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ELECTIONS. ABSTRACTS OF PAPERS.

The Association for Symbolic Logic announces the following elections, each for a term of three years from January 1, 1943: as members of the Executive Committee, Professor Arnold Dresden of Swarthmore College and Professor Frederic B. Fitch of Yale University; as member of the Council, Dr. A. M. Turing of King's College, Cambridge.

The Council and the Executive Committee of the Association have made the following appointments and reappointments, effective January 1, 1943: as Editor of the Journal for a term of three years, Professor Ernest Nagel of Columbia University; as Secretary-Treasurer and as Managing Editor of the Journal, for a term of two years, Dr. Helen C. Brodie; as Vice President, for a term of one year, Professor Charles A. Baylis of Brown University.

The resignation is announced with regret of Dr. J. C. C. McKinsey as Secretary-Treasurer of the Association and Managing Editor of the Journal, and of Professor S. C. Kleene as Vice President of the Association. They are succeeded respectively by Miss Brodie and Professor Baylis.

The meeting of the Association which it was planned to hold at Yale University on December 28, 1942, was canceled because of war-time difficulties of transportation. We print below abstracts of the papers which were to have been presented at this meeting.

A. R. Turquette (introduced by J. Barkley Rosser). *m-valued propositional calculi.* For m-valued propositional calculi ($m \geqq 2$), we may choose from the m possible truth values a set consisting of the values $1, 2, \cdots, s$ $(1 \leqq s < m)$ which will be referred to as "designated." The remaining ones of the m possible truth values will be referred to as "undesignated." We give a generalized axiom set which is "deductively complete" and "consistent" in the sense that every formula is deducible which takes designated values exclusively, and only such formulas are deducible. In setting up such an axiom set we find it necessary to use certain functions denoted by $J_k(P)$ $(1 \leqq k \leqq m)$. We desire of any $J_k(P)$ that it take a designated value when and only when P is assigned the truth value k.

The existence of the functions $J_k(P)$ will be proved for Łukasiewicz-Tarski calculi based on the functions Cpq and Np. In this case it happens that $J_k(P)$ has the additional useful property that $J_k(P)$ takes the value 1 when P is assigned the truth value k, and that $J_k(P)$ takes the value m when P is assigned any truth value other than k.

The generalized set of axioms which we consider is general not only in allowing an arbitrary choice of s subject to $1 \leqq s < m$, but also in not requiring that our primitive truth functions be adequate to build up all possible truth functions. That is, our postulate set can be used for m-valued systems which are "functionally incomplete," such as the Łukasiewicz-Tarski calculus based on the functions Cpq and Np. Certain questions of functional completeness arising from this fact are answered.

A comparison of the m-valued systems for $m=2$ and $m>2$ is made.

Elizabeth Lane Beardsley (introduced by Frederic B. Fitch). *The relation between imperative and indicative sentences.*

The attempt to analyze the logical characteristics of imperatives has in several instances taken the form of attributing those characteristics to associated indicative sentences rather than to the imperatives themselves. Such a subordination of one mood to the other does violence to the unity and autonomy of imperatives as we use them in ordinary speech. The apparent necessity of choosing between a satisfactory treatment of the logical properties and a satisfactory treatment of their linguistic nature breaks down, however, if the relation between imperatives and indicatives is carefully examined.

Both the similarity of content and the difference in form and function between imperatives and indicatives can be accounted for if we regard sentences of the two moods as being different coordinate functions of common entities which are the referents of gerundive phrases. This conception requires a fundamental revision of the classification of the functions of language as "assertion" and "expression."

Although imperatives are independent of indicatives, no "calculus of imperatives" is possible. Logical characteristics of imperatives can be dealt with within the analysis proposed here. An indicative reformulation of imperatives appearing as consequents of conditional sentences is necessary; but this can be shown to be consistent with my treatment.

Carl G. Hempel. *A purely syntactical definition of confirmation.*

The concept of confirmation is used in the methodology of empirical science in statements to the effect that certain empirical findings confirm, or disconfirm, a given hypothesis. Although fundamental for a precise account of the so-called inductive procedure, of scientific testing and explanation, and of empiricism and operationalism, the concept as usually employed is unclear and involves serious logical difficulties. An attempt is therefore made to give a satisfactory definition of confirmation with respect to a model language. The language L chosen for this purpose has the logical structure of the lower functional calculus with individual constants and with predicate constants of all degrees. It is suggested that confirmation be construed as a relation between an observation report (represented by a class of sentences each of which is atomic or the denial of an atomic sentence) and a hypothesis (i.e. a sentence of any form whatsoever in L). Certain logical criteria of adequacy for any suggested definition of confirmation are proposed (among them the requirement that the following statements must be consequences of the definition: If a hypothesis follows from an observation report, then it is confirmed by it; no observation report confirms mutually incompatible hypotheses; if a report confirms every sentence of a given class, then it confirms also every consequence of that class.)

Then, a purely syntactical definition of confirmation is constructed which demonstrably satisfies all the logical criteria of adequacy. Disconfirmation and irrelevance are defined in terms of confirmation. The "intuitive" adequacy of the three concepts thus obtained is then examined, and finally certain generalizations of the given definitions are outlined.

Frederic B. Fitch. *Combinatory foundations for a consistent mathematical logic.*

The theory of combinators as developed by Curry and Rosser is known to be consistent, and so also is the system of logic of Whitehead and Russell's ***Principia mathematica*** even with the addition of the axiom of infinity, provided that the axiom of reducibility is omitted except as required for ordinary mathematical induction. The present paper brings together these two consistent systems into a single consistent system which is powerful enough to provide for the greater part of the differential and integral calculus and which possesses the main advantages of combinatory logic, such as the absence of variables and the presence of the well-known combinatory operators, I, B, T, W, Q, and Σ.

Nelson Goodman. *On the length of primitive ideas.*

The mere number of primitives comprising the extralogical basis of a system is clearly no reliable measure of economy. But we must not conclude that economy itself is unimportant; for the effort to construct any system at all is an effort toward economy.

What seems to be required is a measure of the relative simplicity of ideas; but this problem is so difficult that one can hardly hope for a comprehensive solution.

My proposal here is that by considering only a single aspect of simplicity—a highly generalized notion of the *length* of ideas—we can arrive at a reasonably satisfactory measure of logical economy. This measure will permit comparison, under uniform rules, not only of the lengths of sequences and the degrees of relations—of however diverse types—but also of these with each other and with the length of all ideas that are neither of sequences nor of relations.

The criterion proposed will thus be, in the technical sense, complete. Furthermore, it seems intuitively satisfactory in itself and in its results. While it presupposes the derivation of relation theory from class theory, it gives identical results whether this is done by the Wiener-Kuratowski method or by mine.

SAMUEL EILENBERG and SAUNDERS MACLANE. *Natural isomorphisms in the calculus of relations.*

Throughout abstract mathematics there are many one-one correspondences (isomorphisms) defined in a "natural" manner. The idea of "naturality" is subject to a precise logical analysis which suggests a new chapter in the calculus of relations (cf. Tarski's postulational development of the calculus). The ideas involved may be illustrated by some simple functions of sets, such as the cardinal product $A \times B$, defined on a suitable "collection" of sets. Specifically, a category $\mathfrak{A}$ is a collection of sets A *and* a collection of many-one correspondences α; each α is a correspondence $\alpha: A_1 \rightarrow A_2$ between two sets of $\mathfrak{A}$; given $\alpha_1: A_1 \rightarrow A_2$ and $\alpha_2: A_2 \rightarrow A_3$, the category also contains the relative product $\alpha_2\alpha_1$; $\mathfrak{A}$ contains the identity correspondence restricted to any set A in $\mathfrak{A}$.

The product $A \times B$ is a function on two categories $\mathfrak{A}$ and $\mathfrak{B}$. It determines to each pair of sets $A \epsilon \mathfrak{A}$ and $B \epsilon \mathfrak{B}$ a new set $A \times B$; furthermore it determines to each pair of correspondences $\alpha: A_1 \rightarrow A_2$ in $\mathfrak{A}$ and $\beta: B_1 \rightarrow B_2$ in $\mathfrak{B}$ a new correspondence $\alpha \times \beta: A_1 \times B_1 \rightarrow A_2 \times B_2$. This induced correspondence $\alpha \times \beta$ is defined, for any pair $(a_1, b_1) \epsilon A_1 \times B_1$ by setting $(\alpha \times \beta)(a_1, b_1) = (\alpha a_1, \beta b_1)$. Such "functors" or pairs of functions $A \times B$, $\alpha \times \beta$ occur throughout mathematics.

The commutative law asserts that $A \times B$ is equivalent to $B \times A$; the correspondence is $\tau(a, b) = (b, a)$. With this correspondence, $\tau(\alpha \times \beta) = (\beta \times \alpha)\tau$. This commutation condition, valid for many natural correspondences, is the basis of our general theory of "natural" equivalences.