An Introduction to the Operations with Series, by I. J. Schwatt. Chelsea Publishing Co., New York, 1961. x + 287 pages. $3.95.

A reprint of the 1924 first edition, this encyclopaedic work is largely concerned with developing explicit formulae for the general term of the Maclaurin series expansion of various functions, including arc sin x, sec x, $\sin^p x$, $(\text{arc sec } x)^p$, x cosec x and many more.

The author first develops a technique for obtaining the n^{th} derivative of a function. For example, it is shown that if $y = \phi(u)$ and $u = f(x)$, then

$$\frac{d^n y}{dx^n} = \sum_{k=1}^{n} \frac{(-1)^k}{k!} \sum_{\alpha=1}^{k} (-1)^\alpha \binom{k}{\alpha} u^{k-\alpha} \frac{d^n}{dx^n} u^\alpha \frac{d^k y}{du^k} .$$

The operator $(x \frac{d}{dx})^n$ is introduced; it is shown that

$$(x \frac{d}{dx})^n S = \sum_{k=1}^{n} \frac{(-1)^k}{k!} \sum_{\alpha=1}^{k} (-1)^\alpha \binom{k}{\alpha} \alpha^n x^k \frac{d^k S}{dx^k} .$$

This result is used to compute $\sum_{k=1}^{n} k^p$. The author demonstrates that

$$\sum_{k=1}^{n} k^p = \sum_{k=1}^{p} (-1)^k \binom{n+1}{k+1} \sum_{\alpha=1}^{k} (-1)^\alpha \binom{k}{\alpha} \alpha^p .$$

This leads to expressions for the numbers of Bernoulli and Euler. The author proves that

$$B_n = (-1)^n \frac{n}{2^{2n} - 1} \sum_{k=1}^{2n-1} \frac{1}{2^k} \sum_{\alpha=1}^{k} (-1)^\alpha \binom{k}{\alpha} \alpha^{2n-1}$$

and $$E_n = (-1)^n \sum_{k=0}^{2n} \frac{1}{2^k} \sum_{\alpha=0}^{k} (-1)^\alpha \binom{k}{\alpha} (1 + 2\alpha)^{2n} .$$

This book is filled with examples and illustrations and is a useful reference book on the subject.

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