζ is increasing in e. The expression in brackets is constant if, and only if, $\xi = \eta$; so that we distinguish among three cases. First, it is seen that if v is less concave than u, that is, $\eta < \xi$, then the terms involving $\bar{y}_{t+1}$ will grow without bound, which implies from (34) that $e_t^0 = 1$, and hence that the path is locally stable. Second, in the borderline case $\xi = \eta$, $u'\left(c_t^2\right)/\Omega_t$ is indeed constant. Altruism introduces the additional, constant term $(1-\rho)^{1-\eta}b$ into the said expression, thus inducing an increase in $\zeta(e)$ if $e < 1$. Attaining the state of progress is then more likely than in the absence of altruism. Third, if $\eta > \xi$, the said expression approaches a limit as t

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becomes very large. In effect, altruism ceases and the results of Section 4.2.1 apply. This argument establishes:

Proposition 3. *With altruism and iso-elastic preferences, and given $zh(1) > 1$, the possible steady-state growth paths are as follows.*

(i) If u and v differ, with $\eta < \xi$, then progress is the sole steady-state path that can be supported by families' optimizing decisions. It is also locally stable.

(ii) If $\eta = \xi$, there may exist steady-state growth paths with incompletely educated populations, but the state of progress is also possible as a limiting case.

(iii) If $\eta > \xi$, Proposition 2 applies.

What is the intuition for these findings? A necessary and sufficient condition for Ω_t to fall at the same rate as $u'\left(c_t^2\right)$ in the presence of altruism is $\xi = \eta$. Relaxing that restriction, consider the path $e_t = 1$, along which Ω_t can fall at a rate less than $g(1)$ without violating the conditions for optimality. Investing yields old-age provision and fosters the children's well-being in adulthood. Since $\bar{y}_{t+1}$ is the argument of both $u\left(c_{t+1}^3\right)$ and v, it suffices that v be less strongly concave than u in order to maintain steady-state growth. If parents are perfectly selfish, the concavity of u rules alone, so that $e_t^0 < 1$ is possible.

If u is less strongly concave than v, then along any steady growth path, v' is falling faster than $u'(c_t^2)$ and $u'\left(c_{t+1}^3\right)$. The relative contribution of altruism goes asymptotically to zero as t becomes arbitrarily large, and with it, e_t^0 may slip below 1.

4.2.3. A special case: Log-land

In light of the foregoing results and its ubiquity in applications, the logarithmic case warrants further examination.

Proposition 4. *Suppose $u = \ln c$, $v = \ln[(1-\rho)\bar{y}_{t+1}]$. If*

$$\frac{n}{\left(1-q^2\right)\left[\delta\left(1-q^3\right)+bn\right](1-\rho)} \geq \frac{zh'(0)}{\gamma+w} \tag{15}$$

and, when F is Cobb-Douglas $\left(y_t = A\cdot l_t^{1-\alpha}k_t^{\alpha}\right)$,

$$zh'(1) \geq \alpha\sigma^{\alpha}n^{1-\alpha}wA\cdot\frac{[\zeta(1)]^{1-\alpha}}{(1-q^2-wn)^{\alpha}},$$

then the economy possesses at least three steady-state equilibria: backwardness, progress, and one or more (stationary) states with constant values of $e_t^0 \in (0,1)$. Backwardness and progress are locally stable. Among the third type, that having the smallest value of e_t is unstable.

Proof. See Appendix A. Condition (15) is independent of F. The l.h.s. depends only on fertility and mortality rates, and the social norm and preference parameters ρ, b and δ; the r.h.s. depends only on those representing the costs of education and the associated marginal yield of human capital at $e_t = 0$. □

5. War and pestilence as stochastic events

In reality, mortality and destruction rates are stochastic. This fact rules out steady-state growth paths, but not necessarily bumpy ones. If there is a poverty trap, a more extreme hazard arises: the outbreak of a war or a severe epidemic, especially if sustained for two or more periods, may pitch a hitherto growing economy into backwardness. We formulate the shock as the actual outbreak,

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coupled with the (prior) probability of its occurrence. This prior is assumed to be sharp, in the sense that beliefs about probabilities are certain.[6]

Let $I_t \in \{0, 1\}$ denote the states of peace and war, respectively, in period t; $\pi_{t+1} = Pr(I_{t+1} = 0)$ denotes the probability of peace in period $t + 1$. The survival rate of physical capital is $\sigma_t(I_t)$, where $\sigma_t(1) < \sigma_t(0) \leq 1$. Mortality rates $\mathbf{q}_t$ are likewise dependent on I_t. It is almost surely the case that $q_t^a(1) > q_t^a(0)\ (a = 2, 3)$. By assumption, I_t is known when decisions are made in period t. Full income in the following period, $\bar{y}_{t+1}(I_{t+1})$, for those who survive to enjoy it, depends on the state then ruling, as well as on current investment. The extended family cannot provide insurance against this particular risk.

Young adults' preferences now involve not only the compound lottery arising from the future state I_{t+1}, but also the current realization of I_t if this affects q_t^2.

$$\begin{aligned} V_t(I_t) = {} & u\left(c_t^2\right) + \delta\left[\pi_{t+1}\left(1 - q_{t+1}^3(0)\right) u\left(c_{t+1}^3(0)\right) + (1 - \pi_{t+1})\left(1 - q_{t+1}^3(1)\right) u\left(c_{t+1}^3(1)\right)\right] \\ & + \frac{bn_t}{1 - q_t^2(I_t)}\left[\pi_{t+1}\left(1 - q_{t+1}^2(0)\right) v\left[(1-\rho)\bar{y}_{t+1}(0)\right]\right. \\ & \left. + (1 - \pi_{t+1})\left(1 - q_{t+1}^2(1)\right) v\left[(1-\rho)\bar{y}_{t+1}(1)\right]\right], \\ & I_t = \{0, 1\}. \end{aligned} \tag{16}$$

Exploiting the assumption that F is homogeneous of degree one, we have, from (6),

$$\begin{aligned} c_{t+1}^3(I_{t+1}; I_t) &= \frac{\rho n_t \cdot \bar{y}_{t+1}(I_{t+1})}{\left(1 - q_t^2(I_t)\right)\left(1 - q_{t+1}^3(I_{t+1})\right)} \\ &= \frac{\rho n_t \cdot F\left[\left(1 - q_{t+1}^2(I_{t+1})\right)\lambda_{t+1}(e_t) + n_t\gamma, \sigma_{t+1}(I_{t+1}) s_t / n_t\right]}{\left(1 - q_t^2(I_t)\right)\left(1 - q_{t+1}^3(I_{t+1})\right)}. \end{aligned} \tag{17}$$

Not only does the realized state in period $t + 1$ affect full income in that period, but mortality in period t also affects both the weight of $u\left(c_{t+1}^3\right)$ relative to $v(\bar{y}_{t+1})$ and the share in $\bar{y}_{t+1}$ of those who do survive throughout old age. The weight on future outcomes relative to (certain) current consumption, c_t^2, also changes in a complicated way.

The budget constraint (7) becomes

$$\begin{aligned} & \left[1 - q_t^2(I_t) + \beta n_t\right] c_t^2 + s_t + \rho F\left[\left(1 - q_t^2(I_t)\right)\lambda_t + n_t\gamma, \frac{\sigma_t(I_t) s_{t-1}}{n_t}\right] \\ & \leq F\left[\left(1 - q_t^2(I_t) - w n_t e_t\right)\lambda_t + n_t\gamma(1 - e_t), \frac{\sigma_t(I_t) s_{t-1}}{n_t}\right], \quad I_t = \{0, 1\}, \end{aligned} \tag{18}$$

where the dependence of current decision variables on the current realized state can be (notationally) suppressed without ambiguity.

It should be noted that two steady-state, non-stochastic settings arise as special cases. The state of perpetual peace is the realized sequence $\{I_t = 0\}_{t=0}^{t=\infty}$ expected with certainty; that of perpetual war is $\{I_t = 1\}_{t=0}^{t=\infty}$ expected with certainty; and associated with each are the constant destruction rates $(\mathbf{q}_t(0), \sigma_t(0))$ and $(\mathbf{q}_t(1), \sigma_t(1))$, respectively.

To analyze the economy's behavior in the face of systemic shocks, we proceed essentially as before, noting that the choices of s_t and e_t determine the productive endowments in the next period and hence ζ_{t+1}. The logarithmic forms $u = \ln c_t, v = \ln(1-\rho)\bar{y}_{t+1}$ and F Cobb-Douglas $\left(y_t = A \cdot l_t^{1-\alpha} k_t^{\alpha}\right)$ yield relatively tractable closed-form expressions from the f.o.c. (see Appendix C). These difference equations will be used in Sections 7.2 and 7.3. Otherwise, u, v, and F are not thus restricted to yield Log-land.

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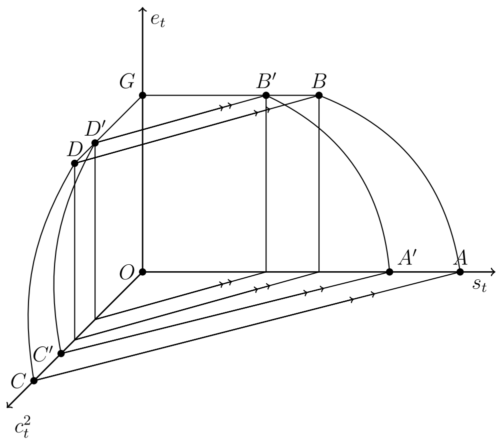

Figure 2. Feasible sets of consumption and investment.

5.1. The occurrence of war

The first step is to examine how I_t and q_t^2 affect the set of feasible current choices, which are independent of π_{t+1}. In state I_t, the budget set is denoted by

$$S(I_t) = \left\{c_t^2, e_t, s_t : (19), c_t^2 \geq 0, e_t \in [0, 1], s_t \geq 0\right\}, \; I_t = \{0, 1\}.$$

It is seen from condition (39) in Appendix B that if the ratio of survival rates, $\sigma_t(I_t)/\left(1 - q_t^2(I_t)\right)$, is independent of the current state, then the outer frontier of the feasible set is affected only by the mortality rate $q_t^2(I_t)$. If the said ratio is indeed independent of I_t, an increase in q_t^2, whether associated with war or not, also makes c_t^2 cheaper relative to s_t.

The extreme allocations of $S(I_t = 0)$'s outer frontier, which involve positive outlays on at most two of savings, education, and consumption, are depicted as the points A, B, C, and D, respectively, in Figure 2. The corresponding allocations in the state of war ($I_t = 1$) are A′, B′, C′, and D′. The allocations on the whole of the frontier are examined in Appendix B. To summarize: A sufficient condition for $\left(dc_t^2/dq_t^2(I_t)\right)_{e_t=s_t=0} < 0 \; \forall \lambda_t$ is $\beta > \gamma$, which is not a very strong requirement. If the ratio of survival rates, $\sigma_t(I_t)/\left(1 - q_t^2(I_t)\right)$, is fixed for each current state I_t and $\beta > \gamma$, the outer frontier of the feasible set $S(I_t)$ will contract inwards everywhere as the mortality rate $q_t^2(I_t)$ rises. If the said ratio is the same for both states, the contraction from $S(0)$ to $S(1)$ represents the effects of an outbreak of war.

To complete the argument, consider the case where $e_t = 1$ is infeasible for sufficiently large values of $q_t^2(I_t)$. Suppose that when $q_t^2(I_t) = 0$, the maximal values of c_t^2 and s_t, respectively, are both positive, as depicted in the figure when ABCD (or A′B′C′D′ when $I_t = 1$) corresponds to a zero level of such premature mortality. As $q_t^2(I_t)$ progressively increases, BD will shift toward G until the allocations B and D coincide at G ($e_t = 1, c_t^2 = s_t = 0$). Further increases in $q_t^2(I_t)$ will reduce the maximal feasible level of e_t below one, with the associated allocation moving downwards along the e_t-axis toward the origin O. Since AC also shifts progressively inwards toward O, the outer frontier of $S(I_t)$ contracts everywhere as $q_t^2(I_t)$ increases.

The contraction of the feasible set established above points to unambiguous income effects, for horizontal lines on the frontier move inwards in parallel and c_t^2 and $\bar{y}_{t+1}$ are normal goods, with $\bar{y}_{t+1}$ increasing in e_t and s_t. Changes in survival rates also imply changes in marginal rates of transformation. As noted above, given the current state I_t, an increase in $q_t^2(I_t)$ makes c_t^2 cheaper relative to s_t, as does an outbreak of war if this event leaves the ratio of survival rates unchanged.

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Turning to the marginal rate of transformation between s_t and e_t, we obtain from (18), on fixing c_t,

$$\mathrm{MRT}_{se}(I_t) = -\left(n_t(w\lambda_t + \gamma)F_1\left[(1 - q_t^2(I_t) - wn_te_t)\lambda_t + n_t\gamma(1 - e_t),\ \sigma_t(I_t)\cdot\frac{s_{t-1}}{n_{t-1}}\right]\right)^{-1}.$$

For any given I_t and e_t, an increase in $q_t^2(I_t)$ will increase F_1 and so reduce $|\mathrm{MRT}_{se}(I_t)|$: s_t will become cheaper relative to e_t, as intuition suggests.

An outbreak of war has ambiguous effects on $\mathrm{MRT}_{se}(I_t)$. Since F_1 is homogeneous of degree zero, $\mathrm{MRT}_{se}(I_t)$ can be expressed in the form

$$\mathrm{MRT}_{se}(I_t) = -\left(n_t(w\lambda_t + \gamma)F_1\left[\lambda_t + \frac{n_t[\gamma - (w\lambda_t + \gamma)e_t]}{1 - q_t^2(I_t)},\ \frac{\sigma_t(I_t)}{1 - q_t^2(I_t)}\cdot\frac{s_{t-1}}{n_{t-1}}\right]\right)^{-1}.$$

Suppose, as before, that the ratio of survival rates is independent of I_t, but with $q_t^2(1) > q_t^2(0)$. It is seen that the associated increase in mortality reduces, leaves unchanged, or increases, the (normalized) input of human capital according to $e_t \lesseqgtr \gamma/(\gamma + w\lambda_t)$. For sufficiently large values of λ_t, the expression on the r.h.s. will be very small, so that war in the current period will make s_t cheaper relative to e_t for all values of e_t. The converse holds when λ_t is close to one; for $\gamma/(\gamma + w\lambda_t)$ is then close to one, and the $n_t/\left(1 - q_t^2(I_t)\right)$ children cared for by each surviving young adult constitute a potentially large pool of labor, relatively speaking. The outbreak of war reduces the opportunity cost of their labor and so makes investment in their education more attractive relative to investment in physical capital. We summarize these findings as

Proposition 5. *The contraction of the feasible set caused by war in the current period reduces both current consumption and investment in both forms of capital. Consumption also becomes cheaper relative to investment in physical capital. Investment in education is likely to suffer especially when λ_t is large, but not when λ_t is small.*

It seems rather unlikely that the associated changes in the marginal rates of transformation will offset the reduction in investment arising from the adverse income effect.

5.2. The probability of war

Intuition suggests that an increase in the prior probability of war in the future will depress investment in the present. It will now be demonstrated that this is indeed so in our framework provided an additional condition holds.

The feasible set in period t, as defined by (18), is independent of π_{t+1}, so that changes in the latter will affect decisions only through $V(I_t)$. Inspection of (16) reveals that the weight on the altruism term ν is increasing in π_{t+1}, since $q_{t+1}^2(1) > q_{t+1}^2(0)$. Where the terms involving old age are concerned, the probability of surviving into full old age is increasing in the probability of peace, π_{t+1}. Weighing against this, the pay-off received by each of the survivors depends on the number of claimants as well as the size of the common pot. It is seen from (17) that for any given (e_t, s_t, I_t), $c_{t+1}^3(I_t;0) \gtreqless c_{t+1}^3(I_t;1)$ according as

$$\frac{F\left[\left(1 - q_{t+1}^2(0)\right)\lambda_{t+1}(e_t) + n_{t+1}\gamma,\sigma_{t+1}(0)s_t/n_t\right]}{F\left[\left(1 - q_{t+1}^2(1)\right)\lambda_{t+1}(e_t) + n_{t+1}\gamma,\sigma_{t+1}(1)s_t/n_t\right]} \gtrless \frac{1 - q_{t+1}^3(0)}{1 - q_{t+1}^3(1)}.$$

The numerator on the l.h.s. is the level of full income in period $t+1$ when peace prevails, and the denominator is the corresponding level when war does so. The r.h.s. is the corresponding ratio of survival rates into old age. Both ratios exceed 1, but it is very likely that the former ratio is the larger; for war is likely to take a proportionally heavier toll on young adults, and it will surely destroy some of the capital stock. It is plausible, therefore, that

$$\frac{F\left[\left(1-q_{t+1}^{2}(0)\right)\lambda_{t+1}(e_t)+n_{t+1}\gamma,\sigma_{t+1}(0)s_t/n_t\right]}{F\left[\left(1-q_{t+1}^{2}(1)\right)\lambda_{t+1}(e_t)+n_{t+1}\gamma,\sigma_{t+1}(1)s_t/n_t\right]}\geq\frac{1-q_{t+1}^{3}(0)}{1-q_{t+1}^{3}(1)} \tag{19}$$

holds. This suffices to ensure that the second term on the r.h.s. of (16), which may be expressed as $E_{I_{t+1}}u\left[c_{t+1}^{3}(I_t;I_{t+1})\right]$, is increasing in $\pi_{t+1}(I_t=0,1)$, and so establishes

Proposition 6. *If (19) holds, an increase in the (prior) probability that war will occur in the next period will depress investment in favor of consumption in the current one.*

The argument in Section 5.1 indicates how the balance between e_t and s_t is affected by the probability of war in the following period depends in a complicated way on the differences in survival rates between the two states. The weight on the altruism term v in V_t is decreasing in the future mortality rate among young adults, and the stronger the degree of altruism, as represented by b, the larger will be the absolute size of the reduction in the said weight. Yet u and v share the argument $\bar{y}_{t+1}$ in the same way, so that even the strong prospect of a rather destructive war is unlikely to induce much substitution between the two forms of investment beyond any differential effect on survival rates.

5.3. Pestilence

An unexpected outbreak of pestilence, such as the Black Death, is an asymmetric shock of a different kind, carrying off much of the population, but leaving the current capital stock untouched. This will be a windfall for the survivors, but it will profit them little if physical and human capital are poor substitutes in production—indeed, not at all if they are strict complements. If, in contrast, they are perfect substitutes, then the windfall will yield a correspondingly large income effect, which may be sufficiently strong to propel an economy out of backwardness onto a growth path, even with perpetual, but not unduly destructive warfare.[7]

A belief that adult mortality will fall in the future will make current investment, especially in schooling, more attractive, thus promoting growth. An outbreak of heavy hostilities or disease that shocks the population into pessimism will therefore have ambiguous effects on accumulation.

6. Stability

When there is perfect foresight, backwardness $\left(e_t^0=0\right)$ and progress $\left(e_t^0=1\right)$ are both locally stable under the conditions established in Section 4. Sufficiently small, favorable, and foreseen changes in mortality and destruction rates will not yield an escape from backwardness, nor will sufficiently small, unfavorable ones upset full schooling or the growth rate, $g(1)$, when progress rules. These findings yield results for stochastic settings in which agents have beliefs, however sharp, but lack foresight.

If backwardness is a locally stable equilibrium when peace always reigns $\left(e_t^0=0, I_t=0\ \forall t\right)$, then once in backwardness, and failing a sufficiently favorable change in its environment, the economy will be perpetually trapped in that state, be there war or peace thereafter, even when families form the unshakeable belief that peace will reign.

Suppose there also exists, when peace always reigns, a set of stationary states with $e_t^0\in(0,1)$. Let $e^*(0)$ denote the smallest such value of e_t^0, so that λ_t is stationary, at $\lambda^*(0)$, where $\lambda^*(0)=zh(e^*(0))\lambda^*(0)+1$. Associated with $e^*(0)$ there is a stationary level of k_t, denoted by $k^*(0)$. Since the state of backwardness is locally stable, this neighboring equilibrium $(\lambda^*(0),k^*(0))$ is unstable. If, at time t, the state variables are such that $(\lambda_t,k_t)<<(\lambda^*(0),k^*(0))$,[8] a descent into permanent backwardness will follow. This conclusion holds *a fortiori* if there is some chance of war. For it is

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established in Section 5.1 that an outbreak of war in the current period will almost surely reduce current investment relative to its level in the state of peace, and in Section 5.2 that an increase in the hazard rate $1 - \pi_{t+1}$ will do likewise.

Consider, therefore, a stationary stochastic environment wherein π_t is constant and firmly believed to be so, whatever episodes of war and peace actually materialize. Then if, at *any* time t in the economy's development, $(\lambda_t, k_t) << (\lambda^*(0), k^*(0))$, backwardness will ultimately follow. Yet this condition is not necessary for such an outcome; for physical and human capital are (imperfect) substitutes in various ways, so that exceeding $k^*(0)$ might not compensate for falling short of $\lambda^*(0)$, and conversely.

Turning to growth, a growing economy's capacity to withstand shocks must be made precise. A *robust* economy can be defined as one in which growth can occur even in a state of perpetual war: $e_t^0 = e^{**}(1), I_t = 1, \pi_t = 0\ \forall t$, where $\lambda_{t+1} = zh(e^{**}(1))\lambda_t + 1 > \lambda_t\ \forall t$. This requires, *inter alia*, that (12) hold at $\mathbf{q}_t = \mathbf{q}(1),\ \sigma_t = \sigma(1)$ when $e_t^0 = e^{**}(1)\ \forall t$. In particular, if $e_t^0 = e^{**}(1) = 1\ \forall t$, then progress, once attained, is robust and locally stable. More generally, if steady growth is possible in a state of perpetual war, growth will also be possible when peace sometimes rules, but it will not be steady, since ζ_t will vary, even if e_t does not.

The starting values of the state variables must be sufficiently favorable for a growth path to be attained. These values depend on the economy's particular history of war and peace. If, at time t', the state variables (λ_t, k_t) are such that, should war become permanent, $e_t^0 \geq e^{**}(1)\ \forall t \geq t'$, then a sustained growth path will be attained for all π. If (λ_0, k_0) lies in a certain interval, the economy's ultimate fate—sustained growth or backwardness—becomes a matter of chance (see Appendix E).

7. Simulations

The results of Section 6 pertain to rather restrictive initial conditions. Extending them to cover economies that are initially close neither to backwardness nor to progress involves some resort to simulations. For in the presence of a poverty trap, whether an economy will withstand one or a whole series of shocks involves complicated transitional dynamics. The first step is briefly to treat a non-stochastic setting. This lays the foundation for simulations wherein mortality and destruction rates are stochastic, in Sections 7.2 and 7.3.

The constellation of parameter values employed in this section is set out in Table 1. For its provenance and the fulfillment of the conditions in Section 7.1, see Appendix D.

7.1. Backwardness and progress: constant destruction rates

The following conditions must be satisfied for both states to be equilibria:

(i) condition (9) must hold as a strict inequality for backwardness $\left(e_t^0 = 0\ \forall t\right)$ to be a locally stable equilibrium;

(ii) $zh(1) > 1$, so that unbounded growth results when $e_t = 1\ \forall t$; and

(iii) condition (12) yields $e_t^0 = 1$ along the steady-state path $e_t = 1\ \forall t$.

Let $u(c_t) = \ln c_t$ and $v[(1-\rho)\bar{y}_{t+1})] = \ln[(1-\rho)\bar{y}_{t+1})]$.

Condition (i). A sufficient condition for (9) to hold is (15). It is seen that there exists a measurable subset of the parameters involved such that (9) will indeed hold.

Condition (ii). Let $h(e) = d_1 \cdot e - d_2 \cdot e^{d_3}$, so that $h(1) = d_1 - d_2$. To illustrate, if $h = e$, then $h'(e) = h(1) = 1$, and $z > 1$ yields $g(1) > 0$.

Condition (iii). Let F be Cobb-Douglas: $y_t = Al_t^{1-\alpha}k_t^{\alpha}$. It is proved in Appendix C that progress $(e_t^0 = 1\ \forall t, g(1) > 0)$ will be an equilibrium path if, and only if,

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Table 1. Log-land: parameter values

Parameter	Value	Variable
n	1.2	Net reproduction rate
q^2	0.1	Mortality rate at the start of young adulthood
q^3	0.3	Mortality rate at the start of old age
σ	0.75	Survival rate of physical capital
γ	0.6	A child's endowment of human capital
d_1	1	A parameter of $h(e)^a$
d_2	0.2	A parameter of $h(e)$
d_3	1.5	A parameter of $h(e)$
z	1.5	Transmission factor for human capital formation
w	0.075	Teacher-pupil ratio
A	1	TFP parameter
α	1/3	Elasticity of output w.r.t. physical capital
δ	0.85	Discount factor
b	(0, 0.1)	Taste parameter for altruism
ρ	0.35	Share of current full income accruing to the old
β	0.325	Share parameter for a child's consumption

[a] $h(e) = d_1 \cdot e - d_2 \cdot e^{d_3}$.

$$\frac{h'(1)}{h(1)} \geq \frac{nw}{(1-q^2-wn)} \cdot \frac{1+\alpha\left[\delta\left(1-q^3\right)+bn\right]}{\left[\delta\left(1-q^3\right)+bn\right]} \cdot \frac{1}{1-\rho\left[\left(1-q^2\right)/(1-q^2-wn)\right]^{1-\alpha}}, \tag{20}$$

where $h'(1)/h(1)$ will be close to 1 if h is weakly concave.

7.2. Transitions to permanent peace or war

We augment the analysis of the stochastic setting of Section 6 by examining the transition to peace or war when these are ultimately established as permanent states. Changing expectations play a central role along these paths.

Suppose war breaks out in period t. Agents form some sharp prior, $1-\pi_{t+1} > 0$, that war will also occur in the next period, and make their investment decisions in t accordingly. Given the resulting normalized endowments and the prior π_{t+1}, households choose (e_t^0, s_t^0). If war occurs again in period $t+1$, the resulting normalized endowments, suppressing the time subscripts for n_t, σ_t and q_t^2, will be

$$\bar{l}_{t+1} = \left(1-q^2(1)\right)\lambda_{t+1}\left(e_t^0\right) + n\gamma \text{ and } k_{t+1} = \sigma(1)s_t^0/\left[\left(1-q^2(1)\right)n\right].$$

Two wars in a row may not be out of the ordinary, so it is quite possible that π is not revised. Indeed, in an environment wherein the discrete variate I_t is i.i.d. and memories are sufficiently long, π_{t+1} will be fixed, a state of affairs analyzed in Section 7.3.

Now suppose that, for some reason or other, peace is confidently expected in period $t+2$ and $\left(e_{t+1}^0, s_{t+1}^0\right)$ are chosen accordingly. Suppose peace also actually rules, so that the resulting normalized endowments in period $t+2$ are

$$\bar{l}_{t+2} = \left(1-q^2(0)\right)\lambda_{t+2}\left(e_{t+1}^0\right) + n\gamma \text{ and } k_{t+2} = \sigma(0)s_{t+1}^0/\left[\left(1-q^2(0)\right)n\right]. \tag{21}$$

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There is the alternative, happier possibility that peace rules in period $t+1$. In that event, the normalized endowments will be

$$\bar{l}_{t+1} = \left(1 - q^2(0)\right) \lambda_{t+1}\left(e_t^0\right) + n\gamma \text{ and } k_{t+1} = \sigma(0)s_t^0 / \left[\left(1 - q^2(0)\right) n\right]$$

and the calculations for period $t+2$ then proceed as before.

Turning from the transition to the long run, suppose peace reigns thereafter and is confidently expected to do so. Given the values of the state variables in period t, $\left(\lambda_t\left(e_{t-1}^0\right), s_{t-1}^0\right)$, it can be checked whether the economy will recover from one or two wars, or fail to do so, whereby the starting endowments at $t+2$ after two wars are given by (21). The same holds, *mutatis mutandis*, for a transition to the regime of perpetual war.

A particular limitation of the two-period phase t and $t+1$ during which war can occur is that its influence on decisions *ex ante* is confined to period t. Two consecutive adverse shocks are possible, but the (exogenous) change in beliefs to the certainty of peace from $t+2$ onwards makes ultimate recovery more likely. Suppose the said phase is extended to three periods, thus yielding an *ex ante* influence on decisions in both periods t and $t+1$. The possible sequences are

$$\{1, 1, 1\}, \{1, 1, 0\}, \{1, 0, 1\}, \{1, 0, 0\}, \{0, 1, 1\}, \{0, 1, 0\}, \{0, 0, 1\}, \{0, 0, 0\}.$$

We concentrate on the grimmest outcome, namely, three consecutive periods of war.

The constellation of parameter values must be extended to cover war and peace. Let the values in Table 1 hold in the state of peace, so there is a poverty trap, even with unbroken peace, {0,0,0}, as the actual outcome in the first three periods. The associated long-run value of ζ_{t+1} along the path of progress, $\zeta_{t+1}(I_t = 0; e_t = 1)$, is 49.26, as yielded by (42), with $I_t = 0\ \forall t$. Let the prior probability that war occurs in periods 1 and 2, $1 - \pi_{t+1}$ $(t = 0, 1)$, be 0.5, and let the mortality rates in that state be $q_t^2(1) = 0.25$, $q_t^3(1) = 0.35$, with $\sigma_t(1) = 0.4$.

If the state variables λ_t and k_t are very large, extremely heavy losses will have to occur in order to reduce the normalized endowments to levels such that even $e_t = 1$ will not be optimal, let alone set in train a certain collapse into backwardness. Suppose, therefore, that the initial values of human and physical capital, λ_0 and k_0, which are inherited from period $t = -1$, are sufficiently small for a sequence of shocks as severe as {1,1,1} to rule out any path to progress. Recalling the results in Section 6, the stationary (critical) values of λ^* are now in play. Under perpetual peace, expected as well as realized, $\lambda^*(0) = 3.1988$. With $\pi_{t+1} = 0.5$, the critical value of λ_0 when the realized sequence is indeed $\{0, 0, 0\}$ is 3.3856; but when the outcome is three periods of war, $\{1, 1, 1\}$, the said value is 3.9976. To complete the initial conditions, let $\zeta_0 = 20$.

The trajectories of λ_t and ζ_t for each of the values $\lambda_0 = 3, 3.6, 4, 4.2$ and 10 are depicted in Figure 3. With the attendant heavy destruction of physical capital, three periods of war generate an immediate and sharp upward spike in ζ_t, regardless of the ultimate outcome. If λ_0 is close to the critical value 3.9976, the trajectories of ζ_t have more than one local extremum. The trajectory for $\lambda_0 = 3.6$ follows a spike at $t = 2$ by first undershooting, and then converging from below to the value under permanent backwardness. That for $\lambda_0 = 4$ also attains a local maximum at $t = 3$, before ultimately and slowly converging from below to the value under progress. That for $\lambda_0 = 4.2$ possesses three extrema and converges from above. These oscillations indicate complicated and long-drawn-out transitional dynamics near critical values of the boundary conditions. These dynamics are less apparent in the trajectories of λ_t. The trajectory for $\lambda_0 = 4$ recovers only extremely slowly, despite the favorable change in beliefs, from the three episodes of war. In contrast, the recovery of the trajectory for the slightly more favorable $\lambda_0 = 4.2$ is complete by $t = 4$, and rapid growth sets in from $t = 6$ onwards, indicating the sensitivity of the long-run path to small changes in λ_0 in the range between 3.6 and 4.2. As for $\lambda_0 = 10$, its trajectory is little affected at any point.

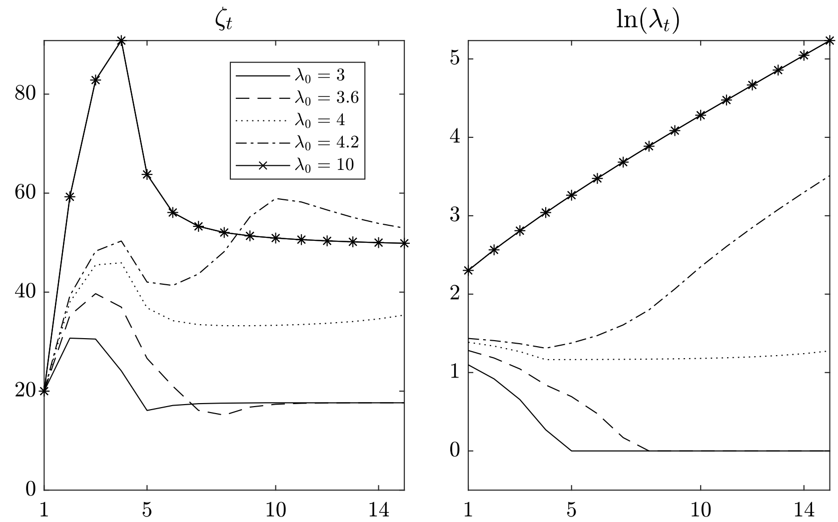

Figure 3. Trajectories of ζ_t and $\ln \lambda_t$: three consecutive periods of war before peace reigns.

7.3. A stationary stochastic environment

We examine the varying fortunes of these five initial configurations as events subsequently unfold in a different setting, namely, one wherein $\pi_t = 0.5$ in all periods, with the associated unchanging, rational belief that $\pi_{t+1} = 0.5$. For each value of λ_0, a draw is made from the binomial distribution $Pr(I_t = 0) = 0.5$ and the resulting values of all relevant outcomes are calculated for period $t = 0$ and the start of $t = 1$. At the latter point in time, a new draw is made, and so on and so forth, up to the start of period $t = n$, thus yielding the realized path $\{\lambda_t\}_{t=0}^{t=n}$. Since the process is i.i.d., there are 2^k distinct, equi-probable sequences $\{I_t\}_{t=0}^{t=k-1}$ up to the start of period k, yielding a distribution of the n-vector variate $\{\lambda_t(I_t)\}_{t=1}^{t=n}$, whose k-th element is $\lambda\left(\{I_t\}_{t=0}^{t=k-1}, \lambda_0, \pi_{t+1} = 0.5\right)$, $\forall k \geq 1$. More than one of these sequences may have landed up in backwardness by the start of period k, in which event, the number of distinct values of λ_k would be correspondingly smaller than 2^k. At all events, the distributions' upper and lower limits are generated by the sequences $\{I_t = 0\}_{t=0}^{t=n-1}$ and $\{I_t = 1\}_{t=0}^{t=n-1}$, respectively.

The resulting distributions of λ_t are depicted in Figure 4. The courses of the upper and lower limits are shown in the left-hand panel; the frequency distributions of λ_{14} are shown in the right-hand panel. Recall from Section 7.2 that when $\pi_{t+1} = 0.5$ $(t = 0, 1)$, the initial realized sequence is $\{0, 0, 0\}$ and there is also peace forever after, the critical value of λ_0 is 3.3856. It follows that when $\pi_{t+1} = 0.5\ \forall t$, all sequences that ever involve $\lambda_t \leq 3.3856$ for some t are doomed to backwardness. This sufficient condition is not, however, necessary for that outcome. When $\lambda_0 = 3$, all sequences will end up in that state by $t = 8$, though ζ_t will continue to fluctuate thereafter with the states of war and peace. The distribution of ζ_{14} comprises four, separate dense clusters, the lowest with values of about 19, the highest with values of about 27.

For values of λ_0 lying in the range 3.6 to 4.2, *Ecclesiastes*, 9:11 sums up the ensuing trajectories aptly: "[...] fortune and chance happeneth to them all."—albeit the chances of ultimately attaining progress are slim when λ_0 is even a little less than 4. When $\lambda_0 = 3.6$, the sequence $\{I_t = 1\}_{t=0}^{t=5}$ yields backwardness for good from period $t = 6$ onwards. At that point in time, about 70% of all the possible 128 sequences yield $\lambda_6 < 3.3856$, rising to 94% of all possible 16,384 sequences at $t = 14$. When $\lambda_0 = 4$, the sequence $\{I_t = 1\}_{t=0}^{t=6}$ yields backwardness for good from period $t = 7$ onwards, and 38% of all possible 16,384 sequences yield $\lambda_{14} < 3.3856$. When $\lambda_0 = 4.2$, that fraction is still

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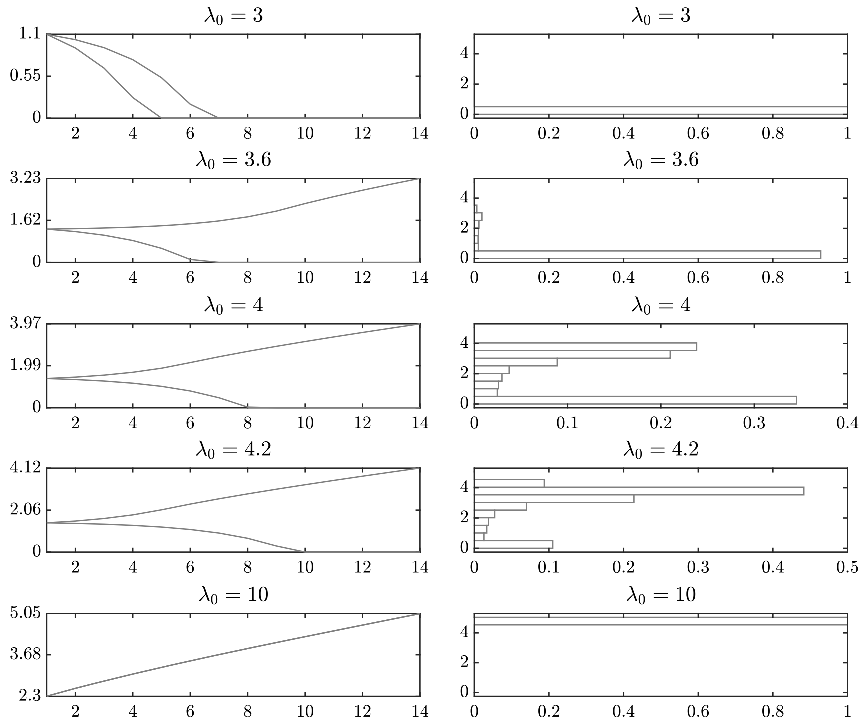

Figure 4. The distributions of trajectories of ln λ_t in a stationary stochastic environment.

12.5%; but about 7 out of 9 attain $\lambda_{14} > 4.004$, the critical value under perpetual war, and hence the assurance of ultimately enjoying progress indefinitely.

Far removed from such starting values, $\lambda_0 = 10$ provides a springboard for robust, but unsteady, growth. Progress is maintained whatever events come to pass $\left(e_t^0 = 1\ \forall t\right)$; but since mortality and destruction rates fluctuate, so do the growth rates of output and investment in physical capital, and hence the level of ζ_t. The distribution of the latter in period $t = 14$ comprises six separate, dense clusters, the lowest with values of about 53, the highest with values of about 87. In the worst possible outcome, war reigns perpetually, and if expectations are revised accordingly, the associated path is classed among those in Section 4.2. The same holds, *mutatis mutandis*, for perpetual peace (see Figure 3 from $t = 3$ onwards.)

8. Kenya, 1910–2070: calibration and simulations

Early in the 20th century, virtually the only formally educated adults in Kenya were colonial administrators, settlers, and missionaries. At Independence in 1964, some progress toward educating the population had been made, and its pace accelerated thereafter. The Mau Mau insurgency in the 1950s was not very destructive—except, ultimately, of colonial rule—and Kenya then remained untouched by war until Islamic State insurgents became active a decade or so ago, albeit always on a limited scale. What did not spare Kenya was the AIDS epidemic, which had broken out in the population at large by 1990 and whose course to date exemplifies a long-drawn-out episode of pestilence.

8.1. Calibration: 1910–1990

The data sources and the series constructed from them are discussed in detail in Bell et al. (2004). The GDP series was taken from the Penn World Tables (hereinafter, PWT), with starting year

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Table 2. Calibration for Kenya, 1920–1990

	1920	1930	1940	1950	1960	1970	1980	1990
First stage:								
Exogenous								
e_t	0.042	0.071	0.119	0.179	0.326	0.407	0.462	0.507
$GDP_t(10^7)$				436	642	1089	2014	3076
y_t				1286	1506	1930	2567	2716
$K_t(10^7)$				32.2	47.9	80.4	148.6	226.7
	α	γ	σ					
	1/3	0.7	0.75					
Solution 1								
λ_t	1	1.25	1.42	1.87	2.48	3.68	2.46	3.45
A_t				224.6	224.6	224.6	254.2	254.2
z_t	5.85	5.85	5.85	5.85	4.39	4.39	1.44	1.44
Solution 2								
λ_t	1	1.69	2.18	4.34	2.52	6.51	2.79	6.26
A_t				170.71	170.71	170.71	197.48	197.48
z_t	16.53	16.53	16.53	3.90	3.90	3.90	1.75	1.75
Second stage:								
Exogenous								
q_t^2				0.175	0.163	0.154	0.141	0.127
q_t^3				0.319	0.305	0.290	0.270	0.248
n_t				2.0	2.0	2.0	1.92	
δ				0.85	0.85	0.85	0.85	
	b	w	ρ	η				
Solution 1	0.092	0.071	0.249	1				
Solution 2	0.024	0.120	0.342	1				

Exogenous values: Bell et al. (2004, 2006a), and text.

1950 (see Table 2). In view of the fairly strong annual fluctuations in GDP, 5-year average levels were employed. The first population census was conducted in 1948, followed by others in 1962, 1969, and then decennially. After some minor smoothing and interpolation, the census data yield fertility and mortality rates, as reported in Table 2. The series for e_t was constructed from the census tables dealing with years of completed education by age cohort. For the present calibration, a series for the physical capital stock, K_t, is also needed. This, too, is taken from the PWT.[9]

8.1.1. First stage

This stage involves only the technologies for producing output and human capital. It holds for all values of e_t, however chosen. The input of human capital employed in producing the aggregate good is

$$L_t = \left(0.9N_t^1 + 0.5N_t^2\right)(1-e_t)\gamma + \left(0.5N_t^2\lambda_t + \sum_{a=3}^{5} N_t^a \lambda_{t+10(2-a)}\right) - w\lambda_{t-10}\left(0.9N_t^1 + 0.5N_t^2\right)e_t, \tag{22}$$

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where, in connection with this step only, N_t^a denotes the number of people in the age cohort a ($a = 0$, ages 0–4; $a = 1$, ages 5–14; $a = 2$, ages 25–34; . . . ; $a = 7$, ages 75+). The resulting level of output of the composite good is

$$Y_t = A_t L_t^{2/3} K_t^{1/3}, \tag{23}$$

whereby it should be remarked that Kenya is an open economy exporting primary products, so that fluctuations in world markets surely affect the TFP parameter A_t.

Turning to the educational technology, there is some evidence that the school system became overburdened from about 1970 [Bell et al. (2006a)], so that z_t, too, may have varied. We choose the simple form $h(e_t) = e_t$ and allow z_t, as well as A_t, to shift over time. Thus,

$$\lambda_{t+10} = z_t e_t \lambda_{t-10} + 1. \tag{24}$$

Given $\lambda_{1900} = \lambda_{1910} = \lambda_{1920} = 1$ [Bell et al. (2006a)] and the values of e_t for 1920–1980, (24) yields the values of $\lambda_{1930}, \lambda_{1940}, \ldots, \lambda_{1990}$ given any values of $z_{1920}, z_{1930}, \ldots, z_{1980}$.

GDP comprises the output of the composite good and investment in education. Valuing teachers' input of human capital at the latter's marginal product, measured in units of the composite good, we have

$$GDP_t = A_t L_t^{2/3} K_t^{1/3} + (2/3) A_t \left(K_t/L_t\right)^{1/3} \left[w \lambda_{t-10} \left(0.9 N_t^1 + 0.5 N_t^2\right) e_t\right]. \tag{25}$$

Table 2 reports two solutions. The large values of z_t up to 1970 reflect the fact that the education of the population began earlier, so that, by definition, Kenya had not remained mired in the state of "backwardness". In both solutions, the second break occurs in 1980, with z_t taking the values 1.44 and 1.75 from then onwards, values implying long-run annual growth rates of 1.84 and 2.84%, respectively.[10]

O'Connell and Ndulu (2000), using entirely different methods and sources in their investigation of the experience of growth in sub-Saharan Africa, also arrive at an estimated long-run annual growth rate for Kenya of 2%. Virtually identical is Sachs and Warner's (1997) estimate of 1.9%. That these estimates lie comfortably in the interval [1.84, 2.84] provides independent support for the values derived here, where solution 2 should be viewed as a somewhat optimistic springboard for developments after 1990.

8.1.2. Second stage

Associated with each of solutions 1 and 2 in the first stage are the parameters $(b, w, \delta, \rho, \eta)$. The method of estimating them rests on the household's f.o.c., which are set out in Appendix A. Those w.r.t. e_t and s_t do not involve the current term $u'\left(c_t^2\right)$; instead, there is a weighted pair of derivatives pertaining to the next period:

$$W_{t+1} \equiv \delta \rho u'\left(c_{t+1}^3\right) + b\left(1 - q_{t+1}^2\right)(1 - \rho) v'[(1 - \rho)\bar{y}_{t+1}],$$

whereby $v = \left[(1 - \rho)\bar{y}_{t+1}\right]^{1-\eta} / (1 - \eta)$. Since $e_t \in (0, 1)$ in the period of calibration, (30) holds as an equality, as does (29), S_t being always positive.

In Log-land ($\eta = 1$), W_{t+1} specializes to

$$W_{t+1} = \left[\delta\left(1 - q_{t+1}^3\right) \frac{N_{t+1}^3}{N_{t+1}^2} + b\left(1 - q_{t+1}^2\right)\right] \frac{1}{\bar{y}_{t+1}},$$

which is independent of ρ, and V_t takes the special form

$$V_t = \ln\left(c_t^2\right) + \left[\delta\left(1 - q_{t+1}^3\right) + \frac{b\left(1 - q_{t+1}^2\right)}{\left(1 - q_t^2\right)} n_t\right] \ln \bar{y}_{t+1} + R,$$

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where R denotes various terms involving parameters, exogenous variables, and logs thereof, terms that have no influence on the optimum. It is seen that the taste parameters δ and b are essentially perfect substitutes; for given fertility and mortality rates, their values can be varied arbitrarily while preserving the value of the expression in brackets, subject only to the requirements $\delta \in (0, 1)$ and $b \geq 0$. We therefore choose the "standard" value $\delta = 0.85$.

It remains to determine the Lagrange multiplier μ_t associated with the budget constraint—see (27). Noting that we have data for y_t and s_t for the period 1950–1990, and so can derive $\bar{y}_t$ from the parameters estimated in the first stage, the assumption that $u = \ln c_t^2$ and (28) yield $\mu_t = 1/\left[\left(1 - q_t^2 + \beta n_t\right) c_t^2\right]$. The normalized form of the family's budget identity is $\left(1 - q_t^2 + \beta n_t\right) c_t^2 + s_t + \rho \bar{y}_t = y_t$. Substituting for c_t^2, we obtain $\mu_t = 1/(y_t - s_t - \rho \bar{y}_t)$, which can then be employed in (30). Rearranging, we obtain

$$\frac{\partial \Phi_t}{\partial e_t} = W \cdot \frac{n_t}{\left(1 - q_t^2\right)} \frac{\partial \bar{y}_{t+1}}{\partial \lambda_{t+1}} \frac{\partial \lambda_{t+1}}{\partial e_t} + \frac{\bar{y}_{t+1}}{\left(y_t - s_t - \rho \bar{y}_t\right)} \frac{\partial y_t}{\partial e_t} = 0. \tag{26}$$

In Log-land, the second term marks the sole appearance of ρ in the system to be estimated. Observe also that the social norm parameter β appears nowhere in the above scheme of equations, and so cannot be estimated.

In 1980, Kenya's people could have had no inkling of the AIDS epidemic that was to befall them later in that decade. Their decisions about education in 1980 were therefore based on expectations about future mortality derived from the experience of falling rates over the previous three or four decades. For this reason, we apply the above procedure using only the 4 years 1950, 1960, 1970, and 1980, and the counterfactual values of q_t^2 and q_t^3 for 1990 estimated by Bell et al. (2006a), as reported in Table 3.

In Log-land, there are just the three parameters b, w, and ρ, whereby w exerts a weak influence on stage 1, which is taken into account. Proceeding from solution 1 in stage 1, the pair of years (1960, 1980) in stage 2 yields one solution wherein ρ is fairly close to α, there is modest altruism ($b = 0.092$) and $w = 0.0710$; this solution is reported in the final panel of Table 2. All the other second-stage solutions from this pair of years involve implausibly small values of ρ or $b < 0$. Of all the other pairs of years, only (1950, 1980) bears examination, and its solutions are marked by $b < 0$. Proceeding from solution 2 in stage 1, the pair of years (1950, 1980) yields the sub-constellation $\rho = 0.342$, $b = 0.024$ and $w = 0.120$ in stage 2, as reported in Table 2. All other sub-constellations arising from this pair involve either $b < 0$ or implausible values of ρ. Other pairs of years fail in one way or another, often yielding implausibly large values of ρ or w.

There remain the variants wherein η is free. It turns out that this additional flexibility is not needed for the purposes of this particular calibration (see Appendix D).

8.2. Transitions to an end of the epidemic

Proceeding as in Section 7.2, we posit a definite end of the epidemic after three 20-year periods, that is, in 2050. We also assume that Kenya will be spared war or a serious insurgency. The full-scale outbreak is revealed in 1990. All agents then form some prior that it will continue in the period starting in 2010, perhaps somewhat attenuated, or mercifully end altogether. In the latter event, the falling course of mortality over the period 1950–1980 would be restored. The agents make their investment decisions accordingly. By assumption, the worst happens again in 2010, and a prior is formed; and once more in 2030, but then with the certainty of full, permanent relief in 2050. The associated mortality rates for each of these two possible states are set out in Table 3. Given what must have been great uncertainty in the minds of Kenya's people in the early 1990s about the course of the epidemic and the likelihood of effective and available antiretroviral therapies, we set the prior probability of a continuation in 2010 and 2030 at 0.5.

Solutions 1 and 2 yield the trajectories of ζ_t and $\ln \lambda_t$ over the period 1990–2070 depicted in Figures 5 and 6, respectively. Although $z_t h(1) > 1$ from 1990 onwards in both solutions, thus satisfying the central necessary condition for long-run growth, $z_t h(1) > 1$ does not suffice to bring

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Table 3. Premature adult mortality rates and fertility: with and without AIDS

		1990	2010	2030	2050
No AIDS	q_t^2	0.127	0.099	0.070	0.041
	q_t^3	0.248	0.193	0.157	0.114
AIDS	q_t^2	0.353	0.270	0.154	0.111
	q_t^3	0.373	0.355	0.249	0.175
	n_t	1.7	1.40	1.15	1.0

Source: Bell et al. (2006a).

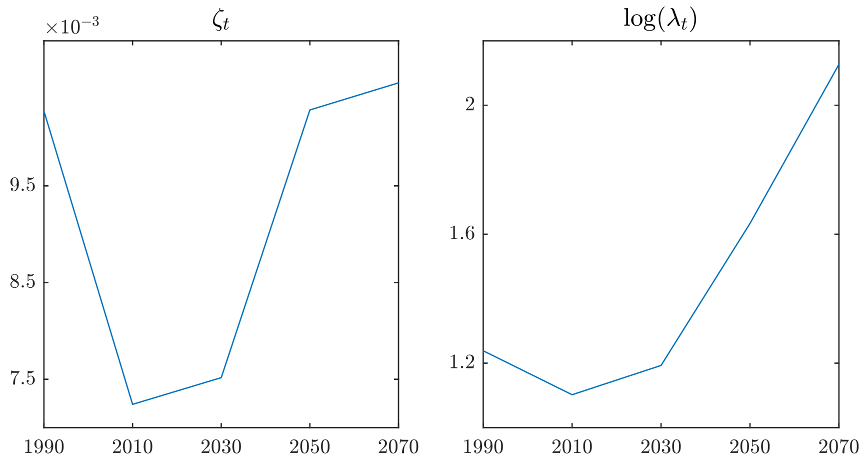

Figure 5. Trajectories of ζ_t and ln λ_t for Solution 1: three realized waves of AIDS.

Figure 6. Trajectories of ζ_t and ln λ_t for Solution 2: three realized waves of AIDS.

about that state: the initial conditions must also be sufficiently favorable. Both solutions involve an initial setback: $\lambda_{2010} < \lambda_{1990}$. In 2030, moreover, recovery of the initial level is still incomplete in solution 1 and scarcely fulsome in solution 2. More encouraging is the attainment of full education for that cohort in both solutions, with λ_t growing at the steady rate $z_t h(1) - 1$ thereafter. Despite the severity of the outbreak early on, a fall into backwardness is not threatened.

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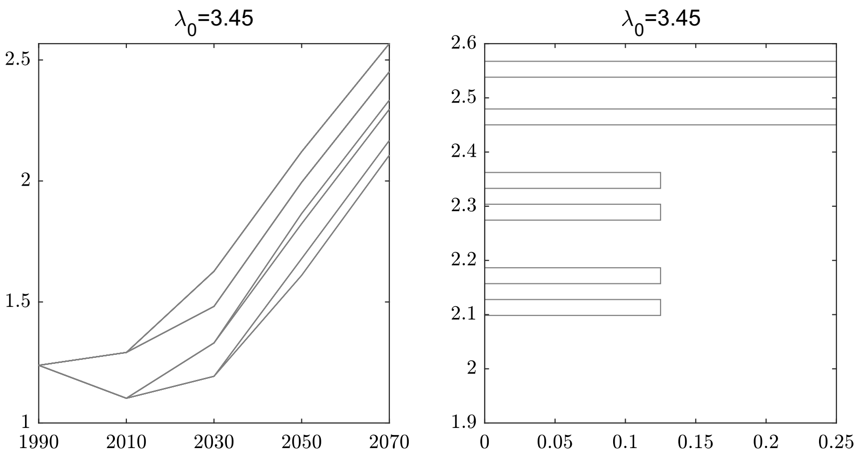

Figure 7. Kenya: trajectories of ln λ_t and the distribution of ln λ_{2070}, solution 1.

It must be emphasized that these trajectories stem from forecasts of mortality that rest on the available evidence and demographic modeling in the period 2002–2004, together with the admittedly speculative prior $\pi_{t+1} = 0.5$. In fact, Kenya's epidemic has indeed continued into the present, with an estimated prevalence rate of 4.9% in 2018 [Kenya (2020)]. Yet changes in behavior and the extensive provision of antiretroviral treatment have held mortality rates in check, at levels well below those in Table 3. As for an early setback, GDP per capita actually fell by about 6% between 1990 and 2001, and gross primary school enrollments also declined in that decade [Kimalu et al. (2001)]. The sharp setback in Figures 5 and 6 is admittedly a rather strong amplification of actual early outcomes, but it captures their essence.

8.3. A permanently stochastic environment

To complete this exploration of Kenya's development in the time of AIDS up to 2070, we proceed as in Section 7.3, treating the stochastic environment as the values taken by the mortality rates described in Table 3. As before, it is assumed that, whereas $\mathbf{q}_{1990}$ was revealed to the population at that time as a fact, their beliefs about $\mathbf{q}_{2010}$ put the respective probabilities of occurrence at one half—a Solomonic judgment in such circumstances. The same is assumed for each of the succeeding periods 2010, 2030, and 2050. As in Section 8.2, war is ruled out, leaving pestilence as the only hazard. We provide the respective trajectories in Figures 7 and 8.

When governed by solution 1, Kenya's immediate prospects were rather poor, even with the counterfactual, no-AIDS mortality profile as the actual outcome in 2010. Yet in all of the $16(=2^4)$ binary sequences, all cohorts of children enjoy a full education from 2030 onwards. In eight of those sequences, λ_{2070} is at least 3.4 times greater than its value in 1990 (see Figure 7). In the two worst, it is just 2.4 times greater—a decidedly modest achievement after 80 years of development. In solution 2, the corresponding ratios are 4.8 and 2.9, respectively (see Figure 8).

In fact, the widespread provision of antiretroviral therapies and changes in behavior have resulted in a course of the epidemic less dire than that forecast in Table 3. If supported by improvements in the conduct of economic policy, and with more sanguine beliefs, the long-term outlook will be a deal better than that portrayed in Figure 8.

9. Discussion

Expectations, the formation of human capital, and their interplay take on leading roles in the framework developed in this paper. Their prominence merits some discussion.

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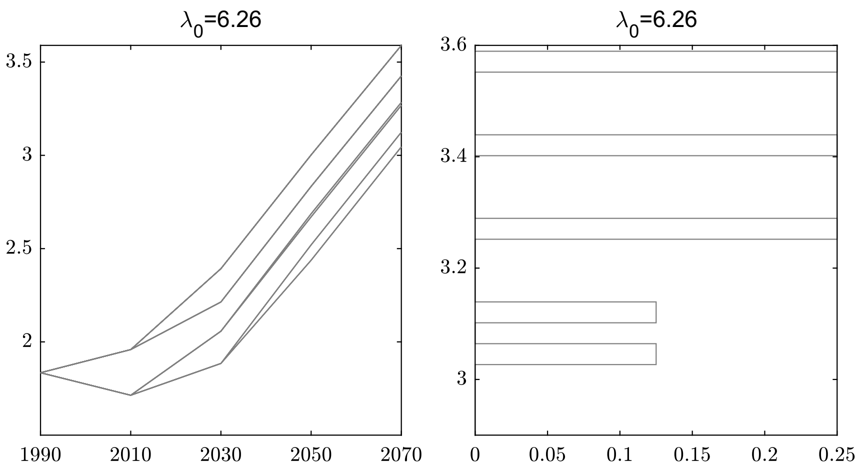

Figure 8. Kenya: trajectories of ln λ_t and the distribution of ln λ_{2070}, solution 2.

Where expectations are concerned, it is instructive to consider Europe's course of development in the 19th century. The battle of Waterloo in 1815 was the final act in the French Revolutionary and Napoleonic Wars, which had been prosecuted with scarcely a break since 1792. Marauding armies had exacted a punishing toll on life and property in Italy, the Iberian Peninsular, much of Western and Central Europe, and the central corridor to Moscow. Only at the close did France suffer the visitation of foreign armies, and then not grievously. The final human reckoning was grim: about 5 million lives—in proportion to population, at least as many as in WWI [Blanning (2008: 670)]. Some 1.4 million of them were French, leaving 857 males to every 1,000 females in 1815 (*ibid.*: 672), and surely a lower ratio still among young and prime-age adults. Other wars had raged, off and on, for the previous 200 years. The Thirty Years' War (1618–1648) had ravaged much of what is now Holland and Belgium, most of Germany and part of northern Italy, and claimed some 5 million German lives,[11] It was succeeded by the series of dynastic wars that followed the Treaty of Westphalia until 1789. With perhaps dim collective memories of the former and fresher memories of the latter, Europe's peoples might, quite rationally, have expected the future to hold more of the same, Waterloo being just the latest bloody event.

Yet things turned out quite differently. Interrupted only by the Crimean War on its periphery and the three brief wars of German Unification, Europe enjoyed peace until the Balkan wars shortly before 1914. Industrialization had already begun in Britain and was stirring in some parts of Western Europe. By 1870, the process of structural transformation was well advanced in Denmark, France, Germany, the Lowlands, and Switzerland, soon to be followed by Austria, Norway, and Sweden [Crafts (1984)]. Despite urbanization, mortality had also declined, albeit rather slowly. Improved public health measures, which owed much to the discoveries of Pasteur and Koch, led to a fairly marked decline thereafter. It is against this backdrop that the outbreak of WWI[12]—and still more its long and murderous course—must have come as a profound shock to Europe's people.

Since our model posits human capital as the engine of long-run growth, it is arguably better applicable to the time from 1870 onwards. Galor (2005) describes how the importance of human capital as an input in production increased rapidly in the second phase of the industrial revolution in the second half of the 19th century, when increasing wages for skilled labor led even capitalists to lobby the government for measures promoting public education. With ongoing adoption of new technologies, workers were required to have more than just manual skills. This complementarity between technology and human capital has continued into the present, as documented by Benhabib and Spiegel (2005). Moreover, there is an extensive empirical literature that

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demonstrates the relevance of human capital for growth in the 20th century [see Pelinescu (2015) and Becker et al. (1990) for overviews].

The impact of human capital before 1870 is empirically unclear, as demonstrated by the debate between Acemoglu et al. (2001) and Glaeser et al. (2004). Using settler mortality as an instrument for institutional quality, the former demonstrates that differences in that quality across colonial settlements can explain long-run differences in countries' wealth. Acemoglu et al. (2005) argue that the accumulation of human capital does not drive growth, but rather that growth follows from good institutions. Glaeser et al. (2004) propose an alternative hypothesis, namely, that the quality of institutions is determined by the abundance of human capital and that initial differences in human capital lead to differences in institutions. While this hypothesis was empirically rejected by Acemoglu et al. (2014), the matter is not settled. Bhattacharyya (2009), for instance, finds a positive effect for institutions and human capital in the 20th century.

10. Conclusions

Unremitting warfare and pervasive communicable diseases, with the privation and destruction that accompany them, would seem sufficient to bring about a Hobbesian existence, even when productive technologies are available. Yet we have established that there are wide-ranging constellations of unchanging war losses and premature adult mortality such that both "backwardness", a state in which there is no investment in human capital through schooling, and "progress", a state in which there is unbounded growth and a fully educated population, are possible equilibria.

Parents' altruism, if sufficiently strong, can rule out backwardness in environments so enduringly hazardous as to keep a selfish population in that condition for good. That is no great surprise. Where attaining—and maintaining—growth is concerned, parents' altruism comes into play in a particular way. If their preferences are such that the sub-utility functions for their own consumption and their children's well-being in adulthood differ—which is highly likely—and the former is more concave than the latter, then the only steady-state path other than backwardness is progress. If, however, the sub-utility function for own consumption is less concave than that for the children's full income net of payments to the surviving members of the older generation, then steady-state growth paths with an incompletely educated population may exist. The same holds if parents are perfectly selfish, so that provision for old age is the sole motive for investment. Thus, altruism may lead to permanently faster growth.

The fact that outbreaks of war and pestilence are stochastic events introduces a central role for expectations. It also raises the question of whether a growing economy can withstand a series of adverse shocks. Mature economies that have experienced growth for long periods will have large per capita stocks of human and physical capital. They will be correspondingly robust, unless the environment itself is destroyed. Economies at an earlier stage of development are more vulnerable. Numerical simulations in which the realized outcome is three consecutive periods of war followed by a confidently expected era of perpetual peace reveal how the initial boundary conditions have a decisive influence on whether this series of shocks will pitch the economy into permanent backwardness or, somewhat less drastically, condemn it to a slow and perhaps enfeebled recovery. Calibrated to Kenya, which continues to experience a severe AIDS epidemic, the model yields results that illustrate how painfully slow such a recovery could be.

To close, we must draw attention to certain important limitations of our analysis. Fertility is treated as exogenous, the old make no current contributions of any kind, and the extended family structure rules out a variety of risks. Individuals are also denied any measures, in the form of either personal or collective action, to mitigate the hazards of disease, endemic, or epidemic. There is ample scope for future work, in various directions.

Notes

1 It should be noted that without efficacious vaccines, individual investment in health may do little to ward off the infectious diseases in question.
2 The determination of β and ρ is discussed in Appendix D. One variant of the rule governing old-age provision is that each of those surviving through old age receives a fixed proportion of the full income of each surviving young adult. This variant is discussed in Appendix E.
3 If only natural children count, the "adjustment" for adopted children $1/\left(1-q_t^2\right)$ drops out.
4 This analytically convenient restriction on h is not easy to square with the fact that there is the need to lay secure foundations early on in schooling in order to enable the rapid development of wider abilities later on. It is therefore arguable that $h'(0)$ is significantly smaller than the average slope, $h(1)$.
5 The marginal rate of technical substitution between human and physical capital.
6 For a vigorous argument that rational actors must have sharp priors, see Elga (2010).
7 For an analysis of this potentially liberating stroke, see Bell and Gersbach (2013), who assume there is only human capital. The assumption that both inputs are necessary for production leaves the matter open.
8 The inequality $\mathbf{x} << \mathbf{y}$ indicates that each component of the vector $\mathbf{y}$ exceeds its counterpart in $\mathbf{x}$.
9 See, in this connection, Feenstra et al. (2015).
10 In view of fluctuations in trading conditions and the performance of the educational system, a strictly monotonic, decadal course of λ_t is hardly to be expected.
11 "[..] die Verlustrate liegt mit Sicherheit dichter an 40 als an 15 Prozent." [Schmidt (2002: 89)]. (".. the rate of loss lies with certainty closer to 40 than 15 per cent.")
12 Clark (2013) provides a compelling account of its origins.
13 Background paper for World Bank (2007).
14 Bell et al. (2006a) assume the iso-elastic form $h = e_t^{\epsilon}$. They arrive at the estimate $\epsilon = 0.57$, which, in view of the Inada condition, rules out backwardness.

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Appendix A: Proofs

A.1. Steady states: a technical preliminary

Rewrite V_t as a function of the decision variables:

$$V_t = u\left(c_t^2\right) + \chi_t u\left(\frac{\rho n_t \bar{y}_{t+1}(e_t, s_t)}{\left(1-q_{t+1}^3\right)\left(1-q_t^2\right)}\right) + \nu_t v[(1-\rho)\bar{y}_{t+1}(e_t, s_t)],$$

whereby (6) is used, and

$$\chi_t \equiv \delta\left(1-q_{t+1}^3\right) \text{ and } \nu_t \equiv \frac{b\left(1-q_{t+1}^2\right) n_t}{\left(1-q_t^2\right)}.$$

The budget constraint (7) can be expressed as $y_t = \left[\left(1-q_t^2\right)+\beta n_t\right] c_t^2 + s_t + \rho \bar{y}_t$. Hence, the associated Lagrangian is

$$\Phi_t = V_t + \mu_t\left[y_t - \left(1-q_t^2+\beta n_t\right) c_t^2 - s_t - \rho\bar{y}_t\right]. \tag{27}$$

Note that y_t depends on the amount of child labor, $n_t(1-e_t)\gamma$. The assumptions on u (the Inada condition at $c_t^2=0$) ensure that $c_t^{2,0}>0$. By assumption, physical capital is necessary for production. Hence, if some young adults survive through three full periods $\left(q_{t+1}^3<1\right)$, so that $\chi_t>0$, then $s_t^0>0$.

The associated first-order conditions (hereinafter f.o.c.) are, noting that $0 \le e_t \le 1$,

$$\frac{\partial \Phi_t}{\partial c_t^2} = u'\left(c_t^2\right) - \mu_t[1-q_t^2+\beta n_t] = 0, \tag{28}$$

$$\frac{\partial \Phi_t}{\partial s_t} = \left(\delta\rho u'\left(c_{t+1}^3\right) + b\left(1-q_{t+1}^2\right)(1-\rho)v'[(1-\rho)\bar{y}_{t+1}]\right)\frac{n_t}{1-q_t^2}\cdot\frac{\partial \bar{y}_{t+1}}{\partial s_t} - \mu_t = 0, \tag{29}$$

$$\frac{\partial \Phi_t}{\partial e_t} = \left(\delta\rho u'\left(c_{t+1}^3\right) + b\left(1-q_{t+1}^2\right)(1-\rho)v'[(1-\rho)\bar{y}_{t+1}]\right)\frac{n_t}{1-q_t^2}\frac{\partial \bar{y}_{t+1}}{\partial \lambda_{t+1}}\frac{\partial \lambda_{t+1}}{\partial e_t} + \mu_t\frac{\partial y_t}{\partial e_t} \le 0,\ e_t \ge 0, \tag{30}$$

$$\frac{\partial \Phi_t}{\partial e_t} = \left(\delta\rho u'\left(c_{t+1}^3\right) + b\left(1-q_{t+1}^2\right)(1-\rho)v'[(1-\rho)\bar{y}_{t+1}]\right)\frac{n_t}{1-q_t^2}\frac{\partial \bar{y}_{t+1}}{\partial \lambda_{t+1}}\frac{\partial \lambda_{t+1}}{\partial e_t} + \mu_t\frac{\partial y_t}{\partial e_t} \ge 0,\ e_t \le 1, \tag{31}$$

where, recalling that F is homogeneous of degree 1, $\Lambda_{t+1} = \left(1-q_{t+1}^2\right)N_{t+1}^2\lambda_{t+1} + \gamma N_{t+1}^1$ and $\zeta_t = \lambda_t/s_{t-1}$,

$$\frac{\partial \lambda_{t+1}}{\partial e_t} = z_t h'(e_t)\lambda_t,$$

$$\frac{\partial y_t}{\partial e_t} = -(\gamma + w\lambda_t)n_t \cdot F_1\left[(1-q_t^2 - w n_t e_t)\zeta_t + \frac{n_t\gamma(1-e_t)}{s_{t-1}}, \frac{\sigma_t}{n_{t-1}}\right],$$

$$\frac{\partial \bar{y}_{t+1}}{\partial s_t} = \frac{\sigma_{t+1}}{n_t}\cdot F_2\left[\left(1-q_{t+1}^2\right)\zeta_{t+1} + \frac{n_{t+1}\gamma}{s_t}, \frac{\sigma_{t+1}}{n_t}\right], \text{ and}$$

$$\frac{\partial \bar{y}_{t+1}}{\partial \lambda_{t+1}} = \left(1-q_{t+1}^2\right)F_1\left[\left(1-q_{t+1}^2\right)\zeta_{t+1} + \frac{n_{t+1}\gamma}{s_t}, \frac{\sigma_{t+1}}{n_t}\right].$$

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A weighted sum of the marginal utilities $u'\left(c_{t+1}^3\right)$ and $v'\left[(1-\rho)\bar{y}_{t+1}\right]$ plays a central role:

$$\frac{\partial V_t}{\partial \bar{y}_{t+1}} = \frac{\rho\delta n_t}{1-q_t^2}\cdot u'\left(\frac{n_t\rho\bar{y}_{t+1}}{\left(1-q_t^2\right)\left(1-q_{t+1}^3\right)}\right) + (1-\rho)\nu_t v'[(1-\rho)\bar{y}_{t+1}] \equiv n_t\Omega_t.$$

Using μ_t from (28) in (29) and (30) yields, respectively,

$$u'\left(c_t^2\right) = \sigma_{t+1}(1-q_t^2+\beta n_t)\Omega_t\cdot F_2\left(\bar{l}_{t+1}, \frac{\sigma_{t+1}s_t}{n_t}\right), \tag{32}$$

which holds for all $e_t \in [0,1]$, and

$$\Omega_t\left(1-q_{t+1}^2\right)\cdot F_1\left(\bar{l}_{t+1}, \frac{\sigma_{t+1}s_t}{n_t}\right) zh'(e_t) \geq \frac{u'\left(c_t^2\right)(w+\gamma/\lambda_t)}{(1-q_t^2+\beta n_t)} F_1\left(l_t, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right),\ e_t \leq 1, \tag{33}$$

where the inequalities are reversed for $e_t \geq 0$. Substituting from the first-order conditions (32) in (33), we obtain, for all interior solutions $e_t \in (0,1)$,

$$\left(1-q_{t+1}^2\right) zh'(e_t)\cdot F_1\left(\bar{l}_{t+1}, \frac{\sigma_{t+1}s_t}{n_t}\right) = (w+\gamma/\lambda_t)F_1\left(l_t, \frac{\sigma_t s_{t-1}}{n_{t-1}}\right)\cdot\sigma_{t+1}F_2\left(\bar{l}_{t+1}, \frac{\sigma_{t+1}s_t}{n_t}\right).$$

A.2. Proof of Lemma 1

Since s_t and λ_t are growing at the steady rate $g(e) = zh(e) - 1 > 0$, n_t and $\mathbf{q}_t$ being constant, we have, from the definitions of c_t^2 and c_t^3,

$$\frac{c_t^2}{c_{t+1}^3} = \frac{\left(1-q^2\right)\left(1-q^3\right)}{\rho n\left(\left(1-q^2\right)+\beta n\right)}\cdot\left(\frac{y_t - s_t}{\bar{y}_{t+1}} - \frac{\rho}{zh(e)}\right),$$

where

$$\frac{y_t - s_t}{\bar{y}_{t+1}} - \frac{\rho}{zh(e)} = \frac{1}{zh(e)}\left(\frac{F[(1-q^2-wne)\zeta, \sigma/n] - zh(e)}{F\left[\left(1-q^2\right)\zeta, \sigma/n\right]} - \rho\right),$$

which is a constant for any given e. Substituting for c_t^2/c_{t+1}^3 from (13), we obtain

$$\frac{\rho\delta\sigma\left(1-q^2+\beta n\right)F_2\left[\left(1-q^2\right)\zeta, \sigma/n\right]}{1-q^2}$$
$$= \left(\frac{\rho n\left(\left(1-q^2\right)+\beta n\right) zh(e)F\left[\left(1-q^2-wne\right)\zeta, \sigma/n\right]}{\left(1-q^2\right)\left(1-q^3\right)\left[F\left[\left(1-q^2-wne\right)\zeta, \sigma/n\right] - \rho F\left[\left(1-q^2\right)\zeta, \sigma/n\right] - zh(e)\right]}\right)^{\xi},$$

which may be rearranged as

$$F[(1-q^2-wne)\zeta, \sigma/n] - \rho F\left[\left(1-q^2\right)\zeta, \sigma/n\right] = \left(1 + B'\frac{F\left[\left(1-q^2\right)\zeta, \sigma/n\right]}{\left(F_2\left[\left(1-q^2\right)\zeta, \sigma/n\right]\right)^{1/\xi}}\right) zh(e), \tag{34}$$

where

$$B' \equiv \frac{n}{(1-q^3)(\delta\sigma)^{1/\xi}}\left(\frac{\rho(1-q^2+\beta n)}{1-q^2}\right)^{1-1/\xi}$$

is a positive constant.

The assumption that F is homogeneous of degree 1, with both inputs necessary in production, implies that ζ is differentiable in e when e is varied parametrically. For continuous changes in e produce continuous changes in the feasible set and V_t, and the isoquant map is smooth everywhere and strictly convex to the origin, and no isoquant intersects either axis.

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A.2.1. Part (i)

By assumption, F is Cobb-Douglas: $y_t = Al_t^{1-\alpha}k_t^{\alpha}$. Substituting into (34) and collecting terms, we have

$$A\left(\frac{\sigma}{n}\right)^{\alpha}\left[(1-q^2-wne)^{1-\alpha}-\rho\left(1-q^2\right)^{1-\alpha}\right]\zeta^{1-\alpha}$$
$$=\left(1+\left(\frac{n}{\alpha\sigma}\right)^{1/\xi}B'\cdot\left(F\left[\left(1-q^2\right)\zeta,\sigma/n\right]\right)^{1-1/\xi}\right)zh(e). \tag{35}$$

Differentiating (35) totally, noting that $\partial F/\partial\zeta = (1-\alpha)F/\zeta$, and collecting terms, we obtain

$$\left[A\left(\frac{\sigma}{n}\right)^{\alpha}\left[(1-q^2-wne)^{1-\alpha}-\rho\left(1-q^2\right)^{1-\alpha}\right](1-\alpha)\zeta^{-\alpha}\right.$$
$$\left.-(1-1/\xi)\left(\frac{n}{\alpha\sigma}\right)^{1/\xi}\frac{(1-\alpha)B'}{\zeta}\cdot\left(F\left[\left(1-q^2\right)\zeta,\sigma/n\right]\right)^{1-1/\xi}\cdot zh(e)\right]\cdot d\zeta$$
$$=\left[\left(1+\left(\frac{n}{\alpha\sigma}\right)^{1/\xi}B'\cdot\left(F\left[\left(1-q^2\right)\zeta,\sigma/n\right]\right)^{1-1/\xi}\right)zh'(e)\right.$$
$$\left.+A\left(\frac{\sigma}{n}\right)^{\alpha}\left[\left(1-q^2-wne\right)^{-\alpha}\right]wn(1-\alpha)\zeta^{1-\alpha}\right]\cdot de.$$

Now, the condition $F[(1-q^2-wn)\zeta,\sigma/n] > \rho F\left[\left(1-q^2\right)\zeta,\sigma/n\right]$ implies that $(1-q^2-wn)^{1-\alpha} > \rho\left(1-q^2\right)^{1-\alpha}$, so that ζ is increasing in e if $\xi \le 1$. By continuity, this result also holds for all F sufficiently close to Cobb-Douglas in form and for all ξ exceeding, but sufficiently close to, 1.

A.2.2. Part (ii)

By assumption, $y_t = A[b_1 l_t^{\epsilon} + b_2 k_t^{\epsilon}]^{1/\epsilon} = A\{b_1\left[\left(1-q^2-wne\right)\lambda_t\right]^{\epsilon} + b_2(\sigma s_{t-1}/n)^{\epsilon}]\}^{1/\epsilon}$, $\epsilon \le 1$: the elasticity of substitution is $(\epsilon-1)^{-1}$, where $\epsilon = 0$ is Cobb-Douglas. Proceeding as before, the term in (34)

$$\frac{F\left[\left(1-q^2\right)\zeta,\sigma/n\right]}{\left(F_2\left[\left(1-q^2\right)\zeta,\sigma/n\right]\right)^{1/\xi}} = \frac{A\left[b_1\left(\left(1-q^2\right)\zeta\right)^{\epsilon}+b_2(\sigma/n)^{\epsilon}\right]^{1/\epsilon}}{\left[b_2(\sigma/n)^{\epsilon-1}A\left[b_1\left(\left(1-q^2\right)\zeta\right)^{\epsilon}+b_2(\sigma/n)^{\epsilon}\right]^{1/\epsilon-1}\right]^{1/\xi}}$$
$$= B_1\left[b_1(\left(1-q^2\right)\zeta)^{\epsilon}+b_2(\sigma/n)^{\epsilon}\right]^{\psi},$$

where $\psi = (1/\epsilon) + (1-1/\epsilon)/\xi$ and $B_1 = (\sigma/n)^{1-\epsilon}/b_2$ is a positive constant. Substituting into (34), noting the derivative of $\left[b_1\left(\left(1-q^2\right)\zeta\right)^{\epsilon}+b_2(\sigma/n)^{\epsilon}\right]^{1/\psi}$ w.r.t. ζ and rearranging as before in part (i), there are two terms on the l.h.s. The first is the partial derivative of $\{F[(1-q^2-wne)\zeta,\sigma/n] - \rho F\left[\left(1-q^2\right)\zeta,\sigma/n\right]\}$ w.r.t. ζ, which is positive if $\epsilon \le 0$ and $F[(1-q^2-wn)\zeta,\sigma/n] > \rho F\left[\left(1-q^2\right)\zeta,\sigma/n\right]$. The second term has the sign of $\psi\cdot\epsilon$. Now, $\psi\cdot\epsilon \le 0$ if and only if $\epsilon+\xi \le 1$. Since both inputs are assumed to be necessary in production, $\epsilon \le 0$, which yields the required result.

A.3. Proof of Proposition 4

Concerning backwardness ($e_t^0 = 0$), $u(c_t) = \ln c_t$ and $v[(1-\rho)\bar{y}_{t+1})] = \ln[(1-\rho)\bar{y}_{t+1})]$ yield the simple forms

$$n\Omega_t\left(e_t^0=0\right) = \left[\delta\left(1-q^3\right)+bn\right]/\bar{y}_{t+1},\ u'\left(c^2\right) = 1/c^2 = \frac{1-q^2+\beta n}{(1-\rho)\bar{y}\left(e_t^0=0\right)-s^b\left(e_t^0=0\right)}.$$

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Recalling (32), it is seen that (9) will hold if, and only if,

$$\frac{n}{(1-q^2)\left[\delta(1-q^3)+bn\right]\left[(1-\rho)-s^b\left(e_t^0=0\right)/\bar{y}\left(e_t^0=0\right)\right]}\geq\frac{zh'(0)}{\gamma+w}.$$

Since $s^b > 0$, a sufficient condition for (9) to hold is (15).

Concerning progress, if F is Cobb-Douglas, condition (12) specializes to

$$zh'(e)\geq\alpha\sigma^{\alpha}n^{1-\alpha}wA\cdot\frac{[\zeta(e)]^{1-\alpha}}{\left(1-q^2-wne\right)^{\alpha}},\ e\leq 1. \tag{36}$$

The l.h.s. is positive and independent of F. If h is concave, $zh'(e)$ is non-increasing in e. (In the limiting case, $h=e$, so that $h'=1$.) Now, Lemma 1 establishes that, along any steady growth path in Log-land, $\zeta(e)$ is increasing in e, so that the r.h.s. is also positive, but increasing in e, and without bound if e is unrestricted. It follows that the equation

$$zh'(e)=\alpha\sigma^{\alpha}n^{1-\alpha}wA\cdot\frac{[\zeta(e)]^{1-\alpha}}{\left(1-q^2-wne\right)^{\alpha}} \tag{37}$$

has a unique, positive solution, denoted by e^*. If $e^*\geq 1$, progress will be the only steady-state growth path; otherwise, progress will not be attainable. If $e^*<1$ and $zh(e^*)>1$, there is also a unique such path, but exhibiting slower growth. If $zh(e^*)\leq 1$, no such path exists. In the limiting case $zh(e^*)=1$, $\lambda_{t+1}-\lambda_t=1\ \forall t$, so that λ_t grows without bound and (37) still holds, but the growth rate tends to zero.

If $zh(e^*)<1$, there remains only the possibility that there exists one or more stationary states in addition to backwardness. Since backwardness is a locally stable equilibrium if (15) holds, and any steady-state growth path, if there exist such, is unique, then by continuity, there exists an $e'\in(0,\min[e^*,1])$ such that $\lambda_{t+1}=zh(e')\lambda_t+1=\lambda_t$, that is, λ_t is stationary at the level $1/(1-zh(e'))\ \forall t$. Substituting into (8) yields, for any stationary path of λ_t,

$$zh'(e)=\frac{\alpha\sigma^{\alpha}n^{1-\alpha}}{1-q^2}[w+\gamma(1-zh(e))]A\cdot\frac{[1-q^2+n\gamma(1-zh(e))]\zeta(e)^{1-\alpha}}{\left[\left(1-q^2-wne\right)+n\gamma(1-e)(1-zh(e))\right]^{\alpha}}, \tag{38}$$

whereby the behavior of $\zeta(e)$ cannot be inferred from Proposition 4. However, in a derivation that is analogous to that of (20), we obtain the expression

$$\zeta(e)=\frac{AQ}{1+Q}\left(\frac{\sigma}{n}\right)^{\alpha}$$
$$\times\left[(1-q^2-wne+n\gamma(1-e)(1-zh(e)))^{1-\alpha}-\rho(1-q^2+n\gamma(1-zh(e)))^{1-\alpha}\right]^{-1}.$$

Since the first expression in brackets is falling faster than the second, $\partial\zeta/\partial e>0$. Yet, it is seen that the r.h.s. of (38) is not necessarily increasing in e everywhere in the interval $(0,e^*)$, so that there can exist more than one $e'\in(0,\min[e^*,1])$ that yields a stationary equilibrium path.

Appendix B: The extreme allocations of $S_t(I_t)$

In what follows, it will be useful to rewrite (18) in the form

$$\left[1+\frac{\beta n_t}{1-q_t^2(I_t)}\right]c_t^2+\frac{s_t}{1-q_t^2(I_t)}+\rho F\left[\lambda_t+\frac{\gamma n_t}{1-q_t^2(I_t)},\ \frac{\sigma_t(I_t)}{1-q_t^2(I_t)}\cdot\frac{s_{t-1}}{n_{t-1}}\right]$$
$$\leq F\left[\left(1-\frac{wn_te_t}{1-q_t^2(I_t)}\right)\lambda_t+\frac{\gamma n_t(1-e_t)}{1-q_t^2(I_t)},\ \frac{\sigma_t(I_t)}{1-q_t^2(I_t)}\cdot\frac{s_{t-1}}{n_{t-1}}\right],\ I_t=\{0,1\}. \tag{39}$$

Allocation A: $c_t^2 = e_t = 0$. Given I_t, s_t is maximal:

$$s_t = (1-\rho)F\left[\left(1-q_t^2(I_t)\right)\lambda_t + n_t\gamma, \frac{\sigma_t(I_t)s_{t-1}}{n_{t-1}}\right], I_t = \{0, 1\}.$$

An increase in $q_t^2(I_t)$ will induce A to shift toward the origin O, as depicted by the point A′. Given that $q_t^2(1) > q_t^2(0)$, the allocations A and A′ also represent those ruling under peace and war, respectively, in period t.

Allocation B: $c_t^2 = 0$, $e_t = 1$. Given I_t and maximum (full time) investment in education, s_t is maximal. We have

$$s_t = F\left[(1-q_t^2(I_t) - wn_t)\lambda_t, \frac{\sigma_t(I_t)s_{t-1}}{n_{t-1}}\right] - \rho F\left[\left(1-q_t^2(I_t)\right)\lambda_t + n_t\gamma, \frac{\sigma_t(I_t)s_{t-1}}{n_{t-1}}\right], I_t = \{0, 1\}.$$

We note that the outer boundary of $S(I_t)$ in the plane defined by $c_t^2 = 0$, AB, is strictly concave in virtue of the strict concavity of F in each argument.

We next establish conditions under which s_t is positive when $c_t^2 = 0$, that is, B lies to the right of G on the plane defined by $e_t = 1$. Since F is homogenous of degree 1,

$$(1-q_t^2(I_t) - wn_t)^{-1}s_t = F\left[\lambda_t, \frac{\sigma_t(I_t)}{1-q_t^2(I_t)-wn_t}\cdot\frac{s_{t-1}}{n_{t-1}}\right]$$
$$- \rho F\left[\lambda_t + \frac{n_t(w\lambda_t+\gamma)}{1-q_t^2(I_t)-wn_t}, \frac{\sigma_t(I_t)}{1-q_t^2(I_t)-wn_t}\cdot\frac{s_{t-1}}{n_{t-1}}\right], \ I_t = 0, 1.$$

The input of human capital in the second expression on the r.h.s. is larger in the proportion $n_t(w+\gamma/\lambda_t)/(1-q_t^2(I_t)-wn_t)$. This proportion is maximal when $\lambda_t = 1$, so that backwardness will therefore yield the best chance that $s_t < 0$, as intuition suggests.

Whether s_t would be positive, conditional on $c_t^2 = 0$ and $e_t = 1$, in that state depends on the numerical values of various parameters. In practice, w is fairly small, say about 1/20, and γ would be about 0.6. In a state of backwardness, $n = 3/2$ and $q_t^2 = 0.2$ are broadly plausible, so that the said proportion of inputs of human capital would be about 4/3. Hence,

$$\lambda_t + \frac{n_t(w\lambda_t+\gamma)}{1-q_t^2(I_t)-wn_t} \le 7\lambda_t/3, \ \forall\lambda_t.$$

Observe, however, that F is strictly concave in each argument alone and ρ is unlikely to exceed 1/3. Comparing the two expressions on the r.h.s., inputs of human capital in the second are slightly more than double those in the first, but the share in the resulting output is at most one-third. It follows that, for plausible values of parameters and demographic variables, $s_t(c_t^2 = 0, e_t = 1) > 0$ for all values of λ_t, and points B and B′ are correspondingly depicted in the diagram.

An increase in $q_t^2(I_t)$ induces a larger movement in B than in A; for the said difference in s_t is

$$F\left[\left(1-q_t^2(I_t)\right)\lambda_t + n_t\gamma, \frac{\sigma_t(I_t)s_{t-1}}{n_{t-1}}\right] - F\left[(1-q_t^2(I_t) - wn_t)\lambda_t, \frac{\sigma_t(I_t)s_{t-1}}{n_{t-1}}\right], \ I_t = 0, 1,$$

which is increasing in $q_t^2(I_t)$ in virtue of the strict concavity of F in each argument.

If the cross-derivative F_{12} is sufficiently small, it is seen that the same claim will hold concerning a comparison of peace and war, respectively.

Allocation C: $e_t = s_t = 0$. Given I_t, c_t^2 is maximal. From (39), we have

$$c_t^2 = \frac{1-q_t^2(I_t)}{1-q_t^2(I_t)+\beta n_t}\cdot(1-\rho)F\left[\lambda_t + \frac{n_t\gamma}{1-q_t^2(I_t)}, \frac{\sigma_t(I_t)}{1-q_t^2(I_t)}\cdot\frac{s_{t-1}}{n_{t-1}}\right], \ I_t = \{0, 1\}.$$

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Suppose the ratio of survival rates, $\sigma_t(I_t)/[1-q_t^2(I_t)]$, is fixed for each I_t. Then,

$$\frac{dc_t^2}{dq_t^2(I_t)} = \frac{(1-\rho)n_t}{(1-q_t^2(I_t)+\beta n_t)^2}\cdot\left[-\beta F + \frac{\gamma(1-q_t^2(I_t)+\beta n_t)F_1}{1-q_t^2(I_t)}\right],\ I_t=\{0,1\},$$

and $\left(dc_t^2/dq_t^2(I_t)\right)_{e_t=s_t=0}<0$ if and only if

$$\beta > \frac{\gamma(1-q_t^2(I_t)+\beta n_t)F_1}{\left(1-q_t^2(I_t)\right)F}.$$

The input of human capital is $\lambda_t + n_t\gamma/\left(1-q_t^2(I_t)\right)$. Its imputed share in output is $1-\alpha\equiv(\lambda_t + n_t\gamma\left(1-q_t^2(I_t)\right)^{-1}F_1)/F$, so that the foregoing inequality can be written

$$\beta > \frac{(1-\alpha)\gamma(1-q_t^2(I_t)+\beta n_t)}{(1-q_t^2(I_t))\lambda_t+\gamma n_t},$$

which certainly holds for all sufficiently large λ_t. The denominator takes its minimum value under backwardness ($\lambda_t=1$), when the inequality becomes

$$(\beta-(1-\alpha)\gamma)\left(1-q_t^2(I_t)\right)+\alpha\beta\gamma n_t>0.$$

Since both inputs are necessary in production, F is strictly concave in both arguments and $\alpha\in(0,1)$. It is plausible that $\alpha<0.5$, but $n\geq 1$, so that the inequality may hold even if $\beta<\gamma$, as in Table 1, for which constellation the inequality holds.

Under the assumption that the ratio of survival rates is fixed for each I_t, we have established that the points C and C′ relate to each other as depicted in the figure, which reveals that, given I_t, there is damage even under a mild mortality shock. If the ratio of survival rates is the same in both states, the points C and C′ also represent the respective allocations in peace and war.

Allocation D: $e_t=1, s_t=0$. Given I_t and maximum investment in (full time) education, c_t^2 is maximal. Analogously to AB, the outer boundary of S in the plane defined by $s_t^2=0$, CD, is strictly concave in virtue of the strict concavity of F in each argument.

Given e_t and I_t, all pairs (c_t^2, s_t) on the outer frontier of S are linearly related and independent of $e_t: ds_t=-(1-q_t^2(I_t)+\beta n_t)\,dc_t^2$. Hence, AC is parallel to BD, and A′ C′ to B′ D′. An increase in $q_t^2(I_t)$ makes c_t^2 cheaper relative to s_t; but since C′ lies closer to O than does C, it follows that D′ lies closer to G than does D. The same holds when the ratio of survival rates is the same in both states.

Appendix C: Analysis for the simulations

The optimization problem under uncertainty is specified by (16)–(18). The term $u\left(c_t^2\right)$ in the objective function is unchanged, but its derivatives with respect to s_t and e_t require close attention. We have

$$\frac{\partial u\left(c_{t+1}^3(I_{t+1})\right)}{\partial s_t} = \frac{u'\left(c_{t+1}^3(I_{t+1})\right)\rho n_t}{\left(1-q_t^2(I_t)\right)\left(1-q_{t+1}^3(I_{t+1})\right)}\frac{\sigma_{t+1}(I_{t+1})}{n_t}\cdot F_2\left[\bar{l}_{t+1},\sigma_{t+1}(I_{t+1})s_t/n_t\right]$$

$$\frac{\partial u\left(c_{t+1}^3(I_{t+1})\right)}{\partial e_t} = \frac{u'\left(c_{t+1}^3(I_{t+1})\right)\rho n_t}{\left(1-q_t^2(I_t)\right)\left(1-q_{t+1}^3(I_{t+1})\right)}\left(1-q_{t+1}^2(I_{t+1})\right)\cdot zh'(e_t)\lambda_t F_1\left[\bar{l}_{t+1},\sigma_{t+1}(I_{t+1})s_t/n_t\right].$$

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After defining $E_t[x_{t+1}] = \pi_{t+1}x_{t+1}(0) + (1-\pi_{t+1})x_{t+1}(1)$ for some variable x and using these expressions in the maximization of (16), we obtain

$$E_t\Big[\Omega_t(I_{t+1})\sigma_{t+1}(I_{t+1})F_2\Big[\bar{l}_{t+1}, \sigma_{t+1}(I_{t+1})s_t/n_t\Big]\Big] = \frac{u'\left(c_t^2\right)}{1-q_t^2(I_t)+\beta n_t} \tag{40}$$

and

$$E_t\Big[\Omega_t(I_{t+1})\left(1-q_{t+1}^2(I_{t+1})\right)zh'(e_t)\lambda_t F_1\left[\bar{l}_{t+1}, \sigma_{t+1}(I_{t+1})s_t/n_t\right]\Big]$$
$$= \frac{u'\left(c_t^2\right)(w\lambda_t+\gamma)}{\left(1-q_t^2(I_t)+\beta n_t\right)}F_1[l_t, \sigma_t(I_t)s_{t-1}/n_{t-1}], \tag{41}$$

with

$$\Omega_t(I_{t+1}) = \left(\delta\rho u'\left(c_{t+1}^3\right) + b(1-q_{t+1}^2(I_{t+1}))(1-\rho)v'[(1-\rho)\bar{y}_{t+1}(I_{t+1})]\right)\frac{1}{1-q_t^2(I_t)}.$$

Hence, (40) and (41) are the stochastic versions of (32) and (33) and are derived analogously.

Since $u = \ln c_t$, (18) and (17) yield

$$u'\left(c_t^2(I_t)\right) = \left(\frac{A\left(l_t^{1-\alpha} - \rho\bar{l}_t^{1-\alpha}\right)\left(\frac{\sigma_t(I_t)s_{t-1}}{n_{t-1}}\right)^{\alpha} - s_t}{1-q_t^2(I_t)+\beta n_t}\right)^{-1}, \quad \text{and}$$

$$u'\left(c_{t+1}^3(I_{t+1})\right) = \left(\frac{\rho n_t}{\left(1-q_t^2(I_t)\right)\left(1-q_{t+1}^3(I_{t+1})\right)}F\Big[\bar{l}_{t+1}, \sigma_{t+1}(I_{t+1})s_t/n_t\Big]\right)^{-1}.$$

With

$$v'[(1-\rho)\bar{y}_{t+1}] = \frac{1}{(1-\rho)F\Big[\bar{l}_{t+1}, \sigma_{t+1}(I_{t+1})s_t/n_t\Big]},$$

we have

$$n_t\Omega_t = \frac{\delta\left(1-q_t^2(I_t)\right)\left(1-q_{t+1}^3(I_{t+1})\right) + bn_t\left(1-q_{t+1}^2(I_{t+1})\right)}{\left(1-q_t^2(I_t)\right)A\bar{l}_{t+1}^{1-\alpha}\left(\frac{\sigma_{t+1}(I_{t+1})s_t}{n_t}\right)^{\alpha}},$$

where the production function is Cobb-Douglas. Substituting the above expression for $u'\left(c_t^2(I_t)\right)$ and Ω_t in (40) yields

$$E_t[Q_{t+1}] = \frac{s_t}{A\left(l_t^{1-\alpha} - \rho\bar{l}_t^{1-\alpha}\right)\left(\frac{\sigma_t(I_t)s_{t-1}}{n_{t-1}}\right)^{\alpha} - s_t},$$

where altruism is operative through $E_t[Q_{t+1}] = \alpha E_t\left[\delta\left(1-q_{t+1}^3(I_{t+1})\right) + bn_t(1-q_{t+1}^2(I_{t+1}))/\left(1-q_t^2(I_t)\right)\right]$. Hence, we arrive at

$$\frac{\zeta_{t+1}(I_t)}{\zeta_t^{\alpha}} = \left[\left(1-q_t^2(I_t) - wn_te_t + \frac{n_t\gamma(1-e_t)}{\lambda_t}\right)^{1-\alpha} - \rho\left(1-q_t^2(I_t)+\frac{n_t\gamma}{\lambda_t}\right)^{1-\alpha}\right]^{-1}.$$
$$\left(\frac{n_{t-1}}{\sigma_t(I_t)}\right)^{\alpha}\left(zh(e_t)+\frac{1}{\lambda_t}\right)\frac{1+E_t[Q_{t+1}]}{AE_t[Q_{t+1}]} \equiv \psi(e_t, I_t;\cdot),\ I_t = \{0,1\}, \tag{42}$$

where we factor out λ_t from l_t and $\bar{l}_t$, use the definition of ζ_t and ζ_{t+1}, and rearrange.

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In the final step, we substitute from (40) in (41) so as to eliminate $u'(c_t^2)$, which yields

$$E_t\left[\Omega_t(I_{t+1})\left(1-q_{t+1}^2(I_{t+1})\right)zh'(e_t)\lambda_t F_1\left[\bar{l}_{t+1},\sigma_{t+1}(I_{t+1})s_t/n_t\right]\right]=$$

$$E_t\left[\Omega_t(I_{t+1})\sigma_{t+1}(I_{t+1})F_2\left[\bar{l}_{t+1},\sigma_{t+1}(I_{t+1})s_t/n_t\right]\right]\cdot F_1[l_t,\sigma_t(I_t)s_{t-1}/n_{t-1}](w\lambda_t+\gamma).$$

Again, we use our expression for Ω_t and the production function to obtain

$$E_t\Big[Q_{t+1}\left(1-q_{t+1}^2\right)\bar{l}_{t+1}^{-1}\Big]zh'(e_t)s_t=E_t[Q_{t+1}]l_t^{-\alpha}\left(\frac{\sigma_t(I_t)s_{t-1}}{n_{t-1}}\right)^{\alpha}An_t\left(w+\frac{\gamma}{\lambda_t}\right),$$

which, if $e_t\in(0,1)$, can be rearranged to yield

$$\frac{\zeta_{t+1}(I_t)}{\zeta_t^{\alpha}}=\left(1-q_t^2(I_t)-wn_te_t+\frac{n_t\gamma(1-e_t)}{\lambda_t}\right)^{\alpha}\cdot\frac{E_t[Q_{t+1}\left(1-q_{t+1}^2\right)(1-q_{t+1}^2+\frac{n_{t+1}\gamma}{\lambda_{t+1}})^{-1}]}{E_t[Q_{t+1}]}\cdot$$

$$\frac{zh'(e_t)\left(\frac{n_{t-1}}{\sigma_t(I_t)}\right)^{\alpha}}{\alpha An_t\left(w+\frac{\gamma}{\lambda_t}\right)}\equiv\phi(e_t,I_t;\cdot),\ I_t=\{0,1\}. \tag{43}$$

The forms of ψ and ϕ are highly complicated, even under the assumption that F is Cobb-Douglas, so it would be as well to untangle their elements, relying rather on intuition. We therefore discuss how the various factors in play influence decisions, but without the said restriction on F.

C.1. *Derivation of condition (20)*

In a steady state, (32) specializes to

$$\frac{nF[\bar{l}_{t+1},\sigma s_t/n]}{c_t^2\left[\delta\left(1-q^3\right)+bn\right]}=\sigma(1-q^2+\beta n)F_2\left[\left(1-q^2\right)\lambda_{t+1}+\gamma n,\sigma s_t/n\right].$$

If F is Cobb-Douglas, then $s_t=\alpha\left(1-q^2+\beta n\right)\left[\delta\left(1-q^3\right)+bn\right]c_t^2$. Hence, from (34),

$$s_t=\alpha\left[\delta\left(1-q^3\right)+bn\right][F(l_t,\sigma s_{t-1}/n)-\rho F(\bar{l}_t,\sigma s_{t-1}/n)-s_t],$$

thus yielding

$$s_t=\frac{QA}{1+Q}\left(\frac{\sigma s_{t-1}}{n}\right)^{\alpha}\left\{\left[(1-q^2-wn)\lambda_t\right]^{1-\alpha}-\rho\left[\left(1-q^2\right)\lambda_t\right]^{1-\alpha}\right\}, \tag{44}$$

where $Q\equiv\alpha\left[\delta\left(1-q^3\right)+bn\right]$.

In the state of progress,

$$\lambda_t/s_t=\frac{\lambda_t}{[1+g(1)]s_{t-1}}=\frac{\zeta(1)}{zh(1)};s_{t-1}/s_t=1/zh(1).$$

Substituting for λ_t/s_t in (44), we obtain

$$1=\frac{QA}{1+Q}\left(\frac{\sigma}{n}\right)^{\alpha}\left\{\left[(1-q^2-wn)\right]^{1-\alpha}-\rho\left[\left(1-q^2\right)\right]^{1-\alpha}\right\}\frac{\zeta(1)^{1-\alpha}}{zh(1)}.$$

Substituting for $\zeta(1)$ and s_{t-1}/s_t in (8), noting that λ_t is large, and rearranging yields (20):

$$\frac{h'(1)}{h(1)}\geq\frac{wn}{(1-q^2-wn)}\cdot\frac{1+\alpha\left[\delta\left(1-q^3\right)+bn\right]}{\left[\delta\left(1-q^3\right)+bn\right]}\cdot\frac{1}{1-\rho\left[\left(1-q^2\right)/\left(1-q^2-wn\right)\right]^{1-\alpha}}.$$

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Appendix D: Parameter values

D.1. Section 7

We draw on some results from a related, but far simpler model, which was calibrated to Kenya for the period 1920–2000, thus covering the first phase of its HIV/AIDS epidemic [Bell et al. (2006a)]. That model lacks physical capital, but it possesses a finer demographic structure (the unit period is a decade), with a distinction between primary and secondary-tertiary education, in keeping with the purposes for which the study was undertaken.[13] Our use of its results in constructing an example for present purposes is therefore selective.

Given the choice $u = \ln c$ in that earlier study, the social norms, as represented by the values of the parameters β and ρ, have no influence on schooling, which is the only form of investment. Since these parameters do exert such an influence in the present paper, they merit discussion. In the state of backwardness, $\lambda_t = 1$, and although a child's endowment γ is smaller, his or her potential contribution to output will be relatively important. If $\beta < \gamma$, the (relative) claim on the common pot is, in a sense, less than the child's potential contribution, thus favoring child labor over education. The values of γ that emerged from the procedure in Bell et al. (2006a) clustered closely around 0.7. Here, β is set at about half that level, which rather favors backwardness as an equilibrium, n being exogenous. Its absence in (20) implies that β has no influence on the existence of progress as an equilibrium.

The old-age generation's claim to the fraction ρ of current full income can be regarded as stemming from its investments in the previous period. Under pure individualism, with no family considerations other than pooling for insurance purposes, this claim comprises the imputed share of physical capital in current output and the return to investments in educating their children. In the state of backwardness, there are no investments in education, so that ρ would then be the said imputed share. In the state of progress, the direct cost of educating each child is $w\lambda_t$ and full income exceeds output, the actual input of human capital being $(1 - q^2 - wn)\lambda_t$. Thus, ρ is a weighted average of physical capital's imputed share in current output and the combined imputed share of physical capital and, neglecting the opportunity cost of the children's endowment, $wn\lambda_t$ units of human capital. Since wn is unlikely to be much greater than 0.1 and altruism enters through v, this way of regarding the norm expressed by ρ argues for keeping its value fairly close to physical capital's imputed share of output in the state of progress; when F is Cobb-Douglas, that share is α. In a well-known empirical study, Mankiw et al. (1992) settle on $\alpha = 1/3$, a value which we adopt here.

Turning to the educational technology, let $h(e) = d_1 \cdot e - d_2 \cdot e^{d_3}$, so that $h(1) = d_1 - d_2$.[14] Let $d_3 = 1.5$, so that h is weakly concave. Since progress is to be a possible equilibrium, we set $z = 1.5$, which implies moderate intergenerational transmission of capacities and lies in the range that emerges in Section 8.1. The estimated discount factor δ in Bell et al. (2006a) is almost 0.8; we adopt the "standard" value 0.85.

Continuing with Kenya where mortality and destruction rates are concerned, q^2 was 0.154 in 1970, and then, reversing a steady earlier decline, it spiked to 0.353 in 1990, before finally peaking at 0.395 in 2000. In the counterfactual of no outbreak of HIV/AIDS, the earlier decline was projected to continue, reaching the value 0.113 in 2000. The latter is taken as the benchmark. Many forms of physical capital have a lifetime much shorter than a human generation. We therefore set $\sigma = 0.75$, which implies fairly substantial longevity. Table 1 sets out the whole constellation of parameter values.

The parameter values for h yield $h(1) = 0.8, h'(0) = 1$ and $h'(1) = 0.7$. Progress is feasible: $zh(1) = 1 + g(1) = 1.2$. Backwardness is an equilibrium; for

$$zh'(0) = 1.5 < \frac{n(\gamma + w)}{\delta\left(1 - q^2\right)(1 - q^3)(1 - \rho)} = \frac{1.2(0.6 + 0.075)}{0.85(1 - 0.1)(1 - 0.3)(1 - 0.35)} = 2.327.$$

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Progress will also be an equilibrium in the absence of altruism if, and only if, from (20),

$$\frac{h'(1)}{h(1)} = \frac{7}{8} > \frac{1.2 \cdot 0.075}{(1 - 0.1 - 0.09)} \frac{1 + (0.85 \cdot 0.7)/3}{(0.85 \cdot 0.7)} \frac{1}{1 - 0.35[(1 - 0.1)/(1 - 0.1 - 0.09)]^{2/3}} = 0.3583.$$

When $b = 0.1$, the said condition is more strongly satisfied, in keeping with intuition: $h'(1)/h(1) = 7/8 > 0.2569$. In view of the size of the two intervals [1.5, 2.327] and [0.3583, 0.875], the scope for making substantial changes to the constellation of values in Table 1, while satisfying the conditions in question, is evidently large.

This example confirms that a poverty trap, coupled with steady-state growth as an alternative equilibrium, will exist for other functional forms. The function $h(e)$, for example, may be weakly convex. If it is strictly convex for all e close to zero, but weakly concave thereafter, it will restrict $h'(0)$ without necessarily making $h'(1)$ too small. Technologies close to Cobb-Douglas will also serve, as will sub-utility functions close to the logarithmic form. The set of admissible values of parameters associated with these functions, together with those of fertility, destruction, and discount rates, also emerges as substantial.

D.2. Section 8

D.2.1. First stage

The seven equations from (24) are joined by the five equations for GDP_t for the years 1950–1990. To save a degree of freedom, we borrow a result from Bell et al. (2006a): let $\gamma = 0.7$, which yields 5 degrees of freedom when setting $z_{1920}, z_{1930}, \ldots, z_{1980}$ and $A_{1950}, A_{1960}, \ldots, A_{1990}$. In the light of the increasing economic problems that set in during the turbulent decade of the 1970s, we allow two values of A_t, with a single break, either in 1980 or 1990. That leaves three values of z_t, the timing of whose two breaks can be varied in search for solutions that exhibit a plausible course of λ_t and a value after the second break that yields a tenable value of the long-run growth rate $g = zh(1) - 1 = z - 1$. It should be noted that the educational parameter w appears in (25). Its value is allowed to vary in the second stage of the calibration and is then entered recursively into the first stage. The interdependence between these steps is very small.

D.2.2. Second stage

Departing from Log-land, if $\eta \neq 1$, then

$$W_{t+1} = \left[\delta(1 - q^3_{t+1}) \frac{N^3_{t+1}}{N^2_{t+1}} + b\big(1 - q^2_{t+1}\big) \, [(1 - \rho)\bar{y}_{t+1}]^{1-\eta} \right] \frac{1}{\bar{y}_{t+1}}.$$

Not only does ρ now appear, but it also interacts with b, accompanied by additional non-linearity through η and the intrusion of $\bar{y}_{t+1}$ within the expression in brackets.

Proceeding from solution 1 in stage 1, the pair of years (1960, 1980) in stage 2 yields solutions close to that for Log-land, as reported in Table 2, when $\eta \in [0.80, 1.04]$; outside this range, b is either non-positive or implausibly large. For other pairs of years, $b < 0$. Proceeding from solution 2 in stage 1, the corresponding interval is $\eta \in [0.80, 1.14]$. For other pairs of years, $b < 0$. We conclude that, despite good, *a priori* grounds for supposing otherwise, there is a strong case for treating Kenya in the period 1950–1990 as an example of Log-land. We attribute this finding to a plethora of parameters in comparison with the count of data points.

Appendix E: Miscellaneous

E.1. Provision for old age: a variant

One consequence of the sharing rule for old-age provision, as specified by (6), is that although the aggregate payment to the old will fall, an outbreak of war will also exact an additional toll on the numbers of those making a claim on it. Consider, therefore, the variation in which all those who survive throughout old age are allocated, not a fixed share of total full income at that time, but each one of them a fixed proportion ρ' of the full income of each surviving young adult. Thus, instead of (6),

$$c_{t+1}^3 = \rho' \bar{Y}_t / \left[\left(1 - q_t^2\right) N_t^2 \right] = \frac{\rho'}{\left(1 - q_{t+1}^2\right)} \cdot F\left[\left(1 - q_{t+1}^2\right) \lambda_{t+1}(e_t) + n\gamma, \sigma_{t+1} s_t / n_t \right].$$

The arguments of F are formally identical, embodying the decision (e_t, s_t) in the previous period. It is seen that if mortality rates vary over time and are sharply forecast, the difference in the multiplicand of $\bar{y}_t$ can yield a different incentive to invest.

An unexpected outbreak of war in period t after an uninterrupted state of peace will leave each of the surviving young adults endowed with less full income than their elders had intended when making decisions in period $t-1$, thus producing an adverse income effect on both forms of investment in period t. The alternative sharing rule will relieve the current loss by reducing the payment to each of the old-age survivors. If, however, war takes a heavier toll on young adults than on their elders, that rule will not necessarily make the economy more robust to such asymmetric shocks.

E.2. Long-run outcomes and chance

Analogously to $(\lambda^*(0), k^*(0))$, suppose there is also a pair $(\lambda^*(1), k^*(1))$ in the state of perpetual war. Since survival rates are higher in peace, $(\lambda^*(0), k^*(0)) = (\lambda^*(1), k^*(1))$ cannot hold, and it is natural to conjecture that $(\lambda^*(0), k^*(0)) << (\lambda^*(1), k^*(1))$. If war and peace are both possible, this conjecture introduces chance into the final outcome in the long run if the initial conditions satisfy

$$(\lambda^*(0), k^*(0)) << (\lambda_0, k_0) << (\lambda^*(1), k^*(1)).$$

To establish this claim, suppose (λ_0, k_0) exceeds, but lies close to, $(\lambda^*(0), k^*(0))$. With some positive probability, the economy will enjoy an uninterrupted run of peace; and if long enough, this run could yield state variables exceeding $(\lambda^*(1), k^*(1))$, and hence ultimately, if the next stationary value of e_t^0 is such that $zh\left(e_t^0\right) > 1$, sustained growth. Then again, there is the grim possibility that (λ_0, k_0) falls short of, but lies close to, $(\lambda^*(1), k^*(1))$, and that this initially tantalizing prospect progressively recedes as the economy endures an unbroken run of wars, an event whose probability of occurrence is also strictly positive. If long enough, such a run could yield state variables short of $(\lambda^*(0), k^*(0))$ and hence, ultimately, backwardness.

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