VARIABILITY OF LAKE ICE GROWTH AND QUALITY IN THE SCHEFFERVILLE REGION, CENTRAL LABRADOR–UNGAVA*

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Abstract. An examination of rates of ice growth, actual ice thickness and the quality of the lake ice profile on Knob Lake, central Labrador—Ungava, showed that all these factors were extremely variable. Three sites were established on Knob Lake from 1957 to 1961, and a similar sequence of sites was established on a neighbouring lake, Maryjo Lake, from 1959 to 1961. Significant variations occurred within a single lake body from site to site, though it was noted that certain patterns of ice growth and quality did occur throughout most years and could be directly related to patterns of snow accumulation on the lake. Variations from lake to lake were found to be least at the centre lake sites, probably because snow cover was always least at these sites and would not affect heat loss so greatly as at the marginal sites. It is suggested that the majority of empirical formulae proposed for lake ice growth are unrealistic for short-term forecasting of ice thickness and growth, and even long-term forecasting, using accumulated degree-days of frost, only gives average values. The only answer is repeated samplings on any lake, until such time that sufficient data are gathered for a full statistical approach based on probability analysis.

Résumé. L'étude des taux de croissance de la glace, de son épaisseur réelle et de la qualité d'un profil sur le lac Knob, situé au centre de la péninsule du Labrador, a montré que tous ces facteurs sont très variables. Trois stations d'observations furent établies sur le lac Knob, de 1957 à 1961, et une série semblable de stations, de 1959 à 1961, sur un lac avoisinant, le lac Maryjo. On a constaté des variations importantes d'une station à l'autre sur un même lac, mais on a noté que certains caractères dans la croissance et la qualité de la glace se produisaient presque toutes les années et pouvaient être directement liés au caractère de l'accumulation de neige sur le lac. On a aussi constaté que les différences entre lacs étaient moindres au centre de ces lacs, probablement parce que le manteau de neige était toujours moins épais pour ces stations et n'influençait pas autant la perte de chaleur qu'aux stations bordières. On pense que la plupart des formules empiriques proposées pour le calcul de la croissance de la glace sur les lacs ne sont pas adaptées pour la prévision à court terme concernant l'épaisseur et la croissance de la glace; même la prévision à long terme, basée sur la somme totale des degrés pour les jours de gel, ne donne que des valeurs moyennes. La seule façon de résoudre ce problème est de répéter les mesures sur tous les lacs jusqu'à ce que des données suffisantes soient rassemblées pour permettre une étude statistique complète basée sur les lois de la probabilité.

Zusammenfassung. Eine Untersuchung über das Ausmass der Bildung, die momentane Dicke und die Beschaffenheit des See-Eises im Profil des Knob Lake, Zentral-Labrador—Ungava, zeigte, dass alle diese Faktoren ausserordentlich stark variieren. Drei Beobachtungsstellen waren am Knob Lake von 1957 bis 1961 eingerichtet; an einem benachbarten See, dem Maryjo Lake, liegen Beobachtungen aus einer ähnlichen Reihe von Stellen von 1959 bis 1961 vor. Deutliche Unterschiede von Ort zu Ort zeigten sich in ein und demselben Seekörper, obwohl zu bemerken war, dass bestimmte Muster der Eisbildung und -beschaffenheit durch die meisten Jahre hindurch auftraten und in direkter Beziehung zur Art der Schnee-Akkumulation auf dem See standen. Die kleinsten Unterschiede von See zu See waren jeweils in Seemitte zu beobachten, vermutlich weil dort die Schneedecke immer am dünnsten war und den Wärmeverlust nicht so stark beeinflussen konnte wie in den Randgebieten. Es ist anzunehmen, dass der Grossteil der empirischen Formeln für das Wachstum von See-Eis einer kurzfristigen Vorhersage der Eisdicke und -bildung nicht gerecht werden kann und dass auch langfristige Vorhersagen, fussend auf der Summation von Kältegraden und Frosttagen, nur Mittelwerte liefern können. Die einzige Lösung sind wiederholte Beobachtungen an jedem See, solange bis ausreichendes Material für eine vollständige statistische Beschreibung auf Grund der Wahrscheinlichkeitsanalyse vorliegt.

Introduction

In recent years an increasing awareness of the world's Arctic and sub-Arctic regions has led to a notable increase in scientific data, in part accomplished by scientific expeditions, the establishment of permanent and semi-permanent research stations, and the collection of data on a regular basis at the established weather stations. Perhaps of special interest are the problems connected with the forecasting of ice growth, ice thickness and ice decay. To date these problems have not been studied in the detail they deserve, though there is every indication that this situation is being rectified, and as a result the available literature is limited. Bilello (1961) has recently published a comprehensive account of the growth and decay of sea ice in the Canadian Arctic Archipelago; Currie (1953) has published reports dealing with ice

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thickness and snowfall in the "Prairie Provinces" and Northwest Territories, while Ryder ([1954]), in a comprehensive survey of the available ice thickness data in the Northern Hemisphere, pointed to the paucity of data from Arctic and sub-Arctic Canada.

This situation is being remedied, and within the last two years the Meteorological Branch of the Canadian Department of Transport has initiated a programme of ice drilling and the recording of ice conditions at selected stations across Canada.

A firm understanding of the nature of ice growth, its controlling variables and the limits of confidence that one might attach to routine observations have a practical significance. Records of ice thickness and quality are required to predict probable ice pressure against dams and other structures; they are necessary in predicting the bearing strength of ice at any geographical locality, with a view to providing dates on the usability of the lakes and sea ice by aircraft and land-based transport vehicles. Estimates of ice thickness are of use to the biologist studying marine and lacustrine life, and to the hydrologist in his predictions of potential water run-off and supply.

The majority of papers related to the study of ice growth and thickness have been primarily concerned with the establishment of empirical formulae, which are intended to provide a means of forecasting the dates of freeze-up and break-up, ice growth and ice thickness at any specific time. However, very few writers have stressed the confidence limits of their predictions, nor has any attention been paid to variations in surface conditions that possibly occur within a locality, variations that could seriously affect transport operations. As little attention has been paid to variations in ice thickness and growth rates upon the same lake, or on lakes within the same area, it is the specific aim of this paper to provide data on such questions. These observations are limited to Labrador—Ungava, though there is no reason to doubt that similar conditions occur elsewhere, and it is suggested that these short-term variations are fundamentally of more importance to northern operations than the long-term predictions based on limited local data, with no provision in the formula for serious local and regional variations in ice growth and ice thickness. Thus, in this paper no empirical formulae are proposed, though their usefulness is discussed. The essence of this paper is a consideration of the usefulness of ice thickness data taken on a routine basis.

Location and the Nature of the Observations

The establishment of the McGill Sub-Arctic Research Laboratory in central Labrador—Ungava in 1954 saw the initiation of an ice observational programme that has continued ever since with some modifications and improvements, and constitutes one of the most comprehensive and long-termed ice observational programmes in Canada. The laboratory is situated close to Schefferville (lat. 54° 48′ N., long. 66° 49′ W.). The results to date have been published by Jones (1958) and Andrews and McCloughan (1961); the last-mentioned paper describes the patterns of ice growth on Knob Lake for the 1957–60 period, and should be consulted if additional information is required.

Measurements were at first confined to one hole on Knob Lake, but in 1957 a new programme was initiated. Three sites were selected along an east—west line and they are designated east, centre and west sites. The sites were marked and all subsequent observations were made within a limited radius of the marker. Measurements were conducted on a regular weekly basis. Information obtained included: total ice thickness, thickness of black and white ice, the nature of the snow surface (density and compactness) and temperature measurements. In 1959 the programme was again expanded and a similar sequence of three sites was established on Maryjo Lake, 1 mile (1·6 km.) to the north-east. Initially the ice drilling was conducted with a Swedish spoon drill but in 1960–61 a SIPRE drill was used, which meant the loss of information concerning the ice quality (amounts of white and black ice in the total ice profile).

It is the observed differences in weekly increments of ice at different localities, observations which are not generally taken, that constitute the *raison d'être* of this paper.

Ice Growth and Observational Errors

The initial stages of ice formation and ice growth are fairly well documented (Devik, [1950]; Callaway, 1954) and need not be discussed here. The average date of freeze-up is 25–29 October on Knob Lake, and from this period on, ice quality and quantity is dependent upon a number of variables. Ice growth is accomplished by the conduction of heat through the ice. Initially, when the ice is thin, the temperature gradient through the ice is steep, heat loss is rapid and the rate of ice growth should, theoretically, be at a maximum. Later on, as the ice increases in thickness and the insulating snow cover blankets the ice, then the temperature gradient is reduced and the ice growth should be correspondingly reduced. The heavy snowfalls of eastern Canada are an important factor in a discussion of ice growth in the area, and probably explain many of the difficulties of ice forecasting that are encountered in this area. The lower snowfalls in the low and high Arctic areas may lead to more homogeneous results, though regional variations in ice measurements have never been published.

The variables determining the rate of ice growth may be defined as: the insulating effects of the snow, which vary as the density and depth of the snow; the quality of the ice and its density determines the rate of conduction and the temperature gradient; while the meteorological factors of wind speed, air temperature and long-wave radiation will determine the heat loss from the snow and/or ice surface.

A serious and as yet uncalculable error in the estimations of total ice thickness can be caused by sudden accretions of surface ice, hereafter called white ice. There is little doubt that white ice is refrozen slush, or wet snow, but the underlying reasons for the appearance of the white ice are not sufficiently well known as yet to allow more than a very approximate estimate to be made of its growth. McCloughan (personal communication), in attempting to rationalize the appearance of the white ice in the total ice profile, found no positive correlation between snowfall in the preceding week, or depth of snow lying on the surface, and the appearance of the white ice. It is suggested, however, that at least one reason for white ice formation is that the weight of the snow on the margins of the lake is sufficient to depress the ice surface below the hydrostatic head so that water is forced to the surface, forming a slush layer, often concealed under the upper snow crust. The appearance of the slush layers, often unsuspected, is a potential hazard to transport. When the slush layer freezes it forms either an opaque ice layer with a higher concentration of air bubbles than in normal or black ice. The bearing strength of white ice is often assumed to be half that of black ice, though no quantitative measurements have been made.

One other, often neglected, difficulty in ice measurements is the actual reliability of the observation. Generally, a site is selected and measurements are conducted within a small radius of the selected point. If the lake water is under pressure, then flooding takes place upon the drilling of a hole and affects the local area. Since observations from these sites are actually composed of a number of measurements, each with a slightly different location, then how far are observed differences in ice growth actually an expression of local differences in the snow/ice surface and the ice/water surface? It is believed that the actual measurements of the ice thickness are accurate to $\pm$ 1 in. ($\pm$ 2·54 cm.*). It is clear that the problems connected with the sampling of ice thickness could best be resolved by the use of an automatic recording device.

Variability of Ice Thickness on the Same Lake

The discussion below is based on the ice measurement data for the period 1957–61. Figure 1 illustrates the variability of ice thickness on Knob Lake by comparing the data from

* Measurements were always made in inches and have been converted to the metric system.

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the centre site with that from the west and east sites for each observational period. In all, 136 observational periods are represented. The actual average deviation of ice thickness is surprisingly small, 2·63 in. (6·7 cm.), but this average conceals a multitude of very real differences in ice quality and thickness. The maximum observed difference between the sites amounted to 11 in. (27·9 cm.), a 25 per cent variation of ice thickness between sites in this instance.

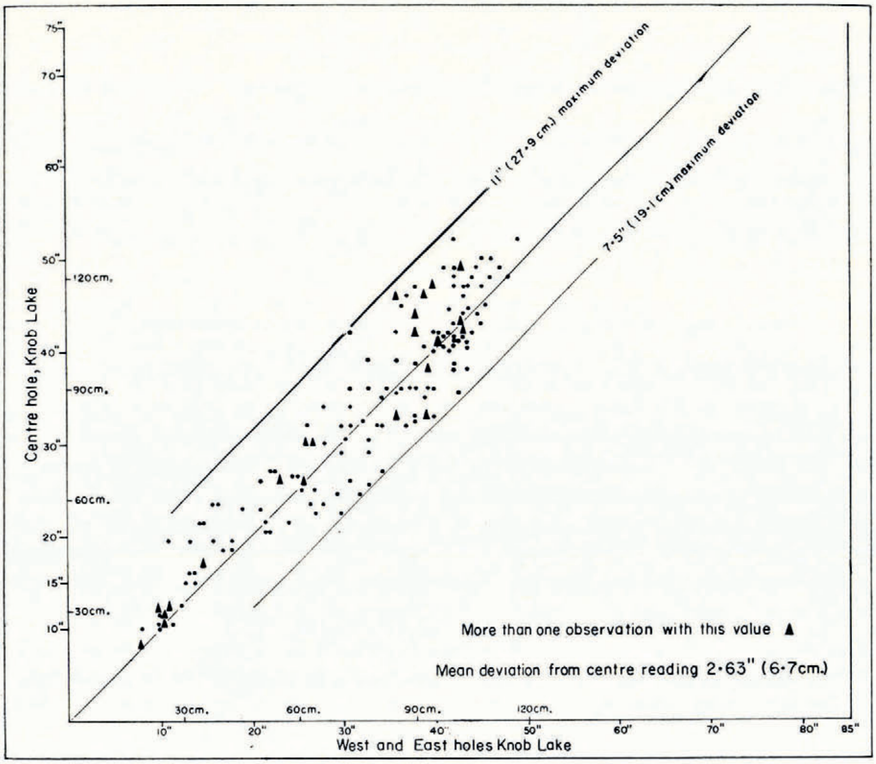

Fig. 1. Total ice thickness for Knob Lake, central Labrador—Ungava, from 1957 to 1961. The centre site is compared with east and west sites

Figure 1 also indicates that, in the early and critical part of the season, the centre of Knob Lake always has a greater ice cover than either the west or east sites, but after this early period there is no recognizable trend.

A more complete analysis of the Knob Lake records (Andrews and McCloughan, 1961) revealed "patterns" of ice thickness in respect to the different sites. For the years 1957–59 and 1960–61 the centre of Knob Lake had a greater total ice thickness than either the east or west sites. When the variations in amounts of black and white ice were examined in more detail it became apparent that the ice profile of the centre site had a tendency (amounting to 81–94 per cent of all observations) to be composed of greater amounts of black ice than the east and west sites, and a smaller amount of white ice. Average differences in the amounts of ice present at the centre and east/west sites were very high and averaged 5 in. (12·7 cm.), and the amount of black ice present at the centre of the lake could vary by as much as 15–13 in. (38·1–33·0 cm.) compared to the marginal sites.

The long axis of Knob Lake is aligned parallel with the prevailing winds and as a result the centre of the lake is often swept clear of snow. At the east and west sites, local topographical irregularities cause wind eddying, resulting in a greater snow cover at the lake margins, as has been shown by observations. The quality of the ice on Knob Lake seems to be primarily a function of the depth of snow cover. It is probable that the greater amount of black ice, and total ice at the centre site, is related to a steeper temperature gradient and heat loss, because of the absence of a deep snow cover. The association of the marginal sites with the greater thickness of white ice is interesting, and serves to support the preceding observation that white ice may possibly be caused by the weight of the snow depressing the ice surface.

Additional support for the thesis that ice thickness varies inversely with snow depth, and that the proportion of white to black ice is directly related to snow depth, was found after an examination of the 1959–60 records. During this year the east and west sites had greater

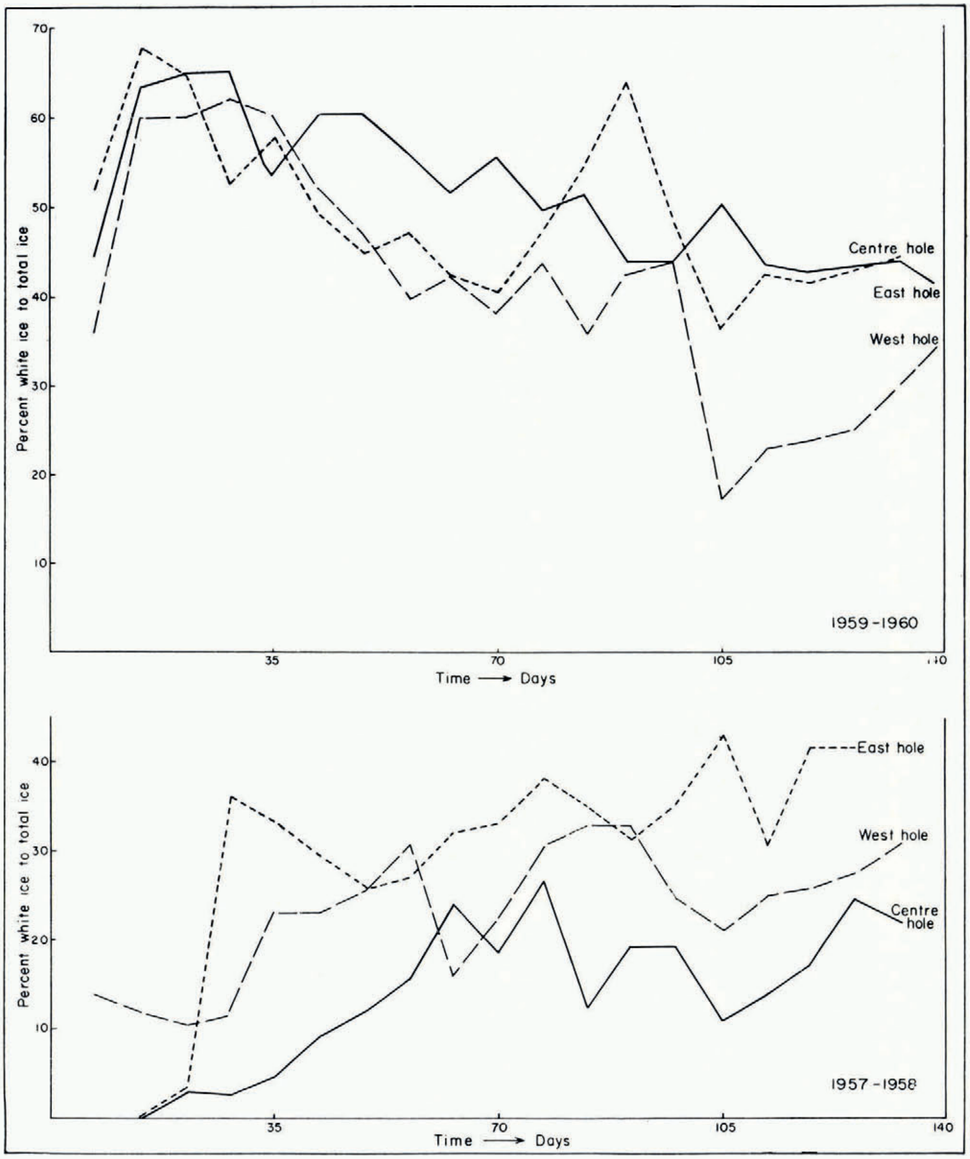

Fig. 2. White ice as a percentage of the total ice thickness, plotted against time in days for 1957–58 and 1959–60

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quantities of black ice, smaller amounts of white ice and a significantly thinner snow cover than the centre site. In other words, the conditions of 1957–59 and 1960–61 were reversed. This change in growth pattern was related to a shift in the prevailing winds of that winter.

The extreme variation in ice quality that can occur upon one ice body during the same season, and variations of this quality from season to season, are illustrated on Figure 2. This figure expresses the quantity of white ice present at any given time as a percentage of the total ice thickness, and it indicates that up to 70 per cent of the total ice thickness can be composed of white ice. Furthermore, growth can be extremely rapid and can commence early in the winter season. This creates a potential hazard for transport, if the available data refer only to ice thickness and not to the quality of the ice. Figure 2 also reveals that white ice is an extremely variable element in lake ice studies; it varies widely from one year to the next and within the same observational period. Variations in the white ice, amounting to 30 per cent of the total ice thickness, can be expected upon the same lake, and even more extreme variations occur from lake to lake within the same local region.

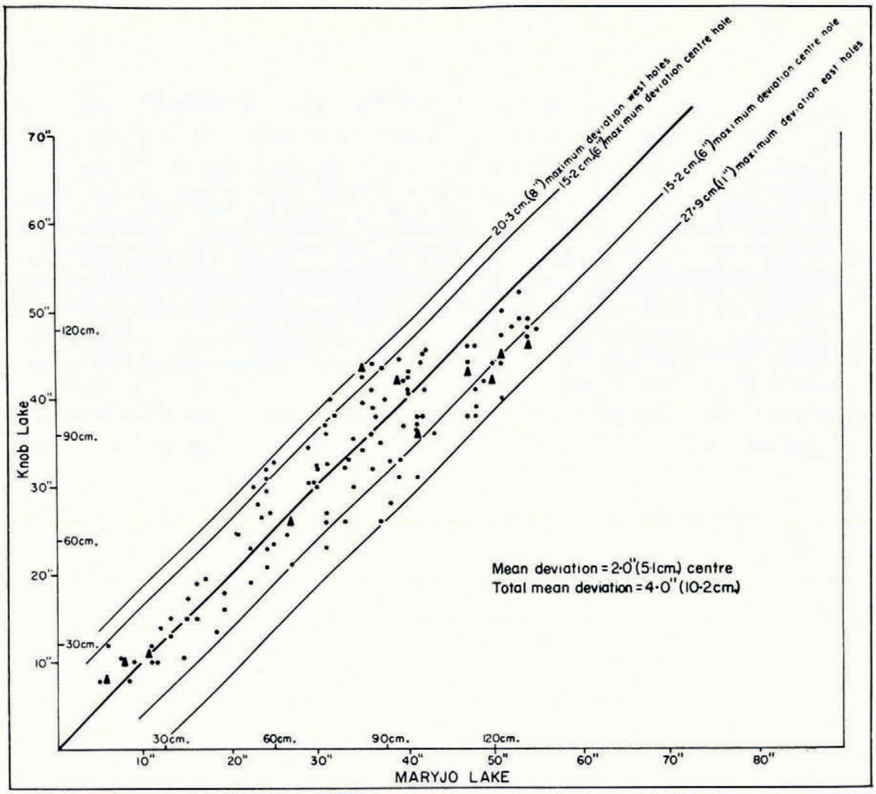

Fig. 3. Total ice thickness data from Knob Lake for 1959–61 compared with observations from Maryjo Lake. Both centre and east/west sites are compared for the same period

In summary, the records from Knob Lake for the 1957–60 ice seasons strongly suggest that widespread variations in all ice parameters can be expected within a single ice body, and that generalizations on ice conditions must of necessity be treated with all due caution. A single reading is insufficient to describe the ice conditions for any one period. Figure 1 suggests that in the early stages, when the lakes are either free of snow or have a very thin cover, variations in total ice thickness and in the snow/ice profile will be small. As the season progresses and as snow cover assumes a constant pattern, this is reflected in widening differences between each site. The available data have been subjected to initial statistical tests but it is considered that more observations are needed before this procedure and its results are warranted. The variations that can and do occur upon a single ice body are sufficient cause to render suspect any use of empirical formulae which attempt to predict ice conditions on a short-term basis.

Local and Regional Variability of Lake Ice Thickness and Quality

A critical factor that generally escapes consideration in papers on the growth and decay of sea or lake ice is the possible variations that occur within ice bodies in a given area. In other words, how far is one measurement or even a series of measurements on one lake representative of all ice bodies in that local or regional area.

In 1959–60 the writer initiated a programme with the specific purpose of providing at least a partial answer to this problem. The programme is being continued by the staff at the McGill Sub-Arctic Research Laboratory. Maryjo Lake was selected; it lies 1 mile (1·6 km.) to the north-east of Knob Lake and has a similar exposure and orientation. Sites selected along an east—west line were designated east, centre and west, and were intended to provide a comparison for the Knob Lake sites. Figure 3 illustrates the actual variations in total ice thickness between the two lakes for the winter seasons 1959–61, by comparing the Knob Lake sites with those on Maryjo Lake. The observations were plotted in such a way that the differences in maximum variation between centre sites on the two lakes and between the east and west sites could be indicated. A comparison of the centre sites of the two lakes shows that observations at any one period had a maximum difference of 6 in. (15·2 cm.) with an average difference between them of 2 in. (5·1 cm.). Thus the differences between the two centre sites are quite low, but this situation contrasts with the observations from the west and east sites on the two lakes. The data show that the difference between these sites can be 8–11 in. (20·3–27·9 cm.) over any one observational period, with an average difference of 4 in. (10·2 cm.). The similarity between the two centre sites is interesting and is probably a reflection of the lack of snow at centre sites, thus leading to a similarity in heat loss at these two sites. Where the pattern of snow accumulation shows an increase in the depth of snow, that is, towards the margins of the lakes, this would reflect in greater variations from lake to lake, varying proportionally to differences in snow depth at any site. These observations indicate that ice drilling, if it is intended to be extrapolated over an area, should be conducted at the centre of the lake and not at the margins.

Because of a change in the drilling equipment, only one year's figures are available to test any significant variations in lake ice quality between the two lakes. The analysis did not reveal any significant differences between the amounts of black ice, white ice and snow depth, but on the basis of only one year's observations no more can be said.

In Canada and in other northern areas it is often necessary to extrapolate ice thickness information over wide areas, a procedure that can only be justified because of the paucity of recording stations. It is therefore instructive to study the wide regional variations in ice thickness and quality, using information from Labrador—Ungava. The Mid-Canada Early Warning System, which was established approximately along the 55th parallel, resulted in a series of observations on ice thickness at the selected sites. The line of sites stretches for approximately 500 miles (800 km.) in an east—west direction. On 17 February 1956, a series of simultaneous measurements showed that ice thickness and quality varied enormously. Total ice thickness varied from a low of 17 in. (43·2 cm.) to a high of 42 in. (106·7 cm.), black ice varied from 36 in. (91·4 cm.) to 11 in. (27·9 cm.) and white ice varied from 2 in. (5·1 cm.) to 17 in. (43·2 cm.). Granted the total distance involved is considerable, but if the sparseness of northern settlement is remembered, then it is obvious that ice thickness data must often be extrapolated over wide areas. As such, the figures from Labrador—Ungava must pose a very serious problem to the accurate forecasting of ice conditions.

Variability of Growth Rates of Lake Ice

In attempting to ascertain the usefulness of empirical formulae designed to predict ice growth rates or total ice thickness over a certain prescribed period, it is useful to examine the Knob Lake records. Figure 4 shows the growth of the ice in inches per week as a percentage of

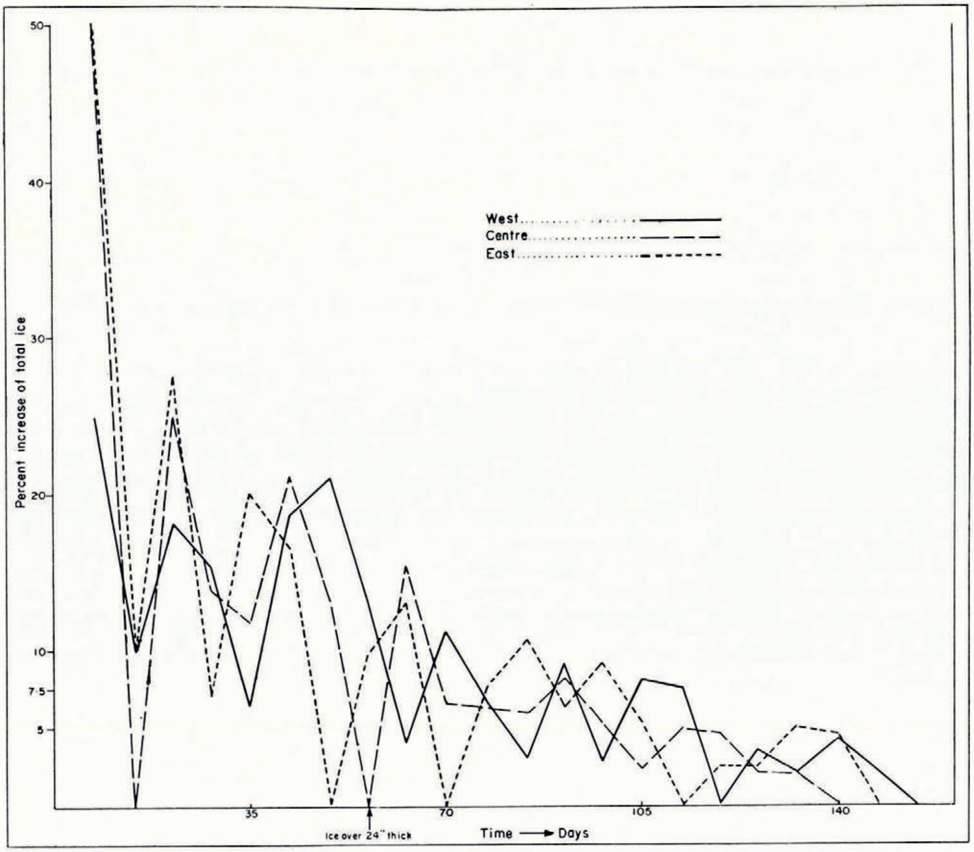

Fig. 4. Ice growth on Knob Lake for 1960–61 expressed as a percentage of the total ice thickness at the preceding observation

the total ice thickness in the preceding week. Growth is thus being graphed as a function of time, and of total ice thickness. As would be expected, the percentage growth declines as the ice season progresses and as the ice profile thickens. The variability of growth also declines from an initial position when the variation could amount to 25 per cent, an extremely significant figure when it is remembered that the ice at this time may be 10–20 in. (25·4–50·8 cm.) thick, to only 2·5–7 per cent near the end of the season, when total ice thickness range is 35–50 in. (88·9–127·0 cm.). These last two figures are the minimum and maximum respectively of extreme ice thickness on Knob Lake, and cover the period 1954–60.

Though Figure 4 illustrates that the ice growth rate is far more critical in the total ice profile in the early part of the season, this should not be taken to imply that actual growth rates diminish, though theoretically one would expect ice growth to decrease as the ice thickens and the temperature gradient becomes less steep. Figure 5 represents cumulative growth curves for the three Knob Lake sites for the 1960–61 season. The form of the curves supports the contention that ice growth does not necessarily decrease with increasing ice thickness and time. The line of least squares has been fitted to the centre site observations and

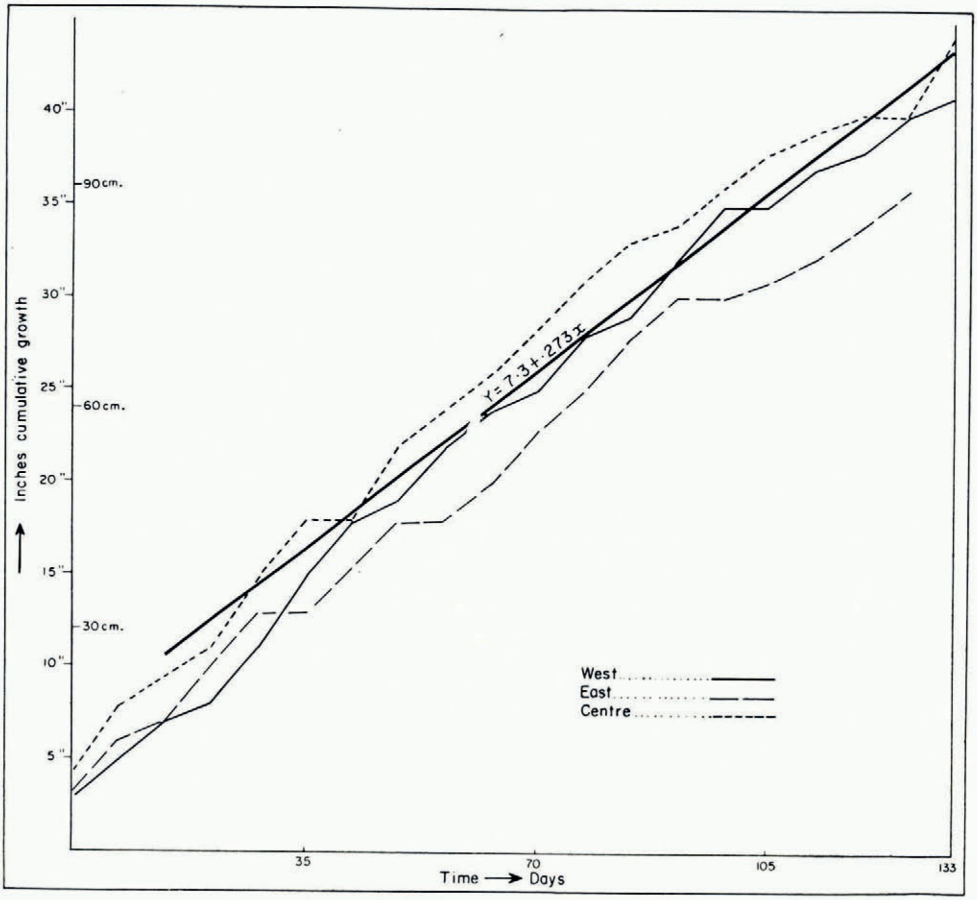

Fig. 5. Cumulative growth curves for the three Knob Lake sites, 1960–61

it was found that $y = 7 \cdot 3 + 0 \cdot 273x$. The line closely approximates the curve of ice growth at the centre and west sites but it is rather different from the calculated growth rate at the east site.

Variations in the actual growth rates on Knob Lake are shown in Figure 6. Variations in growth can be quite extreme, from 2 to 4 in. (5·1 to 10·2 cm.) and, moreover, the amount of weekly ice increment does not fall off towards the end of the ice season. It is possible that the variations in growth rates are due solely to local differences in the accretion of white ice; however, an examination of earlier records indicated that both the growth of white and black ice is irregular, and that widespread variations in growth rates occurred in both ice types. The previous discussion had suggested that the centres of Maryjo and Knob Lakes were moderately similar in total ice thickness and Figure 7, which shows growth rates at these two centre sites, also indicates that the variation between them is less than the variations on the same lake (see Fig. 6).

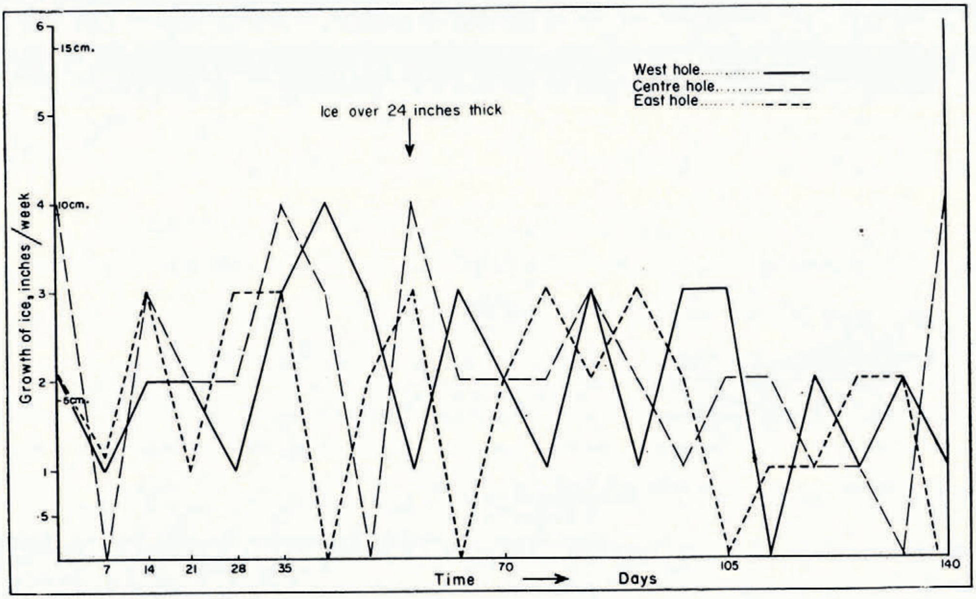

Fig. 6. Ice growth on Knob Lake for 1960–61

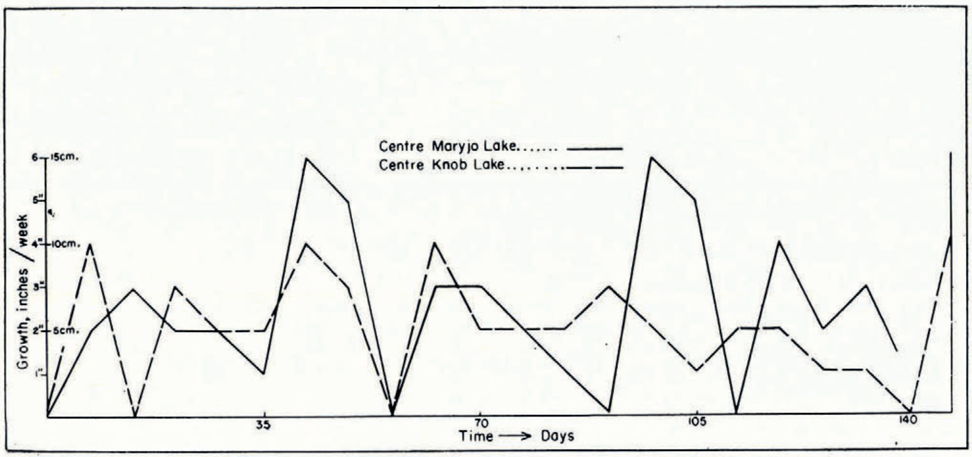

Fig. 7. A comparison of ice growth at the centres of Knob Lake and Maryjo Lake, 1960–61

Usefulness of Formulae in the Prediction of Ice Growth

Because of the complex interrelationship of the factors governing ice growth, it is not likely that any formula will be satisfactory for short-term predictions of ice growth. Many of the formulae and graphical methods so far developed do not take into account snow depth and density (Barnes, 1928; Callaway, 1954), or even the initial ice thickness. These formulae are unreliable. More complex formulae involving all parameters affecting ice growth (Kolesnikov, 1946) have been proposed but this type requires specific information not generally available. McCloughan (Andrews and McCloughan, 1961) related accumulated degree-days of frost to total ice thickness with some measure of success, though he noted that this formula is only applied to mean values. Other workers have used this method (Lee and Simpson, 1954; Bilello, 1961). Assur's formula has the disadvantage that it cannot be applied if white ice is present (Assur, 1956).

It is suggested that the only way to approach the problem of ice growth at the present time is analytically, which would lead to the definition of the average, upper and lower limits of ice growth rates from different areas. This method has been attempted with the Knob Lake results (personal communication from G. P. Williams).

Conclusions

Several conclusions can be drawn from an examination of the Knob Lake data. In order to obtain a representative sample, several measurements should be made on the same lake. Then, the resulting information will not necessarily apply to lakes in the local area, though there is a possibility that centre sites might be strictly comparable. If formulae are used in the short-term prediction of ice conditions, the results will be unsatisfactory, though long-term predictions using degree-days of frost will probably give an *average* estimate of ice growth and thickness.

The final conclusion is that in areas of moderate to heavy snowfall the only sure way of estimating ice conditions is to take an appropriate number of measurements at the site in question.

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