COMPARISON OF LUNAR EPHEMERIDES (SALE AND ELP) WITH NUMERICAL INTEGRATION

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SALE and ELP, analytical theories of the main problem of the Moon, are developed by Henrard (1979) and Chapront-Touzé (1980), respectively. Both theories are compared with numerical integration over one year, which covers about 13 revolutions of the Moon's orbit. The root-mean-square residuals in the distance of SALE truncated at 10^{-5} arcsecond is about 10 cm for series truncated at 10^{-5} arcsecond and 1.2 cm for series truncated at 10^{-6} arcsecond. ELP is also compared with 20 years of numerical integration and the root-mean-square residuals in the distance is about 1.5 cm.

1. INTRODUCTION

Recently analytical solutions-SALE and ELP-of the main problem of the Moon have been constructed by Henrard (1979) and Chapront-Touzé (1980) respectively. Their theories are reviewed by Henrard in these proceedings. Chapront-Touzé and Henrard (1980) compared in detail amplitudes of periodic terms and secular motions of angular variables in their theories. The quadratic mean values of the differences between their theories are 200 cm in longitude, 45 cm in latitude, and 120 cm in distance. Comparison of an analytical solution with another solution obtained by a different method is a good check of a theory. However, it cannot tell which theory is accurate. A theory should be judged only by its agreement with observations. Recent new techniques, such as lunar laser ranging, have enough accuracy to detect the difference between SALE and ELP. However, it is quite difficult to remove from observations planetary perturbations, perturbations due to the figures of the Moon and the Earth, the physical librations of the Moon, and so on. Another way to test a theory is comparison with numerical integration. We discuss comparison of these theories with numerical integration.

O. Calame (ed.), High-Precision Earth Rotation and Earth-Moon Dynamics, 245–255.

2. REPRESENTATION OF SALE AND ELP

Both SALE and ELP are expressed in a trignometric series :

$$\lambda = \text{longitude} = \sum_i A_i \sin H_i$$

$$\beta = \text{latitude} = \sum_i B_i \sin H_i$$

$$p = \sin \text{parallax} = p_0 + \sum_i C_i \cos H_i$$

$$H_i = i_1 D + i_2 l' + i_3 l + i_4 F$$

where D=L=L', L=mean longitude of the Moon, L'=mean longitude of the Sun, l=mean anomaly of the Moon, l'=mean anomaly of the Sun and F=argument of perigee of the Moon. SALE2000 is truncated at 10^{-5} arcsecond. ELP2000 has two versions; one of them is truncated at 10^{-5} arcsecond and the other at 10^{-6} arcsecond. Numbers of terms of each component are listed in Table 1. Both theories give partial derivatives

Table 1. Numbers of terms in SALE and ELP

	SALE	ELP(10^{-5})	ELP(10^{-6})
longitude	1187	1023	1537
latitude	1026	918	1401
sine parallax	669	921	1393
total	2882	2862	4331

with respect to parameters included in their theories, by which we can make orbital improvements. They also give trigonometric series of the normalized distance Earth-Moon and the components of the unit vector pointing from the Earth to the Moon.

3. NUMERICAL INTEGRATION

Equation of motion we integrate are given by the following equation :

$$\frac{d^2 r}{dt^2} = \frac{-k^2(Me+Mm)}{r^3} r + k^2 M_s \left[- \frac{r' + \frac{M_m}{M_m+M_e} r}{\left| r' + \frac{M_m}{M_m+M_e} r \right|^3} + \frac{r' - \frac{M_e}{M_m+M_e} r}{\left| r' - \frac{M_e}{M_m+M_e} r \right|^3} \right]$$

The coordinates r of the Moon are geocentric and the coordinates r of the Sun are referred to the barycenter of the Earth and the Moon. In the main problem of the Moon, the motion of the Sun is assumed Keplerian. Initial position is calculated from the theory itself. Initial velocity is determined by numerical differentiation of the theory.

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We use a Adams-Moulton-Cowell multistep integrator, which is predictor-corrector type integrator and extensively used by Oesterwinter and Cohen (1972) in the orbital improvement of planets. The program of this integrator was written by Nicole Borderies (1975) of GRGS for CDC6400 and converted to the computer at TAO, UNIVAC 1100/80B. All numerical calculations are performed in double precision, which is about 18 digits in floating point calculation. In order to evaluate the precision of the integrator, we made the two following tests : 1) change of stepsize and 2) forward and backward integration. Figure 1 shows the difference between two integrations; integration with one-16th day stepsize and integration with one-8th day stepsize. The difference in radius vector and latitude are less than one millimeter, and the difference in longitude is about one millimeter. We integrated the motion in backward direction with use of the position and velocity at the last date and examined how the position at the initial epoch is close to the initial position. Figure 2 shows the difference between two orbits, forward and backward orbit. The differences in three components are also less than one centimeter. Therefore, we think the integrator we adopted is accurate enough to test an analytical theory of the Moon over one year in centimeter accuracy.

4. COMPARISON OF SALE AND ELP WITH NUMERICAL INTEGRATION

The equations of condition we adopted for differential orbital improvement are

$$\cos\beta\Delta\lambda = \cos\beta \sum_{i=1}^{3} \frac{\partial\lambda}{\partial\varepsilon_i} \Delta\varepsilon_i$$

$$\Delta\beta = \sum_{i=1}^{6} \frac{\partial\beta}{\partial\varepsilon_i} \Delta\varepsilon_i$$

The parameters ε_i in the differential orbital improvement for SALE are ε_1=n=sidereal mean motion of the Moon, ε_2=E=coefficient of sin l in the longitude, ε_3=Γ=coefficient of sin F in the latitude, ε_4=longitude, ε_5=longitude of perigee, ε_6=longitude of node of the Moon at the epoch. The Keplerian elements of the Sun and the mass ratio of the Moon and the Earth are fixed. The parameters for ELP are E/2, Γ/2, and the other 4 parameters are the same as for SALE. In these parameters, ε_4, ε_5 and ε_6 can be chosen freely from a theory. Initial numerical values of ε_4, ε_5, ε_6 are taken from the currently-used ephemerides of the Moon. The weight for λ is $\cos\beta$
In order to improve the six parameters, we use only residuals in longitude and latitude. One reason is that $\Delta\lambda$ and $\Delta\beta$ have enough information for orbital improvement. Another reason is that if orbital improvement with use of $\Delta\lambda$ and $\Delta\beta$ works well, the residuals in radius vector will also diminish as well as in longitude and latitude, which is an independant check of a theory and differential orbital improvement by least squares.

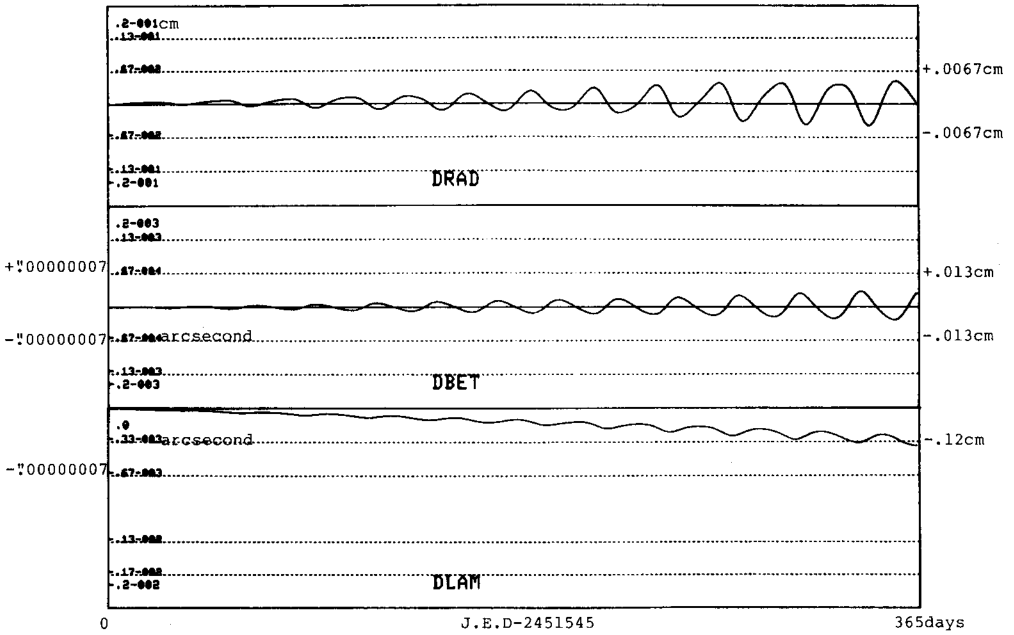

Figure 1. $(N.I.)_{h=1/16} - (N.I.)_{h=1/8}$
N.I.=numerical integration

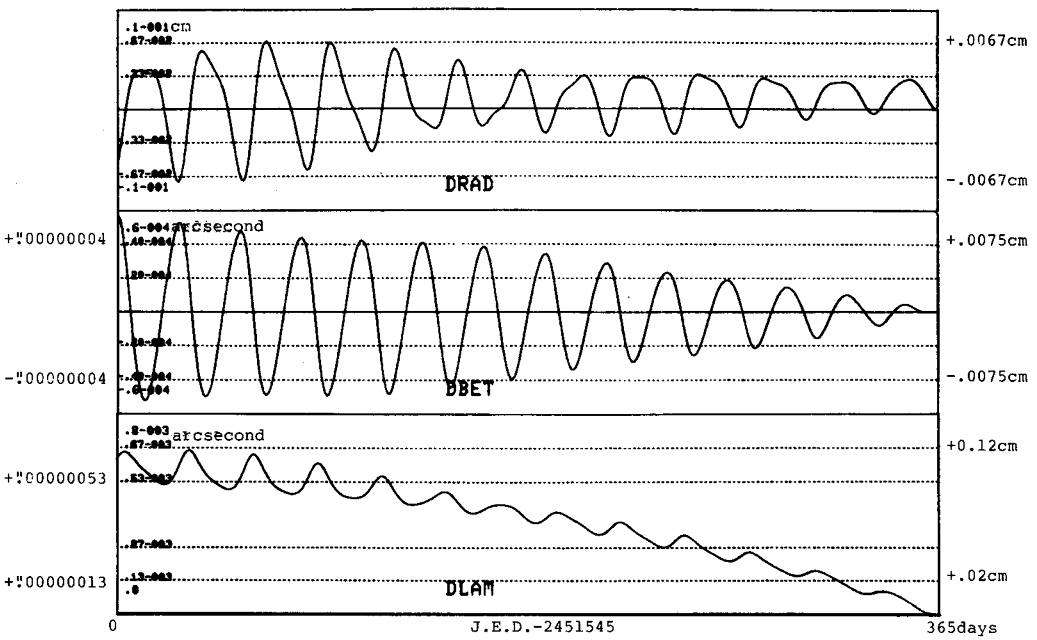

Figure 2. $(N.I.)_{forward} - (N.I.)_{backward}$
N.I.=numerical integration

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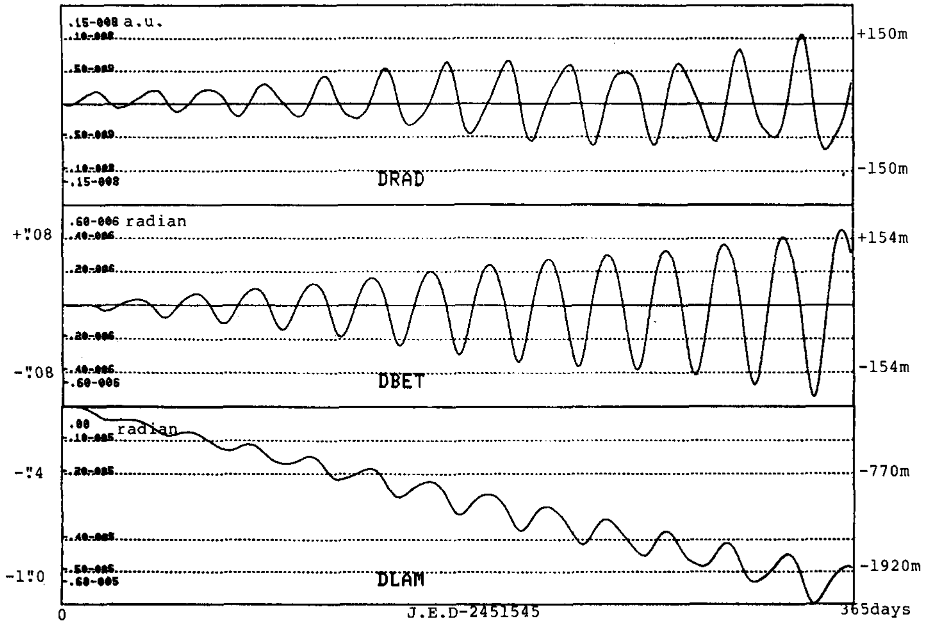

Figure 3. (Numerical Integration)-(SALE)$_0$
coarse residuals

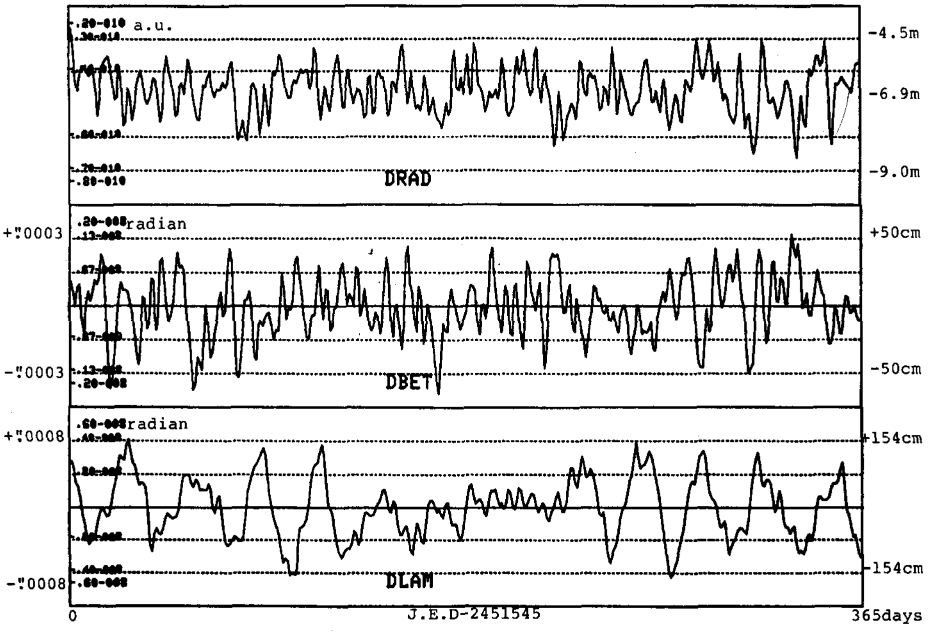

Figure 4. (Numerical Integration)-(SALE)$_1$
residuals after 1st orbital improvement

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Figure 3 shows raw residuals. The secular trend in longitude indicates that the mean motion of the longitude obtained by numerical integration does not agree with that of the theory. Figure 4 shows the residuals after the first orbital improvement. The improvement in residuals is quite remarkable. In calculation of SALE, we re-evaluated the amplitudes of periodic terms and mean motions of angular variables with use of improved n,E, Γ. We did at least three iterative differential orbital improvements.

The amplitude of residuals in longitude is about one meter, and its approximate period is one month. This systematic residual cannot be removed by adjustment of the eccentricity, because the parameter corresponding to the eccentricity is already one of the six parameters in the differential orbital improvement. The amplitude of residuals in latitude and radius vector are about 50 cm and 200 cm, respectively. It is clear that residuals in radius vector have a constant bias, which is about 7 m. It cannot be corrected by the adjustment of the mean motion of longitude, because 7 m displacement of the semi-major axis causes too large secular residuals in longitude. The root-mean-square residuals in longitude, latitude and distance are about 70 cm, 23 cm and 104 cm respectively.

We did another type of differential orbital improvement. The orbit calculated from a theory is considered observation and the orbit obtained from numerical integration is considered calculation. Six quantities we should improve are the initial position and the velocity. The equations of condition are :

$$\cos\beta\Delta\lambda=\cos\beta\left[\frac{\partial\lambda}{\partial r_0}\ \Delta r_0 + \frac{\partial\lambda}{\partial r_0}\ \Delta\dot{r}_0\right]$$

$$\Delta\beta = \frac{\partial\beta}{\partial r_0}\ \Delta r_0 + \frac{\partial\beta}{\partial r_0}\ \Delta\dot{r}_0$$

Partial derivatives with respect to the initial position and velocity are calculated by solving the variational equations of the original equations of motions. Figure 5 shows residuals after orbital improvements by adjustment of initial position and velocity. The residuals in Figure 4 and 5 are in good agreement except sign, because residuals in Figure 4 are (numerical integration) - (theory) and those in Figure 5 are (theory) - (numerical integration). This good agreement indicates that SALE gives precise partial derivatives with respect to the parameter included in the theory.

We now proceed to show the result of comparison of ELP truncated at 10^{-5} arcsecond (see Figure 6). Maximum residuals in three components are about 40 cm. There is no systematic trend in all three components and the residuals in the radius vector do not have a constant bias. The root-mean-square residuals in three components are 17 cm in longitude, 15 cm in latitude and 10 cm in radius vector. Figure 7 shows the residuals of ELP truncated at 10^{-6} arcsecond. The improvement of ELP (10^{-6}) over ELP (10^{-5}) is quite remarkable. The maximum residual in the radius vector is about 4 cm and its root-mean-square residual is about 1.2 cm.

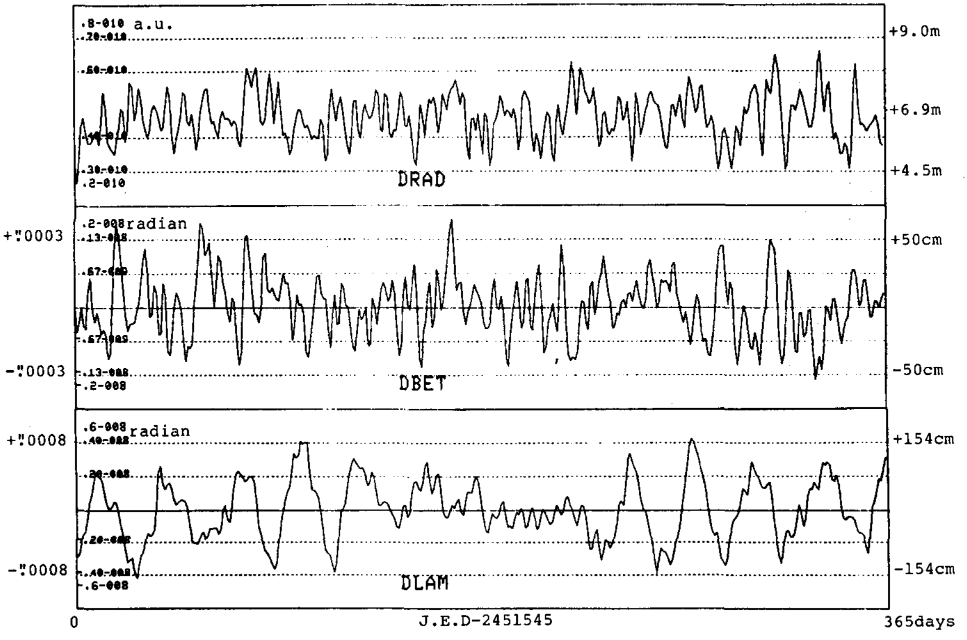

Figure 5. (SALE)-(Numerical Integration)$_2$ residuals after second orbital improvement.

We did make a differential orbital improvement for 20 years time span, which covers one revolution of the ascending node of the Moon. Figure 8 shows residuals over 20 years. The latitude and the distance have no appreciable long-periodic residuals. However, the longitude clearly has long-periodic residuals. Long-periodic terms of which period is longer than 1000 days in ELP are listed in Table 2.

Table 2. Long periodic terms in longitude of ELP(10^{-6})

argument D	l'	l	F	period (day)	amplitude	correction
0	0	2	-2	-1095	-1".372590	1".9x10^{-6}
1	1	-1	0	3233	1.077729	8.4
1	1	1	-2	-1656	-0.001566	4.1
2	2	-2	0	1616	-0.249895	-10.8
2	2	0	-2	-3397	-0.023489	12.6
3	3	-3	0	1078	0.000009	0.1
4	4	-2	2	3084	0.000002	-3.7

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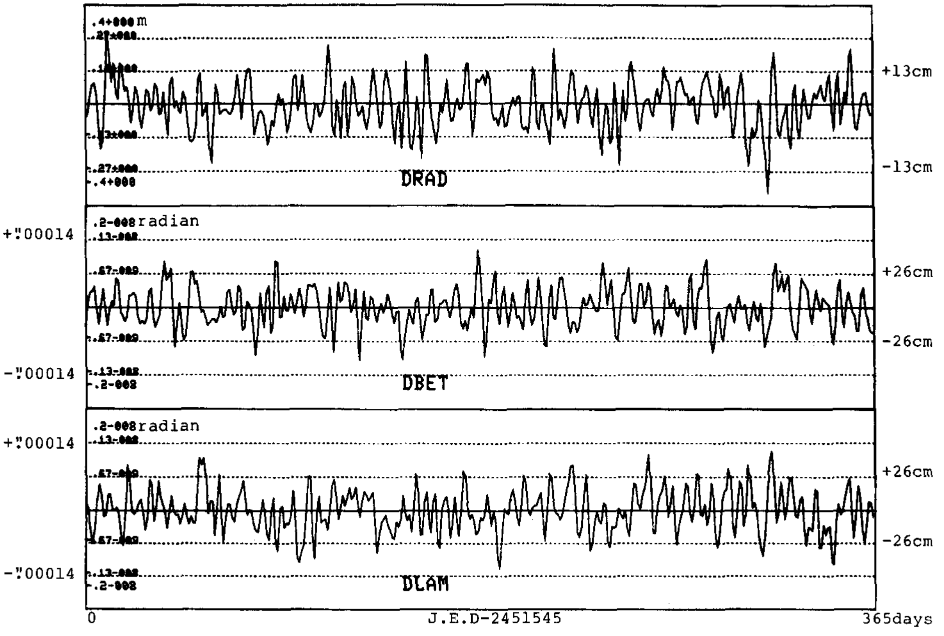

Figure 6. (Numerical Integration)-(ELP(10^{-5})) residuals after second orbital improvement

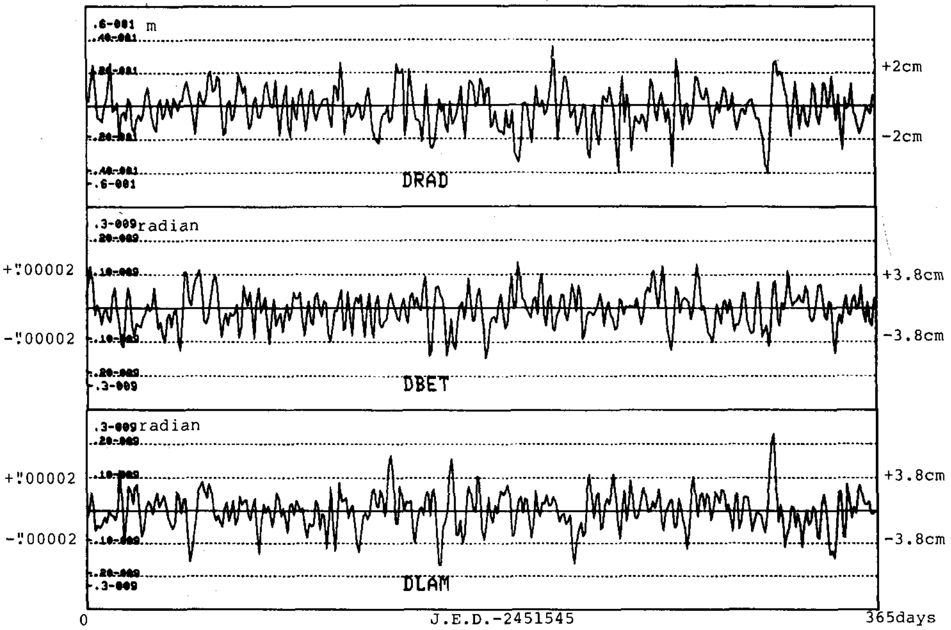

Figure 7. (Numerical Integration)-(ELP(10^{-6})) residuals after second orbital improvement

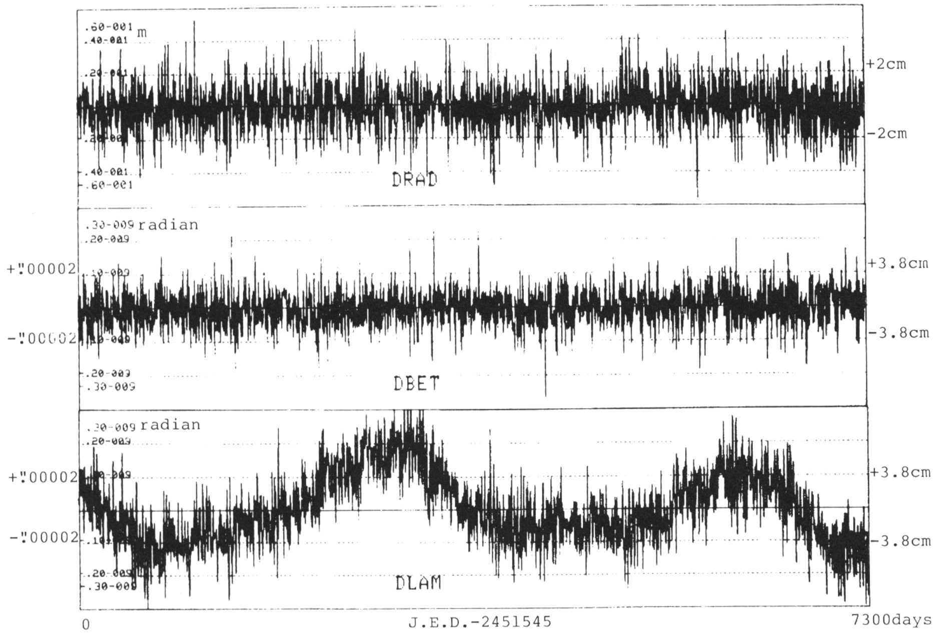

Figure 8. (Numerical Integration)-(ELP(10^{-6})) residuals after second orbital improvement

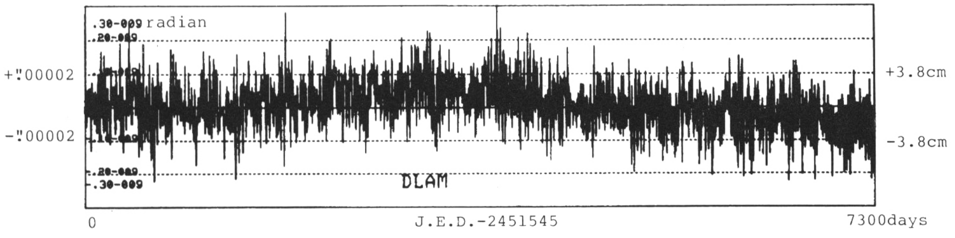

Figure 9.(N.I.)-(ELP(10^{-6})+long-periodic correction) N.I.=numerical integration

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We determined corrections to the amplitude of these terms, which are listed in the last column of Table 2. Figure 9 shows the residuals after applying the corrections. The residuals still have long-periodic component slightly. We think this is due to long-periodic terms which are not picked up by ELP.

Table 3 is a summary of our test of SALE and ELP. Now we have an analytical solution of the main problem of the Moon, of which accuracy is comparable to the accuracy of lunar laser ranging observations.

Table 3. Root-mean-square residuals (one year comparison)

	Longitude		latitude		distance
SALE(10^{-5})	0\".00037	70cm	0\".00012	23cm	104 cm
ELP (10^{-5})	0.00009	17	0.00008	15	10
ELP (10^{-6})	0.00001	2.2	0.00001	2	1.2

From now on we have to construct an analytical theory of planetary perturbations (direct and indirect) of which accuracy is compatible to the accuracy of main problem solution.

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