Again $\sin a = \dfrac{\sin b \sin A}{\sin B}$ by I.

and $\cos a = \dfrac{\cos A}{\sin B}$ by II. Divide

$$\left.\begin{aligned} \therefore\quad \tan a &= \tan A \sin b \\ \text{so}\quad \tan b &= \tan B \sin a \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad 6.$$

Note on Napier's Rules.

By Professor JOHN JACK.

Denote the parts b A c B a of △ ABC (Fig. 9)
by 1 2 3 4 5

then the parts corresponding of the △ BEF, namely,

$$\tfrac{\pi}{2} - c,\quad B,\quad \tfrac{\pi}{2} - a,\quad \tfrac{\pi}{2} - b,\quad \tfrac{\pi}{2} - A$$

will be denoted by $\tfrac{\pi}{2} - 3,\ \ 4,\ \ \tfrac{\pi}{2} - 5,\ \ \tfrac{\pi}{2} - 1,\ \ \tfrac{\pi}{2} - 2$.

Now a third △ can similarly be derived from this second, a fourth from the third, and a fifth from the fourth. But when the process is applied to the fifth, the first △ is obtained. Hence only 5 △s can be obtained, which are the following :—

1	2	3	4	5
$\frac{\pi}{2} - 3$	4	$\frac{\pi}{2} - 5$	$\frac{\pi}{2} - 1$	$\frac{\pi}{2} - 2$
5	$\frac{\pi}{2} - 1$	2	3	$\frac{\pi}{2} - 4$
$\frac{\pi}{2} - 2$	3	4	$\frac{\pi}{2} - 5$	1
$\frac{\pi}{2} - 4$	$\frac{\pi}{2} - 5$	$\frac{\pi}{2} - 1$	2	$\frac{\pi}{2} - 3$
1	2	3	4	5

where the mid-column contains the hypotenuse, the two next to it contain the angles, and the extreme columns the sides of the several right-angled triangles.

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Now *assume* cos hypotenuse = product of cosines of sides
= product of cotangents of angles;

that is sine complement of hypotenuse
= product of cosines of sides
= product of tangents of complements of angles.

Now change the 2nd, 3rd, and 4th columns to complements and we have

1	$\frac{\pi}{2}-2$	$\frac{\pi}{2}-3$	$\frac{\pi}{2}-4$	5
$\frac{\pi}{2}-3$	$\frac{\pi}{2}-4$	5	1	$\frac{\pi}{2}-2$
5	1	$\frac{\pi}{2}-2$	$\frac{\pi}{2}-3$	$\frac{\pi}{2}-4$
$\frac{\pi}{2}-2$	$\frac{\pi}{2}-3$	$\frac{\pi}{2}-4$	5	1
$\frac{\pi}{2}-4$	5	1	$\frac{\pi}{2}-2$	$\frac{\pi}{2}-3$

where, taking any horizontal line,

sine of mid column = product of tangents of adjoining columns
= product of cosines of extreme columns;

and this proves completely Napier's rules, for each horizontal line contains Napier's parts in the same (cyclic) order.

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