FORMATION OF LARGE SACLE STRUCTURES IN THE UNIVERSE BY INVERSE CASCADE: COSMOSYNERGETICS

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ABSTRACT

It is suggested that inverse cascade that may occur in
the turbulent cosmic medium can result in the formation of
very large scale structures in the universe, upto the largest
scales like the Great Wall. Again clustering of galaxies on
all scales is interpreted to be due to these self-organisation
processes occurring in a turbulent medium, the largest struc-
tures being anisotropic and nearly two dimensional, the smal-
ler structures remaining isotropic. The observed fractal dis-
tribution of galaxies is also interpreted on this basis. The
direct proportionality between the flow velocity and the lin-
ear dimension of the structure may show a way out of the dil-
emma of missing matter.

INTRODUCTION

The origin of the observed hierarchy of large scale cos-
mic structures continues to be a challenging problem in con-
ventional cosmology. The observed distribution of galaxies
into very ordered large scale formations strongly suggests
the existence of well defined physical processed giving rise
to self organised structures. A recent survey by Geller and
Huchra of thousands of galaxies in a relatively narrow strip
of space shows that galaxies follow intricate networks of
arcs and segments with huge rounded cells. The conventional
scenarios in which scale invariant density fluctuations grew
in scale and amplitude with the expansion of the universe can-
not form such large structures on a Hubble time scale. The
peculiar velocities of many galaxies indicate that they are
being drawn towards enormous concentrations of matter with

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masses $\sim 10^{17}$ $M_\odot$. As shown by (Burstein et al.,1986) galaxies between the local supercluster and the Hydra Centaurus supercluster share same direction of motion showing peculiar velocities of several hundred kilometres per second, revealing presence of enormously massive structures $\sim 10^{18}$ $M_\odot$ powerful enough to draw several clusters towards it. More recently the Great Wall of galaxies Huchra and Geller (1989) has been identified over scales ~ 100 Mpc!

General observational indications are that the smaller scale structures like the globular clusters (containing the oldest stellar populations) formed first. Recent observations show that galaxies already existed at redshifts near $Z = 4$ thus giving rise to the problem of how the smaller scale structures interacted leading to the formation of large structures such as clusters and super clusters of galaxies. In the conventional warm or hot dark matter scenario (as also in the Zeldovich type pancake fragmentation) it appears that the largest scale structures formed first. Then one is faced with the problem of accounting for smaller scale structures even at $Z = 4$. The cold dark matter scenario has had some measure of success in forming the smaller structures first but its essential ingredient is a particle (i.e. the axion) evidence for which is becoming more and more meagre. The rejuvenation of cosmic turbulence in galaxy formation through energy cascading from larger to smaller scales has been due to Ozernoi and coworkers (1968,1971,1972). Cascading occurs for scales for which the hydrodynamic interaction time is smaller than cosmic expansion time. For an expanding fluid with expansion rate given by the Friedmann equation the minimum scale over which dissipation can occur $\ell_{min} \sim (Vt)^{\frac{1}{2}}$, obtained by equating time scales for dissipation (V) and expansion while the maximum length scale is given by maximum velocity of largest eddy, i.e. $\ell_{max} \sim V_{max} t$; $V_{max} \ll V_s$, V_s is the sonic speed. As a consequence of large Reynolds number and short hydrodynamic times, motions on comoving scales involving $\ell_{min} < \ell < \ell_{max}$ should rapidly acquire the Kolmogorov spectrum characteristic of ordinary turbulence in an incompressible viscous fluid. In an isolated turbulent medium, decay of large eddies into smaller eddies is a common occurrence. The converse, i.e. small eddies coalescing into large ones can happen in an energetically open system, i.e. so called inverse cascading where the physical mechanism may be fusion of small-scale vortices into larger-scale coherent structures, i.e. cluster-vortices. The cluster-vortices may be formed by an aggregation of "elementary" vortices (i.e. vortices with no cluster structure) to the cluster. Clusters of smaller sizes can in turn form through the aggregation of even smaller clusters. Such vortex clusters have been repeatedly observed in natural and numerical experiments. So here we suggest a definite physical mechanism for the large scale structure formation based on inverse

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cascading which quite naturally yields a bottom-up self simi-
lar hierarchical structure. In this picture, large helicity
fluctuations present in a turbulent medium play an important
role in inverse cascading process. The helicity is defined as
$\gamma = \vec{V}.(\vec{\nabla} \times \vec{V})$. For a 3-D hydrodynamic system in addition to
total energy, there is another invariant I which is defined as
$I = C \int [E(K)]^2 \, dK$ for a quasi-normal distribution of helicit-
ies. Using Kolmogorovic arguments, one finds $E(K) \propto K^{-5/3}$ and
$E \propto L^{2/3}$ from energy invariance and from I invariance, $E(K) K^{-1}$
and $E \propto \log K(t)/\ell$ where $L(t)$ is the largest length scale ex-
cited at time t. Levich and Tzvetkov (1985), Krishan(1980a,b),
Krishan and Mogilevskii (1990) and Bruckner et al. (1989).
Analogous to 2-D case (Hasegawa 1985) one expects that in 3-D
the invariant 'I' would cascade towards large spatial scales
and energy towards small spatial scales. The cascading of 'I'
towards large spatial scales essentially enhances the corre-
lation length of helicity fluctuations. We propose that this
regime of turbulence favours formation of galaxy clusters at
smaller scales. We can ask if there is an upper limit to the
size of these three-dimensional isotropic galaxy clusters(the
lower limit being determined by dissipation). A turbulent med-
ium is restricted in the vertical direction by gravity. The
largest dimension of fully 3-D structures is given by the ra-
tio $I/E^2 = L = L_z$ where L_z is the characteristic vertical sca-
le. Here we identify it with the size of clusters of galaxies
= 0.3 Mpc. When the correlation length of helicity fluctuat-
ions reaches the limit L_z, it can only grow in the horizontal
plane, so that the system becomes more and more anisotropic.
Thus the vertical component V_z becomes independent of (x,y,z)
and the horizontal components V_x, V_y independent of Z, the in-
variant I becomes $I \propto V_k^2 = KE(k) \propto L^{2/3}$, or $I(k) \propto k^{-5/3}$ where L is
the largest length scale in the horizontal plane. Thus I(K)
spectrum coincides with energy spectrum of 2-D turbulence
corresponding to inverse cascade. One expects an increasing
fraction of energy to be transferred to large spatial scales
as anisotropy increases and we consider this part of the
turbulence spectrum conducive to formation of superclus-
ters appearing predominantly on large horizontal scales
with above energy spectrum. The intermediate region where
the 3-D system is developing anisotropy the energy spec-
trum going as $E(K) \propto K^{-1}$, can be identified with the gap bet-
ween scales of clusters of galaxies and superclusters. The
growth of large structures in a highly anisotropic turbulence
can be interrupted by symmetry breaking caused by the corio-
lis force, the length scale L_c where the nonlinear Navier-
Stokes term becomes comparable with the coriolis force is
$L_c \sim V/\Omega$, where Ω is the angular velocity. Given sufficient
energy, structures of size L_c must form. At these large spa-
tial scales, the system simulates 2-D turbulence which conser-
ves entrophy and energy. The enstrophy then cascades towards
small scales with a power law spectrum given by $E(K) \propto K^{-3}$ and
$E \propto L^2$. There is a break in the spectrum as energy must cascade

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to large spatial scales as $L^{2/3}$ and to small scales as L^2. If we take L_c, L_{GCG} (size of giant clusters ~ 100 Mpc) and a random velocity of superclusters $\sim 10^4$ km/s one finds an angular velocity $\sim 3 \times 10^{-18}$ rad/sec. If the energy of the larger structures has been inverse cascaded from smaller structures, the energy density in the large scale should not exceed that in the small scale. From the energy spectrum $E \alpha L^2$, energy balance gives then:

$$E_{GCG} \leq \left| \frac{E(L_{SCG})}{L^2_{SCG} \tau_{GCG}} \right|^{2/3} L_{GCG} \leq E(L_{SCG})$$

where $[\quad]^{2/3}$ is the enstrophy injection rate.

For $L_{GCG} = 100$ Mpc, $E(L_{SCG}) \sim (10^4 \ (km/s)^2$ and $L_{SCG} \sim 3$ Mpc, one gets $\tau_{GCG} \simeq 10^{11}$ years. So structures of the size of a few hundred Mpc cannot be formed if the random velocity of a supercluster is less than 10^4 km/s. For a supercluster $L \sim 3$ Mpc and $E(L_{CG}) = (300 \ km/s)$, for random galactic cluster velocities we get $\tau \sim 3 \times 10^9$ years. The giant clusters are formed in a turbulent medium being stirred by random motion of superclusters. The eddy turnover time for the largest structure of say 100 Mpc with the associated fluid velocity of $\sim 10^4$ km/s is $\sim 10^{10}$ yrs, which is smaller than the Hubble time. This is also true for smaller scales. In this theory, the smaller structures are more or less isotropic whereas the larger ones are anisotropic and become nearly two dimensional at the largest scales forming giant sheets like the Great Wall. At the largest scales, $V \alpha L$. This inbuilt linear velocity-distance relation may account for flat rotation curves and velocities increasing with distance in giant clusters without the need to invoke unseen matter. Silk has shown that the effects of expansion causes little deviation in turbulence from incompressible case. We can also generalize the above results to fractal turbulence again considering an inverse cascade, i.e. clustering of vortices. These clusters join with one another into even larger clusters etc. (a hierarchical cascade). It is clear that the energy dissipated in a volume of characteristic size r on vortices of size r (roughly true for galaxies as galaxy size $\sim$ galaxy separation) must be proportional to number of elementary vortices in the fractal cluster vortex. Denoting this by N_d and the dissipated power by W_d then for the isotropic case we can write Monin and Yaglom (1975), Bershadskii (1989):

$$W_d \ \alpha \ r^d \int K^2 \ E(K) dK$$

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where d is the dimension of space in which the motion (here d ≈ 3). Assuming that the scale r^{-1} belongs to the Scaling range where $E \alpha K^{-\alpha}$ is satisfied, we obtain

$$W_d \; \alpha \; r^{d+\alpha-3}$$

For a Fractal cluster Smirnov (1976)

$$N_d \; \alpha \; \ell^D$$

Thus $\alpha = D+3-d$, so that for 3-dim-turbulence $\alpha = D$, i.e. scaling exponent in the spectral law $E \alpha K^{-\alpha}$ for 3-dim isotropic turbulence is equal to the fractal dimension of the cluster vortex correlation. For the fractal dimension of the clusters formed by inverse cascade one may use the value given by Kolb (1984) for hierarchical clustering. This applies to the situation, where both direct cascading and clustering occur simultaneously, i.e. D = 1.72, so that we obtain for three-dimensional isotropic turbulence, the fractal dimension of the correlation as D = 1.72, which is in good agreement with the value of ~ 1.8 generally used empirically for galaxy correlation. Fractal dimension, according to current experiment data, for inverse cascade $\sim 1.69 - 1.72$ Yaglom (1985). For the strictly Kolmogorov case in strictly three dimensions, D = 5/3 = 1.66 (as argued above $\alpha = D$ in the $E^{-\alpha}$ relation)not far from the above value. This shows that Kolmogorov Scaling can be valid for hierarchical galaxy cluster formation. In short we have for the first time the picture of inverse cascading process (with its many attractive features) for formation of a whole hierarchy of large scale cosmic structures.

NOTE ON INFLATION

In the above picture we did not take into account, the earliest epochs of the universe, which according to popular thinking is an exponential expansion of the scale factor as $R = R_o \exp(Ht)$. It has been argued that such an exponential expansion would considerably dilute all initial vorticity and magnetic fields if any. For instance with flux conservation assumed an initial magnetic field would be diluted by a factor of $\exp(2 Ht) \sim e^{60}$ for most injection models. However, this need not necessarily be so. There are very recent models where B decreasing as $1/r^2$ is averted by breaking conformal invariance of electromagnetism by considering non-minimal gravitational couplings of photon field to gravity such as $RA^2, RF^{\mu\nu}$ $F^{\mu\nu}$ etc. These terms in fact results in <u>production</u> of large scale magnetic fields during inflation, Turner and Widrow (1988) also De Sabbata and Sivaram (1988), for post inflation generation of magnetic field through vorticity in general relativity. Since magnetic fields can be coupled to differential rotation of ions and electrons in the plasma prior to recombination.

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This would also generate vorticity in addition to magnetic field. Moreover MHD turbulence aids inverse cascading. So inflationary expansion in the earliest phase need not be a drawback to this picture which gives a scale-invariant description independent of inflation. Since we have only acoustical dissipation of energy (viscosity), it would not couple to the photons of the microwave background (i.e. no thermal effects). So no large scale distortion of the microwave background is expected in this process. (The amount of vorticity needed on the largest scales is consistent with the anisotropy of the microwave background (since we are just translating large scale peculiar velocities with turbulent velocities).

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