THE EPIDEMIC CURVE OF SMALLPOX.

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(With 3 Graphs in the Text.)

When a disease assumes epidemic proportions, it is now generally recognised that certain conditions govern the rise and fall of the epidemic wave. Farr was probably the first person to attempt to describe these conditions in quantitative terms. His theory was "the real law (*i.e.* of the epidemic) implies that the ratio of increase goes on rapidly decreasing until the ratio itself is decreasing." In the Appendix to the second *Annual Report of the Registrar-General* he discusses the progress of the smallpox epidemic which had spread through England and Wales in 1837–9, causing the deaths of over 30,000 persons. "Five die weekly of smallpox in the metropolis when the disease is not epidemic.... Why do the five deaths become 10, 15, 20, 31, 58, 88 weekly and then progressively fall through the same measured steps?" He suggests, "amidst the apparent irregularities of the epidemic of smallpox and its eruptions all over the kingdom, it was governed in its progress by certain general laws." He found that the deaths from smallpox in the quarters of the year during the epidemic increased up to the third quarter very nearly at the ratio of 30 per cent. "The rate of increase is retarded at the end of the third period, and only rises 6 per cent. in the next, where it remains stationary, like a projectile at the summit of the curve which it is destined to describe. The decline of the epidemic was less rapid than its rise." He showed that the fall of mortality took place at a uniformly accelerated rate and calculated a "regular series of numbers" (such that the second differences of the logarithms are constant) for the decline of the epidemic. He compared these with those actually recorded and found "on the whole the agreement is remarkable." Farr did not put his law in the form of an equation nor did he explain the method by which his regular series were obtained, but it has since been shown that his epidemic law is a function of the normal curve. From the fact that the first, third and fifth of the calculated values agree exactly with the observed, for the two regions—Metropolis, Wales and the western counties—it seems probable that some method of differences was employed to find his "constant rate of acceleration." The calculated series found by Farr can be obtained by taking logarithms of the first, third and fifth of the observed values and finding the second difference. This value divided by

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four gives the constant second difference for the series, and taken with positive sign is the logarithm of the rate of acceleration. The logarithm of the second calculated value is obtained by adding to the logarithms of the first and third observed values the constant second difference and dividing by two. The ratio of the first to the second calculated value gives the first rate of decrease and the subsequent rates of decrease are found by multiplying the preceding rate by the "rate of acceleration." This method does not apply to the series for the whole kingdom; in this series only the third value of the observed and calculated are identical and no simple method of obtaining Farr's smoothed series suggests itself. The observed values, Farr's series and the calculated obtained by the method of least squares, for the Metropolis are:

Quarter	Mean quarterly deaths registered	Farr's series	$e^{-0 \cdot 1531x^2+0 \cdot 2811x+6 \cdot 8965}$ *
1	1103	1103	1124
2	959	967	940
3	611	611	579
4	240	278	263
5	91	91	88

* Origin at 0.

Some years later Farr applied his "law" to a prevalent epidemic of cattle plague, which according to a member of the House of Commons would increase from thousands to tens of thousands until all the cattle would probably die of it. He modified his previous mathematical expression by making the third differences of the logarithms constant and predicted the maximum and course of the epidemic. This forecast met with hostile criticism, but later events justified the accuracy of his main contentions. In 1873 Evans tried to extend Farr's method to epidemics of cholera and scarlet fever but without success.

Since Farr's time various studies have been made on the theory of the course of the epidemic curve. Brownlee put forward the hypothesis that the degree of infectivity of the organism decreased, as the epidemic progressed, according to the law of the monomolecular reaction in physical chemistry, *i.e.* in geometrical progression. He maintained that there was no evidence that an epidemic declined because of the lack of susceptibles but he thought the decline was much more likely to be due to the loss of infectivity on the part of the infecting agent.

Since 1919 Greenwood, Topley and their co-workers have been endeavouring to study, under laboratory conditions, the genesis and development of epidemics. These experiments are being carried out under conditions in which many disturbing factors, such as varying environment, nutrition, density, etc., can be standardised.

VARIATION IN TYPE OF SMALLPOX.

The epidemic curve of any disease may be modified by certain factors, chief among these being (*a*) changes in virulence of the organism and (*b*) changes in the resistance of the population. The epidemic curve of smallpox may have been influenced by the first factor; as for the second, smallpox is the only disease for which there was any method of preventive treatment by artificial

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immunisation a hundred years ago. There is no question that the type of smallpox has changed from the classical type of the eighteenth and early nineteenth centuries, both in fatality and age incidence, to the mild form which we experience, in this country, to-day.

Jenner's discovery of vaccination was made public in 1798, and early in the nineteenth century an appreciable proportion of the population had been vaccinated against smallpox. It was not until 1853 that the first vaccination law was passed, and vaccination was made compulsory in 1871. It has been contended that the general decrease in smallpox throughout the nineteenth century was a consequence of the introduction and dissemination of the method of vaccination. For example, Guy, writing in 1882, on the statistics of smallpox for 250 years in London, showed that smallpox was the most formidable epidemic disease in the seventeenth and eighteenth centuries, and yet in the nineteenth century had decreased much more than any of the other important infectious diseases such as measles, scarlet fever, etc. From this he argued, on the assumption that an equal improvement had been wrought by sanitary measures in all epidemic maladies, smallpox included, that the remarkable excess of improvement in smallpox must be attributed to some cause or causes other than sanitary reforms, and that there was only one cause to which it was reasonable to attribute this excess, namely, vaccination. A more recent study by Greenwood is less confident. He concludes from a study of the vaccination problem that vaccination has "saved lives and diminished suffering under conditions which have prevailed in England and may prevail again," but the use of vaccination is "not the sole, perhaps not the most important, factor in modifying the epidemiological history of smallpox during the last hundred years."

We have numerous and fairly accurate records of the time sequences in epidemics of smallpox both before and since the introduction of vaccination and before the present phase of inordinately mild smallpox. It has seemed worth while to inquire whether the form of the epidemic in pre-vaccination days, that is, its evolution in time, contrasts with its form in days when a proportion at least of the exposed to risk had been protected. Even if we assumed that no *recently* vaccinated persons can take smallpox at all, the susceptible population in post-Jennerian days must differ in some respects from those of the early eighteenth century. Some of them will be persons vaccinated many years before the epidemic and they will be intermingled with persons completely protected. Even if these considerations were ignored it would still be of interest to learn whether the form of an epidemic had changed over the long period of observation.

For descriptive purposes, the family of frequency curves developed by Prof. Karl Pearson has in point of flexibility and ease of computation few rivals and naturally attracted the attention of those who wished to graduate curves of epidemics. Pearsonian curves have been fitted by Brownlee, Greenwood and others to data of epidemics; Brownlee in particular used the method extensively. From the standpoint of graduation the results were often satisfactory, but it did not prove possible to classify epidemics on the basis of the types of curves found appropriate.

As is well known, the Pearsonian curves were derived from the integration of the equation

$$\frac{1}{y}\frac{dy}{dx}=\frac{a-x}{c_0+c_1x+c_2x^2},$$

where y is the frequency, x the abscissal value, and a, c_0, c_1 and c_2 are constants. The frequency distribution is taken to be unimodal. Integration leads to twelve types of curves. In order to use the method on the data of smallpox epidemics, the following preliminary adjustments were made:

(1) The small subsidiary rises at the tails of the epidemics of 1716–17, 1770–2 and 1881–2 were omitted from curve fitting, to conform with the fact that the theoretical curve is unimodal.

(2) The abscissa has, in some cases, been arbitrarily shifted to make the observed data tail off at the ends. This was necessary since smallpox had a varying but fairly high endemic level, when the deaths of the eighteenth and early nineteenth centuries were summed in four-week periods. In other words the epidemic has been taken as a phenomenon superimposed on the endemic level.

LONDON BILLS OF MORTALITY, 1700–1800.

When the deaths from smallpox are summed in four-week periods they give an irregular curve. The time between successive maxima varies from one to three years, but there is on the average a two-year period throughout this century. There is no definite seasonal trend such as is shown by most epidemic diseases, half the number of minima occur in spring and early summer and the maxima tend to be scattered over the rest of the year. When only the larger epidemics (where the maxima were 300 or more deaths in a month) are considered, the peaks are, on the average, distributed fairly evenly over three-quarters of the year. The distribution is as follows:

Four-week period	No. of maxima	No. of minima	No. of maxima (over 300)
1–4	3	4	2
5–8	—	3	—
9–12	1	7	—
13–16	1	7	—
17–20	—	10	—
21–24	3	3	2
25–28	6	2	5
29–32	4	—	3
33–36	6	3	3
37–40	7	2	4
41–44	3	1	1
45–48	4	1	1
49–52	10	4	3
Totals	48	47	24

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LONDON, 1840–1931, REGISTRAR-GENERAL'S REPORT.

When the deaths from smallpox are summed in four-week groups, for this period, the suggestion of periodicity shown by the data in the Bills of Mortality for the eighteenth century is absent. The interval between successive maxima varies from $1\frac{1}{2}$ to $6\frac{1}{2}$ years during the period 1840–85. Between 1800 and 1870 the maximum and minimum values are below those of the eighteenth century and the peaks generally are not so definite. London, in common with most European cities, experienced a severe epidemic in 1870–1. This, the worst of the nineteenth century, rose to a maximum of 1048 deaths in the 17th–20th weeks of 1871. A period of low mortality followed this outbreak, only 246 deaths being recorded in the three years 1873–5. Epidemics of smallpox occurred in 1877, 1878, 1881, 1885, the maxima being 381 in the 1st–4th weeks of the year, 241 in the 13th–16th weeks, 330 in the 17th–20th weeks and 239 in the 17th–20th weeks respectively. By the end of 1885 smallpox had practically disappeared as a cause of death, and during 1886–91 only 55 deaths were recorded in London. A slight rise in the number of deaths followed, and from 1892 to 1895 the annual deaths were 41, 206, 89 and 55. For the next five years only 33 deaths were due to smallpox. The last fatal epidemic made its appearance in the summer of 1901, reached a maximum of 289 deaths in the 9th–12th weeks of 1902 and disappeared by the late summer. For the twenty-nine years 1903–31 the deaths from smallpox were 113. The distribution of the maxima and minima for 1840–1902 is:

Weeks of year	No. of maxima	No. of minima
1–4	4	1
5–8	—	2
9–12	1	1
13–16	1	1
17–20	5	2
21–24	1	1
25–28	—	1
29–32	—	2
33–36	—	—
37–40	—	—
41–44	—	3
45–48	1	2
49–52	2	—
Totals	15	16

CURVE OF EPIDEMIC FOR LONDON.

The epidemics have been smoothed by fitting Pearson's Type Curves to the deaths summed in four-week periods. The important constants and equations are given in Table I. While these curves give a good description of the epidemics, they do not "fit" in a statistical sense. Type II, a symmetrical curve, graduates the epidemics for the years 1751–3, 1870–1, 1876–7, 1877–8 and 1880–1. Type I, an asymmetrical curve, describes the outbreaks for 1716–17, 1780–2 and 1862–3; the epidemic of 1780–2 declined faster than it rose, whilst

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Table I. *Constants and equations of the London epidemics. Deaths.*

Year	Mean (4-week periods from start)	μ_2	β_1	β_2	Equation	Origin (4-week periods from start)
1751–3	9·921	14·095	0·0000206	2·626	$y=331{\cdot}2118\left(1-\frac{x^2}{197{\cdot}7948421}\right)^{5{\cdot}5162912}+50$	9·921
1870–1	5·578	5·093	0·03857	2·597	$y=847{\cdot}949\left(1-\frac{x^2}{65{\cdot}5694651}\right)^{4{\cdot}9367302}+200$	5·578
1876–7	9·752	9·635	0·00003	2·848	$y=376{\cdot}9233\left(1-\frac{x^2}{361{\cdot}2574425}\right)^{17{\cdot}2477423}$	9·752
1877–8	6·335	6·822	0·00406	2·323	$y=209{\cdot}5966\left(1-\frac{x^2}{46{\cdot}8083069}\right)^{1{\cdot}9304626}$	6·335
1880–1	6·979	5·632	0·00420	2·643	$y=265{\cdot}2878\left(1-\frac{x^2}{83{\cdot}4785612}\right)^{5{\cdot}9111340}+50$	6·979
1716–17	7·178	7·579	0·1594	2·485	$y=314{\cdot}4602\left(1+\frac{x}{3{\cdot}9592714}\right)^{0{\cdot}9025024}\left(1-\frac{x}{10{\cdot}3979481}\right)^{2{\cdot}3701765}+90$	5·957
1780–2	11·544	14·218	0·4080	3·105	$y=335{\cdot}8209\left(1+\frac{x}{23{\cdot}5349765}\right)^{6{\cdot}5389295}\left(1-\frac{x}{5{\cdot}4237548}\right)^{1{\cdot}5069295}+50$	13·347
1862–3	7·381	8·336	0·0356	2·693	$y=183{\cdot}5409\left(1+\frac{x}{8{\cdot}8029066}\right)^{4{\cdot}5949158}\left(1-\frac{x}{13{\cdot}8402712}\right)^{7{\cdot}2243048}+50$	7·017
1901–2	8·993	6·216	0·0458	3·163	$y=0{\cdot}1960109\left(1+\frac{x^2}{336{\cdot}6103835}\right)^{-34{\cdot}6949177}e^{-32{\cdot}0352183\tan^{-1}\frac{x}{18{\cdot}3469448}}$	17·714
1901–2 (cases)	9·799	7·066	0·01600	3·416	$y=1487{\cdot}172\left(1+\frac{x^2}{116{\cdot}1366055}\right)^{-9{\cdot}7178801}$	9·799

Before analysis x deaths were subtracted from some of the observed distributions to conform with the Pearsonian theory. Thus the series used for 1751–3 was the observed one minus fifty, *i.e.* 22, 59, 59, 142, etc. The smoothed values were obtained in this way and then the constant x added to each term.

the reverse is the case for 1716–17 and 1862–3. The outbreak in 1901–2 conforms to a Type IV and is the only distribution that approaches a statistical fit. Most of the epidemics which have been smoothed are irregular, but some of the epidemics experienced in London give such a ragged distribution that no unimodal curve will smooth them, neither does a parabolic curve of the 4th order describe the distribution (see Table III). In this class fall the outbreaks of 1722–4, 1795–7 and 1884–5; the last exhibits three distinct peaks. A possible explanation of this seemed to be that the disease had made its appearance in hitherto unaffected districts, so that the total was really a summation of several epidemics with different time intervals. To examine this, the deaths were taken out in the five broad groups—West, North, Central, East and South, as defined by the Registrar-General, for the nineteenth-century epidemics. The surmise proved to be incorrect since there is no apparent difference in the commencement of the outbreak in the various divisions, but the interesting discovery was made that the course of the epidemic varied from district to district. The time of the maxima differs, the most outstanding case is in 1871 when the largest number of deaths in the East district occurred in February and in the North in June. The order in which the London districts reach their maximum varies, but for the years examined the North tends to occupy the last position. In 1863 the maximum for London as a whole, the North and South was in the 19th–22nd weeks of the year, whilst the largest number of deaths in the West, Central and East occurred in the preceding four-week period. The outbreak in 1870–2 attained a maximum for London as a whole and the South in the 17th–20th weeks of 1871, whilst that for the West, Central and East was in the 5th–8th weeks. The climax of the epidemic in the North district was reached in the 21st–24th weeks. During the epidemic of 1876–8, which was bimodal, the first maximum for London as a whole and the South was reached in the first four-week period of 1877, in the East in February and in the North in March. The Central and West districts did not have a definite maximum. The second peak for London as a whole, North and East appeared in the 13th–16th weeks of 1878 and that for the South and West in the preceding and succeeding months respectively. In 1881 the peaks of the distribution for London as a whole, West and Central districts were simultaneous, 17th–20th weeks, that for the East, South and North in the 13th–16th, 21st–24th and 25th–28th weeks respectively. The distribution of deaths for the epidemic of 1884–5 was trimodal, and although the maxima of the five districts were not so clearly defined as in the preceding outbreaks, they show the same time irregularities as did the previous epidemics. The majority of the deaths in the 1901–2 outbreak occurred in M.A.B. ships and hospitals at Dartford, so that a similar analysis in districts for this epidemic is of little value. It is not possible to analyse the eighteenth-century data in subdivisions. The notifications for the years 1901–2 and 1928–31 were also examined in the five districts. These two distributions show varying maxima and irregularities similar to those displayed by the deaths for the nineteenth century. The maximum number of cases of smallpox for London as a whole, North and South, for the epidemic of 1901–2, was reached in the 9th–12th weeks of 1902, the maximum of the Central, West and East districts had occurred earlier in the 49th–52nd weeks of 1901, 1st–4th and 5th–8th weeks of 1902 respectively. The distribution of cases of smallpox during 1928–31 is irregular. The largest number of cases for London as a whole, North and Central occurred during the 13th–16th weeks, the East in the 9th–12th weeks of 1930, whilst in the South district the peak of the epidemic was in the 45th–48th weeks of 1929. The West district had very few cases. A Type VII, a symmetrical curve, smoothes the 1901–2 notifications. The observed and smoothed values and the proportion of the total deaths occurring in each four-weekly period, for each of the London epidemics, are given in Tables II, III and IV.

This analysis makes it clear that no precise formal distinction differentiates the London epidemics of the eighteenth century from those of the late nineteenth and early twentieth centuries. There is perhaps a tendency for the modern epidemics to be more concentrated in time. This can be most easily seen by an examination of the proportional distribution as given in Table IV. If we now take the three four-weekly periods of which that containing the maximum is central, then in the three eighteenth-century epidemics these three periods include 34·4, 29·3 and 30·6 per cent. of the total epidemic deaths; the six nineteenth- and twentieth-century epidemics have respectively 34·7, 39·5, 34·9, 41·4, 40·4 and 48·1 per cent. of their totals within the defined limits. There is a suggestion that as we approach more nearly the modern epoch this concentration increases. Graphs I and II illustrate the comparison. This phenomenon could be explained in many ways, but, for the reasons already pointed out, *e.g.* the non-uniformity of distribution throughout the area of London, we have no adequate means of verifying any speculations. All that can fairly be said is that, in respect of evolution in time, no sharp distinction between epidemic smallpox in pre- and post-vaccination times can be established.

TOWNS OTHER THAN LONDON.

It is of some interest to compare the forms of the London epidemics with those of other cities. Glasgow is the only British city for which suitable data were available and a Type II curve graduated the 1863–4 experience.

The 1870 epidemic was common to most European countries and makes comparison possible between different cities. The distributions are given by Prinzing in his *Epidemics resulting from Wars*. A normal curve describes the Hamburg epidemic of 1870–2; Leipzig 1871 yields a Type IV, the fall being more rapid than the rise; Breslau (cases) 1871–2 gives a Type I with steeper decline than rise. The distributions for Danzig and suburbs and Berlin are two-peaked, and a fourth order parabola fails to describe them. The curve for Paris 1869–71 can be regarded as two curves, the first a Type I, with a slower rise than fall, and the second a Type II. The epidemic was apparently declining when it received

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Table II.

London 1751–3

4-week periods	Deaths: Observed values	Deaths: Smoothed values
33–36	72	70
37–40	109	91
41–44	109	122
45–48	192	163
49–52	216	211
1–4	251	262
5–8	303	309
9–12	290	348
13–16	336	373
17–20	387	380
21–24	478	370
25–28	380	343
29–32	295	302
33–36	208	254
37–40	167	203
41–44	150	156
45–48	118	117
49–52	108	87
1–4	78	67
Totals	4247	4228

London 1870–1

4-week periods	Deaths: Observed values	Deaths: Smoothed values
49–52	297	330
1–4	559	492
5–8	852	700
9–12	797	897
13–16	947	1022
17–20	1048	1032
21–24	971	923
25–28	764	733
29–32	440	523
33–36	321	351
37–40	269	248
41–44	236	208
Totals	7501	7459

London 1876–7

4-week periods	Deaths: Observed values	Deaths: Smoothed values
25–28	20	6
29–32	25	17
33–36	28	37
37–40	53	73
41–44	74	124
45–48	210	190
49–52	297	261
1–4	381	324
5–8	369	366
9–12	356	374
13–16	292	349
17–20	291	295
21–24	243	226
25–28	143	156
29–32	104	97
33–36	60	53
37–40	47	26
Totals	2993	2974

London 1877–8

4-week periods	Deaths: Observed values	Deaths: Smoothed values
41–44	45	35
45–48	97	78
49–52	116	124
1–4	146	165
5–8	179	194
9–12	183	208
13–16	241	205
17–20	219	186
21–24	149	152
25–28	87	109
29–32	62	63
33–36	28	23
Totals	1552	1542

London 1880–1

4-week periods	Deaths: Observed values	Deaths: Smoothed values
45–48	56	60
49–52	77	84
1–4	154	127
5–8	205	187
9–12	202	249
13–16	289	297
17–20	330	314
21–24	315	295
25–28	262	247
29–32	149	184
33–36	123	125
37–40	80	83
41–44	72	60
Totals	2314	2312

Table II (*continued*),

London 1716–17

4-week periods	Deaths: Observed values	Deaths: Smoothed values
21–24	95	90
25–28	135	115
29–32	207	252
33–36	325	344
37–40	414	391
41–44	449	406
45–48	383	392
49–52	372	362
1–4	307	321
5–8	231	275
9–12	227	227
13–16	202	183
17–20	161	145
21–24	116	116
Totals	3624	3619

London 1780–2

4-week periods	Deaths: Observed values	Deaths: Smoothed values
37–40	76	66
41–44	101	75
45–48	101	88
49–52	131	106
53–3	126	128
4–7	125	156
8–11	143	188
12–15	166	225
16–19	261	265
20–23	313	304
24–27	386	340
28–31	425	369
32–35	468	384
36–39	366	380
40–43	257	351
44–47	281	294
48–51	176	209
52–3	141	109
4–7	64	52
Totals	4107	4089

London 1862–3

4-week periods	Deaths: Observed values	Deaths: Smoothed values
47–50	58	63
51–2	121	86
3–6	105	121
7–10	132	161
11–14	173	198
15–18	261	224
19–22	269	233
23–26	212	225
27–30	183	202
31–34	176	171
35–38	124	138
39–42	111	108
43–46	84	84
47–50	76	67
51–2	58	57
Totals	2143	2138

London 1901–2

4-week periods	Deaths: Observed values	Deaths: Smoothed values
29–32	2	3
33–36	16	7
37–40	20	17
41–44	26	35
45–48	73	66
49–52	91	111
1–4	162	167
5–8	221	219
9–12	289	247
13–16	230	236
17–20	166	190
21–24	130	126
25–28	67	68
29–32	30	30
33–36	12	11
37–40	5	3
Totals	1540	1536

London 1901–2*

4-week periods	Cases: Observed values	Cases: Smoothed values
25–28	13	11
29–32	17	26
33–36	116	59
37–40	173	128
41–44	303	260
45–48	413	480
49–52	654	790
1–4	960	1135
5–8	1511	1402
9–12	1663	1472
13–16	1241	1313
17–20	893	999
21–24	906	657
25–28	352	380
29–32	139	198
33–36	73	94
37–40	18	42
41–44	14	18
45–48	14	8
49–52	11	3
Totals	9484	9475

* Metropolitan Asylum Board's Report.

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Table III. *Unclassified distributions. London.*

1722–4		1795–7		1884–5	
4-week periods	No. of deaths	4-week periods	No. of deaths	4-week periods	No. of deaths
13–16	61	37–40	89	1–4	15
17–20	93	41–44	138	5–8	11
21–24	123	45–48	210	9–12	23
25–28	132	49–52	218	13–16	46
29–32	153	1–4	169	17–20	85
33–36	219	5–8	192	21–24	155
37–40	254	9–12	166	25–28	185
41–44	308	13–16	172	29–32	97
45–48	397	17–20	216	33–36	61
49–52	414	21–24	346	37–40	51
1–4	425	25–28	374	41–44	97
5–8	344	29–32	400	45–48	164
9–12	198	33–36	325	49–52	208
13–16	191	37–40	406	53–3	226
17–20	186	41–44	265	4–7	219
21–24	258	45–48	207	8–11	122
25–28	243	49–52	202	12–15	152
29–32	269	1–4	83	16–19	240
33–36	242	5–8	42	20–23	206
37–40	231			24–27	127
41–44	205			28–31	63
45–48	148			32–35	35
49–52	112			36–39	35
1–4	81			40–43	12
5–8	60				

Table IV. *London epidemics. Proportion of deaths in each month.*

Month	1716–17	1751–3	1780–2	1862–3	1870–1	1876–7	1877–8	1880–1	1901–2
1st	0·026	0·017	0·019	0·027	0·040	0·007	0·029	0·024	0·001
2nd	0·037	0·026	0·025	0·056	0·075	0·008	0·063	0·033	0·010
3rd	0·057	0·026	0·025	0·049	0·114	0·009	0·075	0·067	0·013
4th	0·090	0·045	0·032	0·062	0·106	0·018	0·094	0·089	0·017
5th	0·114	0·051	0·031	0·081	0·126	0·025	0·115	0·087	0·047
6th	0·124	0·059	0·030	0·122	0·140	0·070	0·118	0·125	0·059
7th	0·106	0·071	0·035	0·126	0·129	0·099	0·155	0·143	0·105
8th	0·103	0·068	0·040	0·099	0·102	0·127	0·141	0·136	0·144
9th	0·085	0·079	0·064	0·085	0·059	0·123	0·096	0·113	0·188
10th	0·064	0·091	0·076	0·082	0·043	0·119	0·056	0·064	0·149
11th	0·063	0·113	0·094	0·058	0·036	0·098	0·040	0·053	0·108
12th	0·056	0·089	0·103	0·052	0·031	0·097	0·018	0·035	0·084
13th	0·044	0·069	0·114	0·039		0·081		0·031	0·044
14th	0·032	0·049	0·089	0·035		0·048			0·019
15th		0·039	0·063	0·027		0·035			0·008
16th		0·035	0·068			0·020			0·003
17th		0·028	0·043			0·016			
18th		0·025	0·034						
19th		0·018	0·016						

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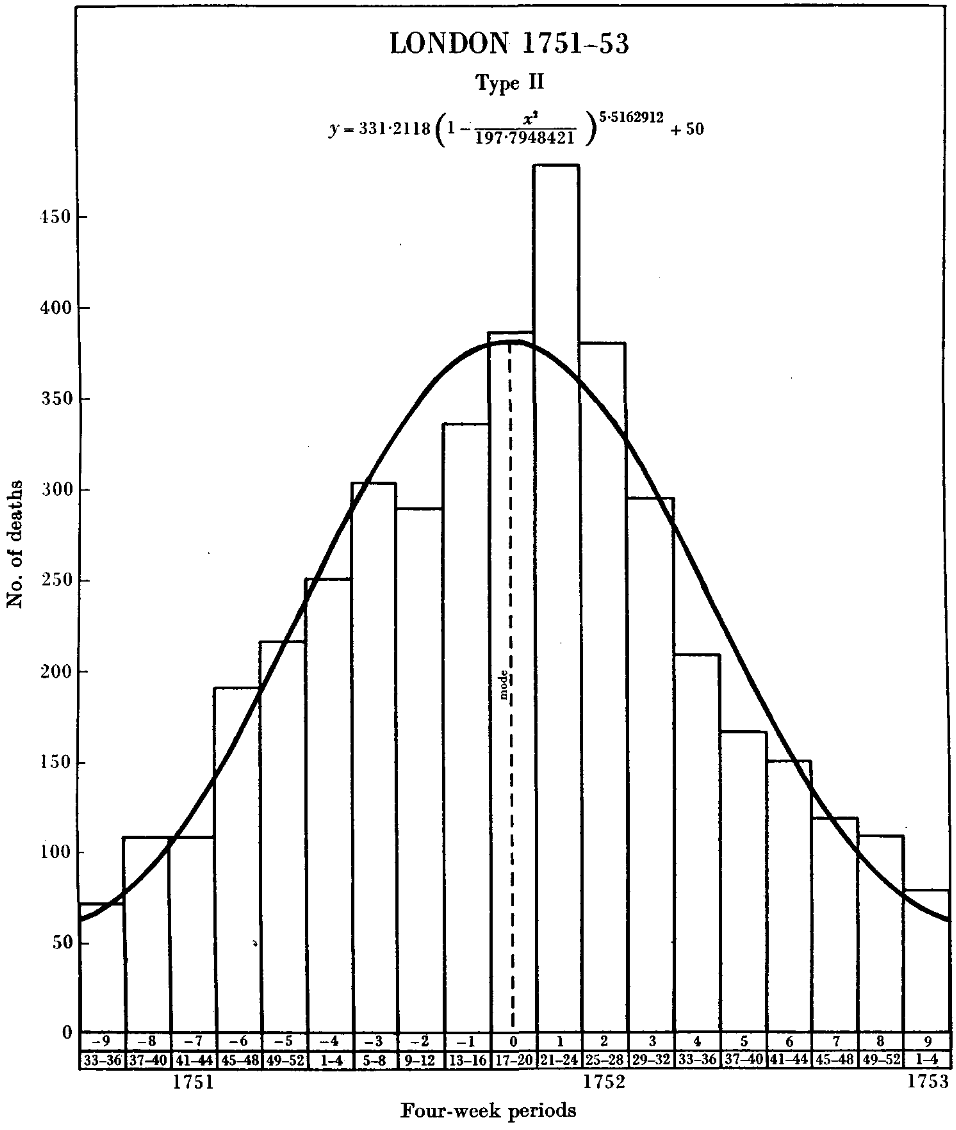

Graph I.

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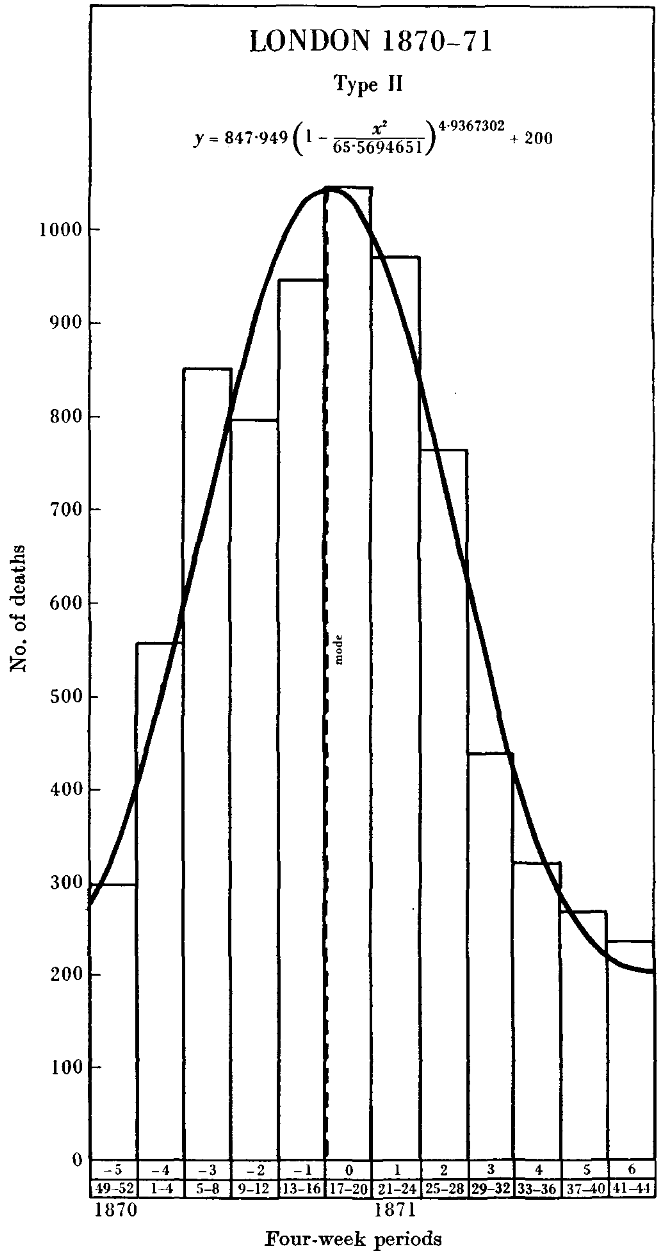

Graph II.

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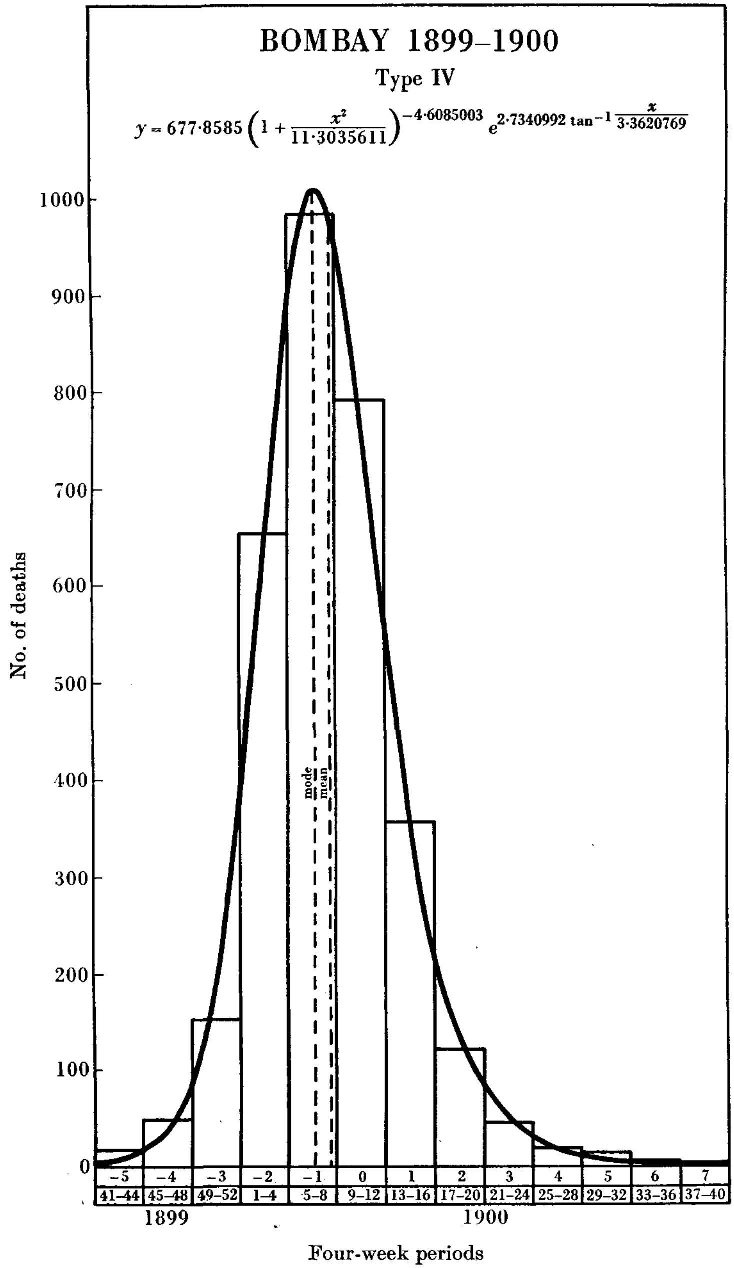

Graph III.

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a stimulus from the arrival, in Paris, during September 1870 of newly organised troops, consisting of young men who had not been vaccinated or revaccinated. The constants, equations, observed and smoothed values for these cities are given in Tables V, VI and VII.

India.

Finally the deaths from smallpox in British India have been analysed, and curves fitted to the data of Bombay and Calcutta. The data as a whole for the 49 years 1882–1930 give a remarkably smooth curve, with a steady increase to a maximum in the spring and a steady decrease to a minimum in the autumn. The peaks of the curve occur as follows:

	Maxima	Minima
	10 in March	3 in September
	22 in April	40 in October
	16 in May	6 in November
	1 in June	
Totals	49	49

There is a suggestion of periodicity in the severity of the epidemics in the Indian data, the maxima following a wave-like trend. The more severe epidemics (and to a less extent the minor epidemics) tend to occur in pairs, in following years. The actual maximum and minimum values for this period are given in Table VIII.

Curves have been fitted to the deaths, summed in four-week periods, for the epidemics in Calcutta, during the years 1905–6 and 1908–9, and for Bombay for 1899–1900 and 1904–5. In each case a Type IV curve gives a good description. In these Indian cities, the concentration around the period of maximum is far greater than in the London experience. Thus in Bombay, 1899–1900, 75·8 per cent. of the deaths fall to the three four-weekly periods in which the maximum is central (see Graph III). In 1904–5, the percentage was 70·5. In the two Calcutta epidemics the proportions were 60·8 and 78·9 per cent. The constants, equations, observed and smoothed values are given in Tables IX and X.

Conclusions.

It is not possible, on the basis taken, to differentiate between the course of the epidemic before and after the introduction of vaccination. The shape of the distribution of monthly totals of the outbreaks of 1870 for various European cities are dissimilar. The majority of the fitted curves are symmetrical or almost so; the most skew distributions, of London, examined are those of 1716–17 and 1780–2. It does not seem practicable, from these data, to make the London epidemics a function of time only. The shorter period for which the epidemic rages in India, the greater severity, the striking seasonal variation —the spring maxima and autumn minima—probably account for the four Indian curves being of the same type and giving a better description of the observed distribution than most of the other curves.

Table V. *Constants and equations of epidemics other than London. Deaths.*

City	Year	Mean (months from start)	μ_2	β_1	β_2	Equation	Origin (months from start)
Glasgow	1863–4	8·284	11·308	0·0152	2·532	$y=65{\cdot}66478\left(1-\frac{x^2}{122{\cdot}2110433}\right)^{3{\cdot}9038676}$	8·284
Hamburg	1870–2	10·494	10·190	0·00027	3·025	$y=501{\cdot}1479\,e^{-\frac{(x-0{\cdot}4935162)^2}{20{\cdot}3801322}}$	10·494
Leipzig	1871	5·295	2·615	0·0368	3·370	$y=219{\cdot}4734\left(1+\frac{x^2}{53{\cdot}1165343}\right)^{-12{\cdot}1269795} e^{4{\cdot}7995130\tan^{-1}\frac{x}{7{\cdot}2881091}}$	3·724
Paris	1869–71	—	—	—	—	$y=994{\cdot}6775\left(1+\frac{(x-0{\cdot}2491650)}{12{\cdot}9975940}\right)^{3{\cdot}4999516}\left(1-\frac{(x-0{\cdot}2491650)}{1{\cdot}5287410}\right)^{0{\cdot}4116546} +1842{\cdot}3488\left(1-\frac{(x-4{\cdot}6263674)^2}{9{\cdot}7294523}\right)$	10·249
Breslau (cases)	1871–2	11·244	8·726	1·059	3·991	$y=1161{\cdot}568\left(1+\frac{x}{24{\cdot}3058614}\right)^{7{\cdot}1160334}\left(1-\frac{x}{1{\cdot}9252639}\right)^{0{\cdot}5636600}$	13·556

Table VI.

Glasgow 1863–4 Month	Deaths Observed values	Deaths Smoothed values
May	12	7
June	14	14
July	18	24
Aug.	34	35
Sept.	43	46
Oct.	62	55
Nov.	74	62
Dec.	76	65
Jan.	51	64
Feb.	59	60
March	44	51
April	35	41
May	27	30
June	23	20
July	16	11
Aug.	8	5
Totals	596	590

Hamburg 1870–2 Month	Deaths Observed values	Deaths Smoothed values
Oct.	10	8
Nov.	24	15
Dec.	34	33
Jan.	69	64
Feb.	107	115
March	163	187
April	226	276
May	364	369
June	503	448
July	554	493
Aug.	578	493
Sept.	373	447
Oct.	311	368
Nov.	229	274
Dec.	170	186
Jan.	158	114
Feb.	74	64
March	47	32
April	16	24
Totals	4010	4010

Leipzig 1871 Month	Deaths Observed values	Deaths Smoothed values
Jan.	16	9
Feb.	42	40
March	103	124
April	255	257
May	367	343
June	311	300
July	161	180
Aug.	68	80
Sept.	35	28
Oct.	16	7
Totals	1374	1368

Paris 1869–71 Month	Deaths Observed values	Deaths Smoothed values
Oct.	39	29
Nov.	93	64
Dec.	119	118
Jan.	174	197
Feb.	293	301
March	406	431
April	561	581
May	786	742
June	914	891
July	1072	983
Aug.	713	889
Sept.	700	585
Oct.	1361	1326
Nov.	1722	1752
Dec.	1837	1800
Jan.	1503	1469
Feb.	763	760
March	230	34
Totals	13286	12952

Breslau 1871–2 Month	Cases Observed values	Cases Smoothed values
Jan.	33	21
Feb.	68	36
March	90	58
April	68	91
May	134	138
June	235	203
July	287	287
Aug.	271	395
Sept.	361	527
Oct.	699	680
Nov.	1026	848
Dec.	1229	1011
Jan.	1311	1131
Feb.	790	1127
March	462	699
April	242	0
Totals	7306	7252

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Table VII. *Unclassified epidemics.*

Berlin 1870–2		Danzig and suburbs 1870–2	
Month	Deaths	Month	Cases
Nov.	9	Oct.	4
Dec.	22	Nov.	13
Jan.	48	Dec.	34
Feb.	80	Jan.	123
March	176	Feb.	129
April	349	March	201
May	430	April	365
June	648	May	459
July	532	June	442
Aug.	528	July	182
Sept.	490	Aug.	130
Oct.	600	Sept.	111
Nov.	660	Oct.	124
Dec.	671	Nov.	136
Jan.	445	Dec.	135
Feb.	256	Jan.	245
March	151	Feb.	222
April	117	March	153
May	76	April	89
June	33	May	34
July	18	June	19
Aug.	10	July	13
		Aug.	8

Table VIII. *India.*

	Monthly deaths			Monthly deaths	
Year	Maxima	Minima	Year	Maxima	Minima
1882	11,612	3,013	1907	13,918	3,327
1883	38,023	4,284	1908	28,773	2,879
1884	67,516	5,139	1909	19,347	2,274
1885	12,620	2,218	1910	7,317	1,931
1886	7,043	1,913	1911	7,895	2,474
1887	8,789	2,829	1912	11,726	3,102
1888	13,277	3,035	1913	15,239	2,164
1889	20,803	2,631	1914	10,958	2,256
1890	18,409	3,407	1915	13,837	1,963
1891	12,676	4,139	1916	8,876	2,112
1892	14,418	3,537	1917	7,284	3,453
1893	9,305	2,168	1918	12,966	3,338
1894	6,305	1,468	1919	20,728	3,440
1895	6,691	1,603	1920	16,625	1,849
1896	22,673	3,599	1921	6,542	992
1897	32,521	2,754	1922	4,783	2,497
1898	9,493	1,822	1923	5,320	1,918
1899	6,270	2,798	1924	7,568	1,918
1900	12,024	3,927	1925	13,439	2,632
1901	12,238	3,889	1926	17,648	2,680
1902	18,348	2,934	1927	18,986	2,306
1903	13,354	2,457	1928	16,109	2,544
1904	7,859	1,852	1929	11,927	1,683
1905	8,540	3,260	1930	11,775	1,238
1906	14,916	3,488			

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Table IX. *Constants and equations for the epidemics of Bombay and Calcutta. Deaths.*

City	Year	Mean (4-week periods from start)	μ_2	β_1	β_2	Equation	Origin (4-week periods from start)
Bombay	1899–1900	5·345	2·079	0·4587	5·274	$y=677{\cdot}8585\left(1+\frac{x^2}{11{\cdot}3035611}\right)^{-4{\cdot}6085003} e^{2{\cdot}7340992\tan^{-1}\frac{x}{3{\cdot}3620769}}$	4·072
Bombay	1904–5	5·964	2·218	0·0245	3·363	$y=471{\cdot}8572\left(1+\frac{x^2}{44{\cdot}1613797}\right)^{-11{\cdot}7455494} e^{3{\cdot}6741099\tan^{-1}\frac{x}{6{\cdot}6454029}}$	4·828
Calcutta	1905–6	6·230	2·377	0·0422	3·421	$y=516{\cdot}7851\left(1+\frac{x^2}{43{\cdot}0567153}\right)^{-10{\cdot}9806122} e^{4{\cdot}3260895\tan^{-1}\frac{x}{6{\cdot}5617616}}$	4·808
Calcutta	1908–9	5·926	1·545	0·0047	3·825	$y=1503{\cdot}733\left(1+\frac{x^2}{10{\cdot}7552967}\right)^{-6{\cdot}1705245} e^{-0{\cdot}4831710\tan^{-1}\frac{x}{3{\cdot}2795269}}$	6·080

Table X.

Bombay 1899–1900

4-week period	Deaths: Observed values	Deaths: Smoothed values
41–44	16	6
45–48	48	39
49–52	151	201
1–4	654	636
5–8	984	979
9–12	792	754
13–16	356	365
17–20	120	141
21–24	44	51
25–28	19	19
29–32	14	7
33–36	4	3
37–40	3	1
Totals	3205	3202

Bombay 1904–5

4-week period	Deaths: Observed values	Deaths: Smoothed values
41–44	6	3
45–48	16	17
49–52	78	80
1–4	238	254
5–8	519	508
9–12	649	613
13–16	426	454
17–20	210	222
21–24	81	78
25–28	33	22
29–32	4	5
Totals	2260	2256

Calcutta 1905–6

4-week period	Deaths: Observed values	Deaths: Smoothed values
41–44	4	3
45–48	12	16
49–52	82	78
1–4	273	265
5–8	575	576
9–12	707	772
13–16	741	646
17–20	331	360
21–24	131	146
25–28	41	47
29–32	26	13
33–36	3	4
Totals	2926	2926

Calcutta 1908–9

4-week period	Deaths: Observed values	Deaths: Smoothed values
42–45	5	1
46–49	24	8
50–53	83	50
1–4	329	267
5–8	851	931
9–12	1360	1449
13–16	780	835
17–20	278	210
21–24	62	34
25–28	17	5
29–32	2	1
Totals	3791	3791

A theoretical objection to taking Type I, which occurs four times, as an epidemic curve is the fact that its range is limited in both directions. The same objection can be raised against Type II, which describes the majority of London outbreaks, since it is a special case of Type I, a symmetrical form. The normal curve and Types VII and IV fulfil the theoretical condition of unlimited range in both directions, *i.e.* the epidemic never absolutely disappears although it becomes negligible. Type VII is a particular case of Type IV arising from a symmetrical form. The normal curve is the transition point between a Type II and Type VII. Type IV, which has been found to fit the majority of other epidemics, is leptokurtic, *i.e.* a narrow top curve, whilst most of the observed distributions are platykurtic, hence this curve is unsuitable for describing the majority of the smallpox epidemics.

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