CORRESPONDENCE.

ON THE PARTIAL COMMUTATION OF PREMIUM.

To the Editor of the Assurance Magazine.

SIR,—The following problem, besides presenting several points of interest, admits also of some useful practical applications. Perhaps you will be able to make room for it in your pages.

PROBLEM.

A person aged x desires to assure his life for the sum A, the Office premium for which is P_x. He proposes to pay only Q (less than P_x), and consents to an equivalent abatement in the sum assured during the next t years. Required X, the amount of the abatement.

There are various cases, according to the form taken by the abatement. I shall consider two.

Case 1. The abatement uniform during the term.

The Office gives up, in premium, $P_x - Q$, the value of which is,

$$\frac{(P_x-Q)N_{x-1}}{D_x};$$

and it takes back, in assurance during the next t years, X, the *net* value of which is,

$$\frac{XM_{x|t}}{D_x}.$$

The commission or loading, say l per unit, on the whole sum nominally assured, is included in P_x. The portion of the assurance taken back by the Office will therefore be allowed for at the same rate. Hence the *Office* value of this portion will be,

$$\frac{(1+l)XM_{x|t}}{D_x}.$$

Equating, we get,

$$X=\frac{(P_x-Q)N_{x-1}}{(1+l)M_{x|t}}.$$

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Otherwise:—

The benefit is a whole-life assurance of A, less a temporary assurance of X for t years; and its value is,

$$\frac{(1+l)(\mathrm{AM}_x-\mathrm{XM}_{x|t})}{\mathrm{D}_x}.$$

Also, the premium payable being Q, its value is,

$$\frac{\mathrm{QN}_{x-1}}{\mathrm{D}_x}.$$

Equating,

$$(1+l)(\mathrm{AM}_x-\mathrm{XM}_{x|t})=\mathrm{QN}_{x-1};$$

whence,

$$\mathrm{X}=\frac{(1+l)\mathrm{AM}_x-\mathrm{QN}_{x-1}}{(1+l)\mathrm{M}_{x|t}}.$$

And introducing P_x into this expression, by means of the relation,

$$\mathrm{P}_x=\frac{(1+l)\mathrm{AM}_x}{\mathrm{N}_{x-1}},$$

we have finally,

$$\mathrm{X}=\frac{(\mathrm{P}_x-\mathrm{Q})\mathrm{N}_{x-1}}{(1+l)\mathrm{M}_{x|t}},$$

as before.

It may be noted that X decreases as Q increases, and vanishes if $\mathrm{Q}=\mathrm{P}_x$. If Q exceed P_x we should have X negative, implying that in this case A, instead of undergoing a diminution, would receive an augmentation.

Example. Let $x=30$, A=£1000; then, using the H^{M} Table, at 3 per-cent, with a loading of 20 per-cent, ($l=\cdot20$), we have $\mathrm{P}_x=22\cdot554$. And if Q=21 and $t=10$, the equation becomes,

$$\mathrm{X}=\frac{1\cdot554\mathrm{N}_{29}}{1\cdot20\mathrm{M}_{30|10}}.$$

N_{29}	. . .	log 5·887942	page 10.
	1·554	„ 0·191451	
		6·079393	
M_{30}	14521·0		
M_{40}	11870·7		
	2650·3		
×·2	530·1		
	3180·4	„ 3·502482	
X	377·495	„ 2·576911	

That is, the abatement being £377. 10*s*. 0*d*., the sum assured will be £622. 10*s*. 0*d*. during the next ten years, and £1000 during the residue of the life of (x).

The following small table shows the results arising from giving to t the values in the first column in succession:—

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t	X	A—X
10	377·495	622·505
20	195·054	804·946
30	128·279	871·721
40	92·811	907·189
50	74·704	925·296
60	69·348	930·652
*ω	68·900	931·100

Case 2. The abatement commencing at X, and decreasing annually by one tth part of X.

The Office here gives up, as before, of premium, P_x-Q, the value of which is,

$$\frac{(P_x-Q)N_{x-1}}{D_x};$$

and it takes back an assurance commencing at X, and decreasing annually by $\frac{X}{t}$ till extinction, the value of which is (*Journal*, vol. xii, p. 343),

$$\frac{X\{M_x-\frac{1}{t}(R_{x+1}-R_{x+t+1})\}}{D_x}.$$

Multiplying by $1+l$ and equating,

$$X=\frac{(P_x-Q)N_{x-1}}{(1+l)\{M_x-\frac{1}{t}(R_{x+1}-R_{x+t+1})\}}.$$

Example. Let $x=30$, A=£1000, $t=10$, all as before. And P_{30} being 22·554, let also Q=21, as before.

The numerator here is the same as in last example; and its logarithm therefore is 6·079393.

The denominator is,

$$1{\cdot}20\{M_{30}-\tfrac{1}{10}(R_{31}-R_{41})\};$$

and it is computed as follows:—

R_{31}	392498·7	
R_{41}	262043·4	
	130455·3	
$\frac{1}{10}$	13045·5	
M_{30}	14521·0	
	1475·5	
×·20	295·1	
	1770·6	log 3·248120
Numerator . . .		„ 6·079393
X	678·067	„ 2·831273

* The limiting value of the table.

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Hence the sum assured during the first year is 1000−678·067= 321·933; and the amount for each succeeding year is found by adding 678·067÷10=67·807 to that for the year preceding. The sum assured during the eleventh year is thus £1000; and it remains at this amount during the rest of life.

The deduction at the outset seems here somewhat heavy; but it rapidly diminishes, and vanishes at the end of ten years. Were the term extended to twenty years the deduction at the outset would be only 365·053, and the assurance would consequently commence at 634·947.

I must defer till another opportunity the development of the schemes here shadowed forth. I will now merely mention, that they find their practical applications in cases in which it is arranged that a party who has been "rated up," instead of paying additional premium, shall be subjected to a temporary abatement of assurance.

I am, Sir,
Your most obedient servant,

London, 21 *Oct.* 1872. P. GRAY.

ON THE RELATION BETWEEN THE VALUE OF A POLICY AND THE RATE OF INTEREST.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the paper on "Extra Premium," by Mr. J. R. Macfadyen, in the current volume of the *Journal*, that gentleman has given, in a footnote on p. 89, a demonstration intended to show that "in any given case it is practically certain that the value of a policy by a higher rate of interest must always be less than by a lower." Having, sometime ago, myself arrived at a similar result to Mr. Macfadyen's by a rather different process, I venture to send it you, with the hope that it may be of interest to some of your readers.

We have, by a well-known formula,

$$ {}_nV_x=1-(1-V_x)(1-V_{x+1})\ldots\ldots(1-V_{x+n-1}); $$

consequently, it will be sufficient to consider how the value of a policy one year old is affected by increasing or diminishing the rate of interest at which it is calculated.

Now,
$$ V_x=1-\frac{1+a_{x+1}}{1+a_x} $$
$$ =1-\frac{a_x}{vp_x(1+a_x)} $$

or, omitting the subscript x,

$$ V=1-\frac{a}{vp(1+a)}. $$

Differentiating this with respect to v, we have