From square of b take $4ac$;
Square root extract, and b subtract;
Divide by $2a$; *you've x alway.*

Another mnemonic for the same values, due to Mr N. D. Beatson Bell, is

When you have written $-b$,
The double sign put down;
Then $b^2 - 4ac$
With square-root mark you crown;
Beneath it all a line you trace,
Beneath which line $2a$ *you place.*

The value of the co-efficient of refraction of light in two important cases is got from the following:—

When rays do pass from air to glass,
The value of μ *is three by two;*
But when they pass from air to water,
The value of μ *is one by three-quarter(s)* !

Eighth Meeting, June 12*th*, 1885.

THOMAS MUIR, Esq., LL.D., F.R.S.E., in the Chair.

Summation of certain Series.

By Professor TAIT.

[*Abstract.**]

The attempt to enumerate the possible distinct forms of knots of any order, though unsuccessful as yet, has led me to a number of curious results, some of which may perhaps be new. The general character of the methods employed will be obvious from an inspection of a few simple cases, and any one who has some practice in algebra may extend the results indefinitely.

* This Abstract is part of the paper read in June, entitled "On the detection of amphicheiral knots, with special reference to the mathematical processes involved." I have unfortunately mislaid the MS.—P.G.T.

Take, for instance, the series

$$r^m - n(r+s)^m + \frac{n.\overline{n-1}}{1.2}(r+2s)^m - \&c.$$

where the coefficients are the terms of $(1-1)^n$, and the other factors are the m^{th} powers of the terms of an arithmetical series :—m being a positive integer. The well-known properties of exponential series give us an easy method of summing all expressions of this form. For we have

$$(\epsilon^{px} - \epsilon^{qx})^n = \epsilon^{npx} - n\epsilon^{(\overline{n-1}p+q)x} + \frac{n.\overline{n-1}}{1.2}\epsilon^{(\overline{n-2}p+2q)x} - \&c.$$

which may be written in the form

$$\left((p-q)x + \frac{p^2-q^2}{2!}x^2 + \frac{p^3-q^3}{3!}x^3 + \&c.\right)^n$$

$$= \Sigma\frac{1}{m!}\left(\overline{np}^m - n(np+q-p)^m + \frac{n.\overline{n-1}}{1.2}(np+2\overline{q-p})^m - \&c.\right)$$

Make $np = r$, $q-p=s$; and p and q are known.

The required sum is then the coefficient of x^m in the expansion of

$$m!\left((p-q)x + \frac{p^2-q^2}{2!}x^2 + \ldots\ldots\right)^n.$$

It vanishes therefore, so long as $m \angle n$; and for $m=n$ its value is

$$m!(p-q)^m = (-)^m m! s^m.$$

When the coefficients in the given series are the *alternate* terms of $(1-1)^n$, we have only to treat, as above, the expression

$$(\epsilon^{px} + \epsilon^{qx})^n \pm (\epsilon^{px} - \epsilon^{qx})^n.$$

Such results may be varied *ad libitum*, by introducing two or more quantities in place of x, and comparing coefficients of like terms :—*e.g.*, as in finding, by the two methods of expansion, the term in $x^r y^s$ of the quantity

$$(\epsilon^{px} - \epsilon^{qy})^n.$$

But it suffices to have called attention to processes which can give endless varieties of results, some of which may have useful applications.