3

Classical Bits

3.1 Binary Questions

A *binary question* is a *yes or no* question. A typical binary question $\overline{\boldsymbol{Q}}$ will be of the form $\overline{\boldsymbol{Q}} \equiv$ **Is it true that?**, asked of some proposition $\boldsymbol{P}$ such as

$$\boldsymbol{P} \equiv \textbf{There is a signal in this detector}.$$

Assuming compatibility of question and proposition, the answer $\overline{\boldsymbol{Q}}\boldsymbol{P}$ will be either *one* or *zero*, interpreted as *yes* and *no*, respectively.

The *answer set* associated with any binary question has two elements, denoted 0 (zero) and 1 (one). Usually the element 0 will be contextually interpreted as *no* and element 1 will be interpreted as *yes*, but we could choose to interpret the answer set elements the other way around.

We will base most of this book on the mathematical structures associated with binary questions. Classical binary questions are discussed in this chapter and their quantum analogues discussed in Chapter 4.

Example 3.1 A trial in an English or American court of law can be thought of as a process that answers the binary question $\overline{\boldsymbol{Q}} \equiv$ **Is it true that?** of the proposition $\boldsymbol{P} \equiv$ **This person committed that crime**.

The *negation* $\overline{\neg \boldsymbol{Q}}$ of a binary question $\overline{\boldsymbol{Q}} \equiv$ **Is it true that?** is defined to be $\overline{\neg \boldsymbol{Q}} \equiv$ **Is it false that?**.

3.2 Question Cardinality

The English and American legal systems are based on the ancient Roman principle of *in dubio pro reo* (when in doubt, judge in favor of the accused). This presumption of innocence gives a binary, or dichotomic, flavor to the proceedings: assuming a verdict is reached, then verdicts in such courts can only be either *guilty*, corresponding to *yes*, or else *innocent*, corresponding to *no*.

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

In Scotland, however, there is a third possible verdict, known as *not proven*. This and other examples leads us to define the *question cardinality* $\#\overline{\boldsymbol{Q}}$ of a question $\overline{\boldsymbol{Q}}$ as the number of elements in its answer set, *before* that question is asked. According to this definition, a *rhetorical question* has cardinality 1, a binary question has cardinality 2, and a *Scottish law trial* answers a question with cardinality 3.

The cardinality of a question is contextual, in that it depends on the compatibility of a given question and a given proposition or state.

3.3 Classical Binary Questions

Binary questions (*yes/no* questions) are the fundamental building blocks of the quantized detector network (QDN) approach to physics discussed in this book. The reason has to do with the way experiments are conducted. Although modern experimentalists acquire vast amounts of data electronically and interpret them in terms of sophisticated theories, what goes on at the most basic level in any experiment is that a number (possibly vast) of binary questions are answered. For instance, the question of whether a photon detector has clicked or not is a binary question.

Another reason for choosing to work with binary questions is the principle that any empirical question, no matter how complicated, can always be expressed in terms of some number (possibly infinite) of binary questions. This will be referred to as the *bitification principle*. We shall return to this topic in Chapter 5 as it leads directly to the concepts of classical and quantum registers. These are the central mathematical constructs in terms of which the ideas of this book are expressed.

For the rest of this chapter we shall review some of the properties of single binary questions in classical mechanics. In that context, they are referred to as *classical bits*, terminology that comes from the theory and application of classical computation. When quantum rules are factored in, binary questions become *quantum bits*, or *qubits*. These are discussed in the next chapter. A fundamental difference (but not the only significant difference) between classical bits and qubits is that the former have cardinality 2, whereas the latter have infinite cardinality in a particular sense to do with the interpretation of quantum mechanics.

3.4 Classical Bits

We turn now to the mathematical representation of binary questions known as bits. A *classical bit*, or *bit* for short, is a set with two elements denoted **0** and **1**.

The word *bit* is short for *binary digit*. The first recorded use of the word in this context has been dated to 1947 in a Bell Labs memo written by John W. Tukey. It first appeared in public in 1948 in Claude E. Shannon's landmark paper on information theory (Shannon, 1948). Bits are used extensively in classical

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

computation and information theory and are central to our implementation of quantized detector networks.

Bits represent the most elementary, useful form of data variable, one with only two possible, mutually exclusive values. But that is a mathematical statement, insufficient for our purposes. In the sciences, including the theory of computing, a bit is generally more than a mathematical set with two elements; bits generally have some associated context that gives some physical meaning or interpretation to each of the two possible values. For example, the two elements of a bit in two-valued logic might be thought of as *true* and *false*. With context, mathematical data becomes physical information.

3.5 Signal Bits

The sort of experiment we are mainly interested in this book typically involves one or more single-click detectors, each detector having two possible outcome states. These states are associated with a classical bit as follows. If a detector is found in its ground state, or no-click state, then that state is represented by the bit state **0**, whereas if that detector is found in its signal state, or click state, then that state is represented by the bit state **1**.

Single-click detectors should not be thought of as simple. Typical detectors such as Geiger counters involve cascade processes that are irreversible and complex. What is significant is that, like avalanches on mountainsides, they are triggered by the smallest of effects, and that is where their value lies.

A *signal bit* $B \equiv \{\mathbf{0}, \mathbf{1}; C_B\}$ is a set with two elements, denoted **0** and **1**, together with a context C_B that gives each element contextual physical significance relative to the observer involved. A *signal bit state*, or *bit state* for short, is any one of the two elements, **0** or **1**, of a signal bit.

Complexity rules the real world and experiments are vastly complex processes involving many degrees of freedom. Signal bits are best thought of as *equivalence classes* defined by a context with two clear alternatives. For example, in the standard quantum mechanics (QM) description of the Stern–Gerlach (SG) experiment, electrons with arbitrary momentum and position but with spin *down* are identified with bit state **0** while electrons with arbitrary momentum and position but with spin *up* are identified with bit state **1**. Typically, these equivalence class will not depend on the color of the observer's shirt or other factors regarded by the observer as inessential to the experiment. Such equivalence classes are defined by the observer's chosen criteria, which may appear subjective.

In the original SG experiment, the detecting screen was a photographic emulsion film, acting as a battery or register of signal detectors (Bernstein, 2010). At the start of the experiment, the film was prepared by the observers by blowing cigar smoke onto it. Then, after many silver atoms had passed through the main magnet of the SG device and impinged onto the screen, the observers noticed that there were two relatively crude but nevertheless separate spots where the bulk

of the atoms had landed. It was the observers' decision to interpret these spots as of empirical significance. Stern and Gerlach did not know that their observed spots were a marker of electron spin. In 1922, electron spin was unknown; Stern and Gerlach were trying to validate Bohr's Old Quantum Mechanics theory of the atom. The electron spin interpretation came into focus a few years later.

Classical bits are sets, not vector spaces, because there is no obvious mathematical or physical meaning to the multiplication of a bit state by a real or complex number, or to the addition of two bit states. This is no longer the case when we generalize bits to their probabilistic counterparts, where they are known as *stochastic bits* (or *s-bits*) and to their quantum counterparts, where they are known as *quantum bits* (or *qubits*). Nevertheless, it is useful and convenient to represent bits via two-dimensional complex vector spaces. This allows us to represent bit states as vectors and bit operators as matrices, but it should be kept in mind that classical bit states cannot be added in principle. An exception is in Boolean algebra, where rules such as $1 + 1 = 0$ have a contextual meaning.

3.6 Nodes

QDN analyzes experiments in the simplest form possible, which is in terms of binary questions and answers. We will show in later chapters how any given apparatus is represented in QDN as a collection of binary questions and answers forming a *stage network*, a collection of *nodes* connected by links across which quantum information is transmitted.

Nodes come in two forms, *external* and *internal*.

External Nodes

External nodes correspond to physically existing equipment and come in two varieties: *sources* (preparation devices) and *detectors*. Observers input contextual information into stage networks via sources and extract signal information via detectors.

Internal Nodes

These occur in the *information void*, the region of space and time where no information is extracted. Internal nodes can be thought of as virtual detectors, as they usually correspond to places in a network where a real detector could have been placed, if the observer had so chosen.

Example 3.2 In the double-slit experiment, the two slits are identified with internal nodes, while the source of the incoming beam and the detecting screen are identified with external nodes.

Nodes are not necessarily localized in space. An example involving highly nonlocalized nodes would be an apparatus for the measurement of particle momentum.

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

While it will always be easy to identify external nodes in any stage diagram, that will not always be the case for the internal nodes. The rule for assigning internal nodes is that any such node represents a potential *opportunity* for information extraction, *if the observer so chooses*. So for instance in the double-slit experiment example considered above, not only could the observer place detectors at any of the two slits, but the observer could conceivably fill the space between source and screen with a vast number of internal nodes, if that was needed. In the limit of extremely large numbers, the QDN description would begin to look more like quantum field theory. The art in QDN is to find the simplest possible description of an experiment in terms of a limited number of nodes.

3.7 Dual Bits

In anticipation of subsequent developments, we introduce here the notion of a *dual bit*. For every bit $B \equiv \{\mathbf{0}, \mathbf{1}\}$ we postulate the existence of another bit, denoted $\overline{B} \equiv \{\overline{\mathbf{0}}, \overline{\mathbf{1}}\}$, referred to as the *dual bit*, or simply the dual, of bit B. The two elements of a dual bit will be referred to as *dual bit states*.

Bits and their duals are related as follows: $\overline{\mathbf{0}}$ is the dual of $\mathbf{0}$ and $\overline{\mathbf{1}}$ is the dual of $\mathbf{1}$. In anticipation of subsequent developments, we define a function $\overline{\boldsymbol{i}}\boldsymbol{j}$ from the Cartesian product $\overline{B} \times B$ into the set $Z \equiv \{0, 1\}$ by the rule

$$\overline{\boldsymbol{i}}\boldsymbol{j} \equiv \delta^{ij}, \qquad i, j = 0, 1, \tag{3.1}$$

where $\overline{\boldsymbol{i}}$ is an element of $\overline{B}$, $\boldsymbol{j}$ is an element of B, and δ^{ij} is the Kronecker delta.

This notation will be extended to *classical registers*, that is, collections of bits. If $B^k \equiv \{\mathbf{0}^k, \mathbf{1}^k\}$ is the kth bit in a register, then its dual $\overline{B^k}$ is given by $\overline{B^k} \equiv \{\overline{\mathbf{0}^k}, \overline{\mathbf{1}^k}\}$. Then rule (3.1) becomes $\overline{\boldsymbol{i}^k}\boldsymbol{j}^k \equiv \delta^{ij}$.

If $k \neq l$, then $\overline{\boldsymbol{i}^k}\boldsymbol{j}^l$ is undefined. The interpretation of this is that an observer cannot expect to extract any information from one detector by looking at any another detector. Exceptions can occur, provided the right context is in place, such as charge conservation.

3.8 The Interpretation of Bits and Their Duals

In the previous chapter we discussed the role of questions and answers in physics, and in this chapter started to link this to the notion of a detector. We now tie in these two ideas with bits and their duals as follows. The two bit states $\mathbf{0}$, $\mathbf{1}$ in a bit do not individually "have" absolute truth values per se: such truth values are only contextual, relative to the questions asked of the associated detector.

The two questions that could be asked of a detector are

$$\begin{aligned} &\overline{\mathbf{0}} \equiv \textbf{Is this the ground state?}, \\ &\overline{\mathbf{1}} \equiv \textbf{Is this the signal state?}. \end{aligned} \tag{3.2}$$

It should be now clear why we have chosen our bit state notation in the given form. In our notation, the question $\overline{\boldsymbol{i}}$ asked of the bit state $\boldsymbol{j}$ is written in the

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

form $\bar{\boldsymbol{i}}\boldsymbol{j}$, and then the answer is given by δ^{ij}. If $\delta^{ij} = 0$ (the number *zero*), then that means that the answer is *no*, whereas if $\delta^{ij} = 1$ (the number *one*), then that means that the answer is *yes*. We shall refer to each side of expressions such as (3.1) as a *classical answer*.

3.9 Matrix Representation

We introduce here a convenient *matrix representation* of bits and their duals. The rule is that bit states are represented by two-component column matrices as follows:

$$\mathbf{0} \underset{R}{=} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{1} \underset{R}{=} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{3.3}$$

where $\underset{R}{=}$ denotes "is represented by." For the dual bits, we have the row matrix representation

$$\overline{\mathbf{0}} \underset{R}{=} \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \overline{\mathbf{1}} \underset{R}{=} \begin{bmatrix} 0 & 1 \end{bmatrix}. \tag{3.4}$$

There is a small technical point concerning this representation that we clarify now. The classical answer $\bar{\boldsymbol{i}}\boldsymbol{j}$ is a number, either zero or one. However, according to the rules of matrix multiplication, the action of a two-dimensional row matrix on a two-dimensional column matrix is a 1×1 *matrix*, not a number. For instance,

$$\overline{\mathbf{0}}\mathbf{1} \underset{R}{=} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \neq 0. \tag{3.5}$$

We resolve this problem by interpreting the left-hand side of (3.5) as the *component* of the 1×1 matrix on the right-hand side. Henceforth we shall ignore this technical point.

3.10 Classical Bit Operators

The process of asking a binary question $\bar{\boldsymbol{i}}$ of a bit state $\boldsymbol{j}$ gives the answer $\bar{\boldsymbol{i}}\boldsymbol{j}$, which is a number (either zero or one). The ordering here is significant: the binary question $\bar{\boldsymbol{i}}$ is to the left and the bit state $\boldsymbol{j}$ is to the right. It turns out to be useful to define objects known as *transition bit operators*, which have the ordering interchanged, forming an object known as a *dyadic*. There are four such operators, defined as $\boldsymbol{T}^{ij} \equiv \boldsymbol{i}\bar{\boldsymbol{j}}$ for $i, j = 0, 1$. The application rules of these operators are as follows:

Action on Bit States

The action of $\boldsymbol{T}^{ij}$ on bit state $\boldsymbol{k}$ is from the left and is given by

$$\boldsymbol{T}^{ij}\boldsymbol{k} \equiv (\boldsymbol{i}\bar{\boldsymbol{j}})\boldsymbol{k} \equiv \boldsymbol{i}(\bar{\boldsymbol{j}}\boldsymbol{k}) = \delta^{jk}\boldsymbol{i}. \tag{3.6}$$

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

Action on Dual Bit States

The action of bit operator $\boldsymbol{T}^{ij}$ on dual bit $\overline{\boldsymbol{p}}$ is from the right and is given by

$$\overline{\boldsymbol{p}}\boldsymbol{T}^{ij} \equiv \overline{\boldsymbol{p}}(\boldsymbol{i}\overline{\boldsymbol{j}}) \equiv (\overline{\boldsymbol{p}}\boldsymbol{i})\overline{\boldsymbol{j}} = \delta^{pi}\overline{\boldsymbol{j}}. \tag{3.7}$$

The transition operators have the following matrix representations:

$$\boldsymbol{T}^{00} \underset{R}{=} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ \boldsymbol{T}^{01} \underset{R}{=} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ \boldsymbol{T}^{10} \underset{R}{=} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \ \boldsymbol{T}^{11} \underset{R}{=} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{3.8}$$

These four matrices form a basis for the four-dimensional vector space of complex 2×2 matrices, so we can use them to construct other useful matrices. There are four bit operators, labelled $\boldsymbol{I}$, $\boldsymbol{F}$, $\boldsymbol{D}$, and $\boldsymbol{U}$ here, that are occasionally useful. They are defined and represented as follows.

The Bit Identity Operator $\boldsymbol{I}$

This operator is defined as $\boldsymbol{I} \equiv \boldsymbol{T}^{00} + \boldsymbol{T}^{11}$. It leaves bit elements unchanged, i.e.,

$$\boldsymbol{I0} = \boldsymbol{0}, \quad \boldsymbol{I1} = \boldsymbol{1}. \tag{3.9}$$

The Bit Flip Operator $\boldsymbol{F}$

This operator is defined as $\boldsymbol{F} \equiv \boldsymbol{T}^{01} + \boldsymbol{T}^{01}$. It switches bit elements, i.e.,

$$\boldsymbol{F0} = \boldsymbol{1}, \quad \boldsymbol{F1} = \boldsymbol{0}. \tag{3.10}$$

In quantum computation, $\boldsymbol{F}$ is known as the *NOT gate* and denoted X (Nielsen and Chuang, 2000).

The Bit Down Operator $\boldsymbol{D}$

This operator is defined as $\boldsymbol{D} \equiv \boldsymbol{T}^{00} + \boldsymbol{T}^{01}$. It forces all bit states into the ground state $\boldsymbol{0}$, i.e.,

$$\boldsymbol{D0} = \boldsymbol{0}, \quad \boldsymbol{D1} = \boldsymbol{0}. \tag{3.11}$$

The Bit Up Operator $\boldsymbol{U}$

This operator is defined as $\boldsymbol{U} \equiv \boldsymbol{T}^{10} + \boldsymbol{T}^{11}$. It forces all bit states into the signal state $\boldsymbol{1}$, i.e.,

$$\boldsymbol{U0} = \boldsymbol{1}, \quad \boldsymbol{U1} = \boldsymbol{1}. \tag{3.12}$$

The four operators $\boldsymbol{I}$, $\boldsymbol{F}$, $\boldsymbol{U}$, and $\boldsymbol{D}$ will be used in bit state dynamics, and then it will be convenient to define $\boldsymbol{O}^1 \equiv \boldsymbol{I}$, $\boldsymbol{O}^2 \equiv \boldsymbol{F}$, $\boldsymbol{O}^3 \equiv \boldsymbol{D}$, and $\boldsymbol{O}^4 \equiv \boldsymbol{U}$.

3.11 Labstates

Our objective in this book is to interpret quantum mechanics via signal states of apparatus instead of states of systems under observation (SUOs). In order to keep this in mind, we shall use the term *labstate* whenever we refer to the former, reserving the term *system state* for the latter.

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

In this book, we shall deal with three kinds of labstate associated with a single detector: *classical labstates*, *stochastic labstates*, and *quantum labstates*. Classical and stochastic labstates are discussed in this chapter, quantum labstates are discussed in the next chapter. Each form of labstate has its own dynamical evolution rules, which we shall discuss separately in some detail. Because only one detector is involved in these preliminary discussions, such dynamics will be referred to as *rank one*. If two detectors were involved, then we would be discussing rank two dynamics, and so on.

Given a single detector, the observer would find it either in its signal ground labstate **0** or in its signal labstate **1**, assuming the detector existed, was not faulty, and that the observer actually looked.

3.12 Time and the Stages Concept

Before we can discuss the QDN approach to dynamics, we need some more precision in our modeling of the processes of observation, because detectors are distributed not only in space but also in time. Time is a necessary ingredient in our discussion. Once we start to incorporate that element into the discussion, we are led naturally to the *stage* concept that underpins QDN.

In QDN, time is defined relative to an observer and is generally measured in discrete steps called stages. This requires some explanation. It does not mean that we have dispensed with time as conventionally modeled in standard physics, that is, as a continuous real number–valued parameter via which velocities and other temporal derivatives are calculated.

Contrary to what is implied in conventional formulations of quantum mechanics, such as Schrödinger wave mechanics, the time in the laboratory required to complete any observation of a signal state is *always* nonzero. There are in fact no actual continuous time measurements possible in physics. Any references to continuous time observations, such as in *quantum Zeno* (also referred to as *nondemolition*) experiments (Itano et al., 1990), are to contextually incomplete mathematical approximations that often have great validity and usefulness, but only up to a point and under specific assumptions. Continuous time has much the same status in experimental physics as the concept of temperature: a useful and powerful emergent concept representing a great deal of contextuality, but otherwise not an objective thing in its own right.

In QDN, the concept of events in continuous spacetime is replaced by the concept of stage network.

Definition 3.3 A *stage network* is a conceptual collection of external and internal nodes representing apparatus distributed over time and space in a laboratory. Each node is connected by temporal links to other nodes or to *modules*. Modules represent processes between nodes that influence

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

the transmission of classical or quantum information. Nodes are indexed by integers called *labtimes*. Each labtime is associated with a *stage*, the QDN analog of a hypersurface of simultaneity in relativity.

Quantum state preparation occurs at the source nodes and generally takes place over some contextually "small" or negligible interval of labtime (time as measured in the laboratory). States then evolve undisturbed over temporal links and are detected by the observer at the detectors, again over contextually "small" or negligible intervals of labtime.

The power of QM in general is that detailed modeling of what actually goes on at the nodes seems to be less significant than the detailed modeling of the transition amplitudes evolving over temporal links. For example, Feynman diagrams are used in relativistic quantum field theory to calculate those amplitudes, with virtually no modeling of the detection equipment that would be needed in practice.

This view of quantum processes was taken to an extreme with the development of the Multiverse paradigm (Deutsch, 1997). In that paradigm, only evolution of the wave function for the Multiverse in the information void is asserted to be significant. But because real, empirical information is extracted in the laboratory at nodes only, it should not come as a surprise that the Multiverse concept turns out to be empirically vacuous. So where does QDN stand in relation to these nodes and links?

QDN is an attempt to investigate the nodal aspect of quantum physics more than has been hitherto the case. As with scattering matrix (S-Matrix) theory (Eden et al., 1966), QDN provides a framework for discussing the spatiotemporal architecture of observation but does not provide the dynamical details of the amplitudes involved.

Our aim in this book is to discuss a multi-detector approach to quantum physics. By this we mean to discuss real experiments that involve perhaps many preparation channels; large numbers of modules such as beam splitters, mirrors, and suchlike; and batteries of outcome detectors. In such circumstances, we naturally find ourselves encountering the dictates of special and general relativity (GR). To date there has been no empirical evidence that the principles of relativity and of quantum mechanics are incompatible. There is in practice "*peaceful co-existence*" between GR and QM, and that has to be respected in QDN.

The grouping of nodes into sets called *stages* reflects *classical causality*, the notion that an event can influence some events dynamically but not others. In relativity, events outside each other's light cones cannot be causally related. The analogous concept in QDN is that nodes in the same stage cannot transmit or receive quantum information from each other.

There is a subtlety here, however, to do with *shielding*. This needs some explanation. Consider two detectors, A and B, that are, in relativistic parlance, timelike separated. This means that one of the detectors, say B, is inside the forward light cone of A, viewed from the conventional relativistic perspective. Therefore, by standard causal physics, B could in principle be affected dynamically by whatever was done at A. But suppose B was shielded in some way from any dynamical effects from A (such as being placed inside a Faraday cage, in the case of electromagnetic interactions). Then for all practical purposes, we could regard A and B as if they were dynamically independent. It would not then be inconsistent to assign them to the same QDN stage, even though they were not on any hypersurface of simultaneity in physical spacetime. On this basis the QDN definition of simultaneity is contextual.

Example 3.4 Consider an SG experiment where an electron passes through an inhomogeneous magnetic field and is expected to land on one of two possible sites on a screen. Provided there was no tampering with the screen after the electron had passed through the magnetic field, then the observer could take their time in looking at the two sites to see where the electron actually had landed. The observer could in fact look at one site immediately after the electron had passed though (that is, after it had been calculated to have passed through), and then look at the other site 20 years later. The acts of looking at the two sites would take place 20 years of real time apart, but provided the screen had not been tampered with over those 20 years, the two site examinations could legitimately be regarded as having taken place in the same stage.

In fact, all experiments are conducted in this way. Signals are registered irreversibly in detectors, and the observer generally looks at those memories usually much later.

The stage concept is designed to reflect the inherent certainty/uncertainty dichotomy in any experiment: an observer may be quite sure that a signal has been detected in a detector (simply because they looked and found a signal), but the actual exact laboratory time when the signal was triggered could be quite uncertain. Indeed, it is a vacuous concept to imagine that signals trigger instantaneously. How could that be proved? At best, approximate time intervals of triggering could be determined.

The stage concept is naturally tuned in to the notion of wave-function collapse, or state reduction. Nothing physical actually collapses when an observer looks at a detector and finds a positive signal there: it is true that there will be some quantifiable changes, both in the apparatus and in the observer's information store, but these changes are not manifestations of anything that happened in the information void, merely interpreted as evidence that something had happened.

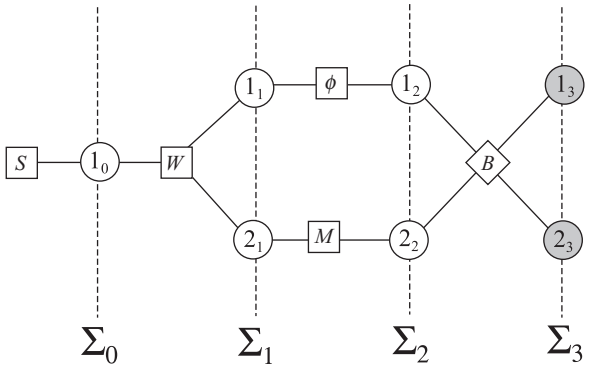

Figure 3.1. A typical stage diagram. Dotted lines represent individual stages, labeled by subscripts, the nth stage being denoted Σ_n. Circles represent nodes, where information either enters or leaves the network, or passes on to other nodes. Shaded circles represent actual outcome detectors. Boxes represent modules such as the source (S), a Wollaston prism (W), a phase-changer (ϕ), a mirror (M), and a beam splitter (B). Solid lines represent transmission in the information void between nodes and modules.

3.13 Stage Diagrams

All experiments have a spacetime architecture that can be represented diagrammatically. In QDN, we use *stage diagrams*. These are simplified diagrams showing the information flow between components of apparatus over the course of an experimental run. Figure 3.1 is a typical stage diagram. Numbered circles represent individual information gates or nodes, either real or virtual (explained later). Shaded circles represent real detectors, that is, nodes that the observer actually extracts signal information from. Boxes represent various modules, such as mirrors and beam splitters. Modules are discussed in Chapter 11. Dotted lines represent the stages, indexed by subscripts.

3.14 Measurements and Observations

To explain more fully the points we are making, we first need to pin down some of the terms we shall use to describe any experiment.

Intervention

An *intervention* is a single act of information extraction from a set of detectors at a single stage.

Run

A *run* is the complete process involved in a given sequence of actions, which starts with initial labstate preparation and ends with final labstate detection.

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

Experiment
An *experiment* consists of a given number of runs, each following an identical protocol, or experimental procedure.

Runtime
Runtime is time required to perform a given run, as measured by the observer's laboratory clocks.

Measurement
A *measurement* is the statistical result of data accumulated over one or more runs of a given experiment.

3.15 Transtemporal Identity

With the introduction of time, we come to an important question in the development of our formalism: does an detector have a *transtemporal identity* (an identity that persists over some interval of time), or is it something that exists only at a specific time?

Such a question goes to the heart of an ancient debate concerning the nature of reality. In *The Way of Truth*, a surviving fragment from a poem of the ancient Greek philosopher *Parmenides* (ca. 520–450 BCE), it was argued that change is impossible and that existence is timeless, uniform, and unchanging. Parmenides also suggested that the world of appearance (or by our interpretation, *observation*) is false and deceitful. He claimed that truth could not be known through sensory perception and that only pure reason could lead to a proper understanding of reality. In essence, this was an argument for not doing experimental physics.

Parmenides's ideas had an enduring effect on subsequent philosophy, physics, and mathematics. One of his pupils, *Zeno of Elea* (ca. 490–430 BCE) took Parmenides's line of reasoning further and created a number of paradoxes about motion, such as the race between the Tortoise and Achilles. Zeno's paradoxes could not be fully resolved until the modern mathematical understanding of the limit concept was developed. Parmenides also denied the existence of *nothingness*, or the *void*, which stimulated *Leucippus* to propose the existence of atoms.

In contrast to Parmenides, *Heraclitus* suggested that *everything flows, nothing stands still*, and that *change is the only constant*.

We can make sense of some of the ideas of Parmenides, Leucippus, and Heraclitus provided we avoid introducing absolute truths and base our discussion on contextuality, the proper basis for theories of observation of reality.

Any answer to the question of temporality determines the way in which dynamics is thought about and represented. If SUOs and/or detectors have enduring temporal identities, then it is reasonable to imagine that they "evolve" over time in fixed spaces, changing perhaps their states but retaining sufficient attributes to

justify giving them their particular identities. This is related to the phenomenon of *persistence*, which depends on the time scales over which objects can be taken reliably to have significance.

Such an approach is taken in conventional classical mechanics (CM), where SUOs are represented by points moving about phase space, and in QM, where state vectors move about in Hilbert space.

The alternative to this is to imagine, like Heraclitus, that everything changes, nothing persists. According to this view, objects such as SUOs and apparatus only appear to persist because sufficient patterns of mass and energy are repeated sufficiently unchanged over certain time scales as judged by some observer, and it is this that gives that observer the impression that "objects" exist in the universe. We shall adopt the Heraclitian point of view, because we need to consider the possibility that observers and apparatus can be created and destroyed. This will mean changing, over time, the dimensions of the mathematical spaces used to model processes of observation.

Therefore, when we discuss a rank-one labstate evolving from stage Σ_M to stage Σ_N, we shall think of it as a succession of detectors, denoted, say, by $\Delta_M, \Delta_{M+1}, \ldots, \Delta_N$. Each detector Δ_n is associated with a classical bit B_n and its dual $\overline{B}_n$, and that detector exists only at stage Σ_n. But if persistence is assumed, then the whole set $\{\Delta_i : i = M, M+1, \ldots, N\}$ of detectors may be thought of as a single detector with an enduring, transtemporal identity existing over the time interval $[t_M, t_N]$.

It is important not to mix different theoretical spaces associated with different times. While we can ask the question $\overline{\boldsymbol{i}_n}\boldsymbol{j}_n$ for $M \leq n \leq N$, we are not allowed to ask the question $\overline{\boldsymbol{i}_n}\boldsymbol{j}_m$ for $n \neq m$. The reason is obvious when stated in words: *we cannot observe today what does not yet exist or what used to exist.* All actual observations are done in process time. Past and future are inferred from the data so acquired: archaeologists do not dig up the past – they dig up traces of the past embedded in the present. Improper questions such as $\overline{\boldsymbol{i}_n}\boldsymbol{j}_m$, $n \neq m$, are not defined mathematically either, in the same way that dual vectors (one-forms) are defined only by their action on their individual, associated vector space.

3.16 Typical Experiments

A typical rank-one experiment of the classical type starts with a definite initial labstate $\boldsymbol{j}_M$ at initial stage Σ_M. Usually we shall take $M = 0$, but this is not essential. We shall not ask how this initial labstate was created but, in this section, require it to be definite; that is, there is no element of probability here other than certainty. There are therefore only two possible initial labstates in any such experiment: $\boldsymbol{j}_M$ can take the value 0 or the value 1.

Next, we imagine that the observer has no further interaction with "the" detector until stage Σ_{M+1}, but by this time, something will have acted on it to possibly change its signal state. We shall represent this by the action of one

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

of the classical bit operators $\boldsymbol{O}^k$, $k = 1, 2, 3, 4$, defined in Section 3.10, i.e., we suppose the transition

$$\boldsymbol{i}_M \to \boldsymbol{i}_{M+1} \equiv \boldsymbol{O}^{k_M}_{M+1,M} \boldsymbol{i}_M, \tag{3.13}$$

where $\boldsymbol{O}^{k_M}_{M+1,M}$ is given by

$$\boldsymbol{O}^{k_M}_{M+1,M} \equiv \sum_{a=0}^{1} \sum_{b=0}^{1} \boldsymbol{a}_{M+1} O^{k_M}_{ab} \overline{\boldsymbol{b}_M} \tag{3.14}$$

and $O^{k_M}_{ab}$ are the ab components of the corresponding classical bit matrix. This operator takes us from the two-dimensional vector space $\mathcal{Q}_M$ in which we have embedded $\mathbf{0}_M$ and $\mathbf{1}_M$ into the two-dimensional vector space $\mathcal{Q}_{M+1}$ in which we have embedded $\mathbf{0}_{M+1}$ and $\mathbf{1}_{M+1}$.

This process can be continued. Specifically, for any time n such that $M \leq n < N$ we may write

$$\boldsymbol{i}_n \to \boldsymbol{i}_{n+1} \equiv \boldsymbol{O}^{k_n}_{n+1,n} \boldsymbol{i}_n, \tag{3.15}$$

from which we find

$$\boldsymbol{i}_M \to \boldsymbol{i}_N \equiv \boldsymbol{O}_{N,M} \boldsymbol{i}_M, \tag{3.16}$$

where the complete evolution operator $\boldsymbol{O}_{N,M}$ is given by

$$\begin{aligned} \boldsymbol{O}_{N,M} &\equiv \boldsymbol{O}_{N,N-1} \boldsymbol{O}_{N-1,N-2} \dots \boldsymbol{O}_{M+1,M} \\ &= \sum_{a=0}^{1} \sum_{b=0}^{1} \boldsymbol{a}_N \left[\boldsymbol{O}^{k_{N-1}} \boldsymbol{O}^{k_{N-2}} \dots \boldsymbol{O}^{k_M} \right]_{ab} \overline{\boldsymbol{b}_M}. \end{aligned} \tag{3.17}$$

If now at time N, the observer stepped in and looked at the detector, they would normally ask the question $\overline{\mathbf{1}_N} \equiv$ **Is there a signal here?** The answer is given by $\overline{\mathbf{1}_N} \boldsymbol{i}_N$, where a value one represents an answer *yes* while a value zero represents an answer *no*.

Each set of integers $\{k_M, k_{M+1}, \dots, k_{N-1}\}$ in (3.17) represents a specific "operator chain" of labstate dynamical changes. Each of the integers k_n can be 1, 2, 3, or 4, so there is in total 4^{N-M} different operator chains. However, because the multiplication of the operator matrices is closed, the net result is that there are only four possible overall complete evolution operators, given by

$$\boldsymbol{O}^k_{N,M} \equiv \sum_{a=0}^{1} \sum_{b=0}^{1} \boldsymbol{a}_N O^k_{ab} \overline{\boldsymbol{b}_M}, \quad k = 1, 2, 3, 4. \tag{3.18}$$

This form of dynamics is relatively simple but there is a surprising aspect to it: it is entirely deterministic, in that a given present labstate unambiguously determines the labstate at any time in the future. However, if any one or more of the $N - M$ matrices in (3.17) is of $\boldsymbol{D}$ or $\boldsymbol{U}$ type, then the complete dynamics is irreversible. What this means is that even if the observer knew the final labstate and every detail of the evolution operator from initial to final times,

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

but not the initial labstate, they could not say for sure what that initial state was. Retrodiction is therefore impossible if a $\boldsymbol{D}$ or $\boldsymbol{U}$ transition occurs even once.

The mathematical reason why the occurrence of just a single $\boldsymbol{D}$ or $\boldsymbol{U}$ transition generates irreversibility is that each of these maps is two-to-one, and so their matrix representations are singular.

3.17 Rank-One Stochastic Evolution

In the real world, classical certainty is an exception and we have to use concepts of probability to discuss most situations. In this section we extend the ideas of the previous section to incorporate this requirement.

There are two competing philosophies or schools of thought about probability: the *Frequentist school* and the *Bayesian school*. These are in principle quite different in their core philosophies of what probability means, although some convergence of thinking appears to be taking place among the experts, and the differences are at times too subtle to be of much significance to us here.

There is however one clear difference, analogous to the difference between the CM noncontextual view of reality and the contextual QM view. Frequentists talk about probabilities as if they were intrinsic to the events taking place, such as a fair coin "having" a probability of $\frac{1}{2}$ landing on a head. Bayesians require a context to be supplied before they presume to make such an assertion. In an absence of such context, *Bayesians* will make a *prior*, or educated starting guess, such as $\frac{1}{2}$ for the probability p of a head. A Bayesian would then throw the coin a few times and make some outcome observations. With this new information, the Bayesian would make an updated estimate of p based on a well-known formula attributed to Bayes.

For Bayesians, probabilities are contextual. For example, a Bayesian who had thrown a coin 10 times and not observed a single tail would calculate the probability of the eleventh throw landing on a tail to be much less than $\frac{1}{2}$. This is because the 10 observations (i.e., the 10 runs of the basic experiment) had provided new information about the SUO (the coin) that could not be discounted. The specific details of any calculation as to the likely outcome of the eleventh throw would depend on the sort of assumptions made, such as whether successive throws were truly independent. We shall not discuss those details further here. Suffice it to say that *prior information*, or as we would put it, the *context* of the eleventh throw, would have significant bearing on the probability calculation. Also, *how* the coin was thrown would have an important bearing on the probability outcome calculations and this is part of the context as well.

The pragmatic view of a person unfamiliar with the rules of probability would be that the observation of ten successive heads is reasonably convincing evidence for the hypothesis that the coin is in fact double-headed. On that basis, the probability of getting a tail on the eleventh throw would be zero. A more careful

analysis based on particular "reasonable" assumptions about the prior gives a probability of about 0.95 of getting a head on the eleventh throw.

We see here a similarity between our approach to observation and the Bayesian approach to probability. In both cases, prior information or context is crucial to the predictions. Our first principle of observation is that all truths in physics are contextual. The equivalent principle in Bayesian statistics is that all probabilities are conditional.

Probability in process physics has a different flavor compared with probability in block world physics. In block world physics, probability has to be discussed in terms of limits of ratios of large numbers of outcomes, presumably counted by some unspecified exophysical observer over some number of runs, or repetitions, that were embedded in the Block Universe. This is the Frequentist approach to probability and is typical of the way probability is discussed in standard QM.

In *Process Time*, however, we may find ourselves in a situation where we have no more than one opportunity to throw a coin. Real life is usually like that and it is that aspect of physics that we are trying to develop. Under such circumstances, the term *propensity* may be used rather than probability. Propensity is a gambler's view of probability as opposed to an accountant's view. We shall use the term "probability" to represent both kinds of concept.

Randomness and uncertainty enter into our discussion in two ways: the observer may be uncertain as to the initial labstate of the detector, and also be uncertain as to which dynamical operators are acting. We need to discuss both aspects. We consider first random labstates.

3.18 Stochastic Bits

Previously, we represented the ground state **0** and signal state **1** by column vectors, as in (3.3). Now suppose the observer was unsure as to which initial labstate they had started with, to the extent that they could only assign a probability of p for it to be in the ground state **0** and a probability $q \equiv 1 - p$ to be in its signal state **1**. We shall represent such an uncertain labstate by

$$\mathbf{\Psi} = p\mathbf{0} + q\mathbf{1} \underset{R}{=} \begin{bmatrix} p \\ q \end{bmatrix}. \tag{3.19}$$

Now suppose that the observer wanted to know if there was a signal in the detector. They need to ask the question

$$\overline{\mathbf{1}} \equiv \textbf{Is the detector in its signal state?} \tag{3.20}$$

In our formalism, the answer is given by

$$\overline{\mathbf{1}}\mathbf{\Psi} \underset{R}{=} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = q, \tag{3.21}$$

where we interpret the right-hand side as a number. In this case, the answer is not zero or one as in the classical case but interpreted as the probability of there

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

being a signal in the detector. Likewise we find $\overline{\mathbf{0}}\boldsymbol{\Psi} = p$, which is the probability that the detector would be found in its ground state.

The sum of the elements of the column vector (3.19) is unity. Such a vector will be called a *stochastic vector* and the associated bit will be called a *stochastic bit*, or s-bit. For such bits, we shall refer to expressions such as $\bar{\boldsymbol{i}}\boldsymbol{\Psi}$ as *stochastic answers*, as they are generally neither zero nor unity, and are interpreted as probabilities.

3.19 Left-Stochastic Matrices

We turn now to the other possibility where randomness may occur: the dynamics affecting an detector may be random. We suppose now that the observer is uncertain as to which of the four possible evolutions $\boldsymbol{I}$, $\boldsymbol{F}$, $\boldsymbol{D}$, or $\boldsymbol{U}$ actually has occurred. Consider the operator $\boldsymbol{S}$ defined by

$$\boldsymbol{S} \equiv p\boldsymbol{I} + q\boldsymbol{F} + r\boldsymbol{D} + s\boldsymbol{U}, \tag{3.22}$$

where p, q, r, and s are probabilities summing to unity. With the matrices as defined previously, we find

$$\boldsymbol{S} \underset{R}{=} \begin{bmatrix} a & b \\ 1-a & 1-b \end{bmatrix}, \tag{3.23}$$

where $a \equiv p + r$ and $b \equiv q + r$ are in the interval $[0, 1]$.

Such a matrix is a *left-stochastic matrix*, i.e., one that has the property that each element lies in the interval $[0, 1]$ and the sum of elements in each column is unity.

3.20 Stochastic Jumps

We now consider stochastic labstates jumping under the influence of left-stochastic operators, from stage to stage. The labstate at stage Σ_n is given by

$$\boldsymbol{\Psi}_n = p_n^0 \mathbf{0}_n + p_n^1 \mathbf{1}_n = \sum_{i=0}^{1} p_n^i \boldsymbol{i}_n, \tag{3.24}$$

where $0 \leqslant p_n^i \leqslant 1$ and $\sum_{i=0}^{1} p_n^i = 1$. This state evolves to $\boldsymbol{\Psi}_{n+1} \equiv \boldsymbol{S}_{n+1,n}\boldsymbol{\Psi}_n$, where $\boldsymbol{S}_{n+1,n}$ is a left-stochastic operator given by

$$\boldsymbol{S}_{n+1,n} \equiv \sum_{i=0}^{1}\sum_{j=0}^{1} \boldsymbol{i}_{n+1} S_n^{ij} \overline{\boldsymbol{j}_n}, \tag{3.25}$$

where S_n^{ij} are the components of the left-stochastic matrix S_n, where

$$S_n \equiv \begin{bmatrix} a_n & b_n \\ 1-a_n & 1-b_n \end{bmatrix}. \tag{3.26}$$

Then $\boldsymbol{\Psi}_{n+1}$ is also a stochastic labstate. The determinant $|S_n|$ of the stochastic matrix S_n in (3.26) is given by $|S_n| = a_n - b_n$, which means that stochastic evolution is irreversible for $a_n = b_n$.

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press

Exercise 3.5 Prove that the product of any two left-stochastic matrices is also a left-stochastic matrix. Prove also that the inverse of a nonsingular left-stochastic matrix is also a left-stochastic matrix.

3.21 Stochastic Questions

In much the same way that we can ask a definite binary question about a stochastic bit state, we may consider the possibility of asking *stochastic binary questions*. By this we mean the following.

Suppose an observer has prepared a stochastic state $\boldsymbol{\Psi}$ of a system under observation and could ask two different binary questions $\overline{\boldsymbol{Q}^1}$ and $\overline{\boldsymbol{Q}^2}$ in principle. Suppose that for some reason outside their control, every time the observer asked a question, there was a probability p that it was actually $\overline{\boldsymbol{Q}^1}$ being asked and not $\overline{\boldsymbol{Q}^2}$, and a probability $q \equiv 1 - p$ that it was actually $\overline{\boldsymbol{Q}^2}$ being asked and not $\overline{\boldsymbol{Q}^1}$. Suppose further that the observer asked a large number of such questions, not knowing precisely which question was being asked each time, but observing the answer each time. Then the average of the observed answers would be given by $p\overline{\boldsymbol{Q}^1}\boldsymbol{\Psi} + q\overline{\boldsymbol{Q}^2}\boldsymbol{\Psi}$, which we could use to define the stochastic binary question $\overline{p\boldsymbol{Q}^1 + q\boldsymbol{Q}^2}$.

https://doi.org/10.1017/9781009401432.004 Published online by Cambridge University Press