Monads and Chaos: The Vitality of Leibniz's Philosophy

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Leibniz's work resembles its author. A. Robinet has called it "an intellectual storm." In its two hundred thousand pages of manuscript (most of it still unpublished) there are philosophical works that have nourished the thoughts of thinkers from generation to generation; mathematical texts of fundamental import (we all know of Leibniz as the founder – or rather co-founder – of infinitesimal calculus, but this triumph ought not to obscure his other contributions; for example, his being a precursor in the field of formal logic and the inventor of *analysis situs*); treatises on physics that have been relegated to obscurity by Newtonian mechanics but which may be in the process of being given new life because of the problems that the classic paradigm has encountered in the twentieth century; an impressive correspondence (approximately fifteen thousand letters, addressed to more than a thousand corespondents); significant contributions to fields as varied as theology, jurisprudence, history, politics and even technology (Leibniz did not scorn practical problems, and we find, alongside the most abstract of metaphysical systems, notes concerning the problem of Venice's sinking or the production of cognac).

In spite of its polymorphous character, Leibniz's work is profoundly unified. The various elements of his system reflect and communicate with one another.[1] His monadology can be discerned in infinitesimal calculus just as divine omniscience is part of his projective geometry. Each theme is developed and translated into a series of different languages: those of logic, geometry, physics, biology, metaphysics, theology, and others. The entire system, in some sense, can be seen in each of its parts. It should therefore come as no surprise that various scholars have attempted to describe the system based upon a single area of study (L. Couturat, B. Russel, and, to a certain extent, B. Mates, have done so on the basis of Leibniz's logic, and M. Guérolt based his description of the system on Leibniz's physics). This approach can work. Indeed, as Serres might say, it can work no matter which area is chosen. The Leibnizian world is full of mirrors. The laws of the metaphysical universe are projected upon the physical one; compounds are analogous to simple substances; and the world of nature reflects the world of grace.

Serres himself has illustrated some of these "translations," these correspondences among various parts of the system (for the most part – although not exclusively – he has used mathematical "models"[2] to carry out the translations).

One could therefore, following Serres, describe the network of correspondences that organizes the totality of the system. Another possibility would be to extend the method, that is, to seek translations of the system outside of Leibniz's writings themselves. Among recent examples of this approach are *La Nouvelle Alliance* (The New Alliance), in which I. Prigogine and I. Stengers attempt to translate principles of dynamics, and a work by G. Deleuze, *Le Pli, Leibniz et le baroque* (The Crease, Leibniz, and the Baroque), in which various aesthetic translations are offered. Moreover, it would not be impossible to defend the idea that Borges offers us a Leibnizian translation (that is, a translation of the means of tale-telling); and the same kind of argument could be made about the Leibnizian themes found in the work of Escher.[3]

This approach to Leibniz's system seems quite fertile to me. It also seems particularly helpful if we wish to highlight those features of the system that still remain vital. (Perhaps the vitality of a philosophical system can be measured by the extent to which it continues to generate new "translations."[4])

What I propose to do here is examine several translations of a specific theme: that of the relations between the finite and infinite, or, more precisely, of the envelopment of the finite by the infinite. This theme was a constant concern of Leibniz's, one that runs throughout his work and is particularly prominent in its most problematic sections. Leibniz liked to say that there were two labyrinths for the human mind. The first concerned the make-up of the continuous; the second, the nature of freedom. And these two labyrinths, Leibniz wrote, *"ex oedem infini fonte oriuntur."*

There are two particular translations, one concerning fractal geometry and the other chaos theory,[5] that we will take up in detail. These theories (which in fact are linked) have, over the last ten years, been an area of spirited and intense investigation. The importance, for philosophy, of the infatuation with these questions is the extent to which these inquiries bear witness to an evolution in the way scientists envisage the role of science. For example, we are seeing the emergence of a new way of understanding complexity: rather than ignoring or trying to reduce it, science is learning how to identify and describe it; rather than trying to force complexity into a mathematical framework to which it is unsuited, science is trying to create new frameworks, that is, mathematical tools that are better adapted to the facts.

I should say at the outset that it is in no way my intention – for this would only convict me of naiveté – to try to prove that there are golden veins of chaos embedded in the sterile mines of Leibnizian physics. Nor do I pretend to have discovered some brilliant prefiguration of contemporary theories in the texts of Leibniz. I do believe, however, that the general spirit of Leibniz's philosophy, and in particular his philosophy of nature, is startlingly similar to the spirit that drives the specialists of today's "new science," that is, the physics of chaos.[6] Although Leibniz's physical theories may in large measure be outdated (science ages much more rapidly than does philosophy), the vitality of the philosophical principles upon which he built his physics are not. Indeed these principles have often been applied and modified, remade and enriched, but also reconfirmed, by successive waves of scientific theory.

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At the heart of Leibniz's system is the idea that each particle of the universe, however small, contains (in a sense that it will be necessary to define) the infinite.

Leibniz's universe, it is well known, is made up of monads, that is, simple, undivided, and indivisible substances, veritable "atoms of nature" that have no natural beginning or end (without God's intervention). We must, however, guard against the idea that this is a form of atomism. In fact, nothing is farther from the spirit of Leibniz than those primordial particles of matter. (We must not be deceived by the word atom; the "atoms" that come up in monadology are incorporeal, *metaphysical* atoms.) Matter is infinitely divisible, and every particle of matter contains an infinity of substances, an infinity of monads.

> [A]ny quantity of matter is not only infinitely divisible, as the ancients realized, but in fact infinitely subdivisible, each particle divisible into

> further particles . . . if this were not the case, it would be impossible for each particle of matter to express the entire universe. Any quantity of matter can be conceived of as garden full of plants, a lake full of fish. But each branch of the plant, each limb of an animal, each drop of its vital fluids, is itself that garden, that lake.[7]

The world of Leibniz is full, and each region contains an infinite number of beings, "the smallest grain of sand contains a world with an infinite number of creatures." It is well known that Leibniz was extremely impressed by Leeuwenhoek's discovery of "animalcules," that is, those living organisms in numberless forms, colors, and sizes that teem in the smallest drop of water; for Leibniz, this discovery was decisive evidence in favor of his theory of the linkage between living organisms and the infinite (as well as his theory of the preformation and the interlocking of germs). Although these ideas were based on what are now outdated biological concepts, it would be wrong to condemn them to oblivion for this reason alone. Perhaps they are bearers of something deeper. Indeed, I would like to try to demonstrate this by elaborating several aspects of fractal geometry.

The first attempts at defining sets of fractal objects can be traced back to the end of the last century, although these in fact were isolated inquiries that only interested a handful of mathematicians. More recently, it was B. Mandelbrot who called attention to these forms; not only did he name and offer a theory to account for them, he also showed that many natural objects can be represented in mathematical terms with the help of these fractal forms (for example, a fern, a fluvial basin, a lung, Brownian motion, the arrangement of the galaxies, and, more broadly, any object sufficiently complex, sufficiently "irregular," sufficiently fragmented or ramified so as not to allow description within the structures of classical geometry). Until Mandelbrot, these kinds of objects were seen as little more than mathematical curiosities, "monsters" and chimeras, that only served to confirm the extraordinary creative power of the human spirit and to show that the wealth of mathematics far outstripped that of nature. But this, of course, was a rather presumptuous notion; in the meanwhile these "pathological" structures have been found in a variety of familiar objects. In fact, a new geometry of nature seems to be developing. And Mandelbrot, trying to come to terms with it, has quoted Pascal: "it is more likely that the imagination will tire of its conceptions than nature will tire of furnishing them."

Mandelbrot, in one of his works,[8] proposes an "intuitive" definition of the adjective "fractal":

> An object or geometrical figure can be said to be fractal when:
> 1) its parts have the same form or structure as the whole, except that they are on a different scale, and can be slightly distorted.
> 2) its form is either extremely irregular, extremely fragmented or discontinuous throughout, whatever the scale of analysis.
> 3) it contains "distinctive" elements, whose scales are extremely varied and cover a wide gamut.

Number 1 corresponds to what Mandelbrot calls the "scaling" character of fractals.

Koch's curve (the "snowflake" curve: see figure 1) constitutes a particularly simple and clear example of a fractal curve.

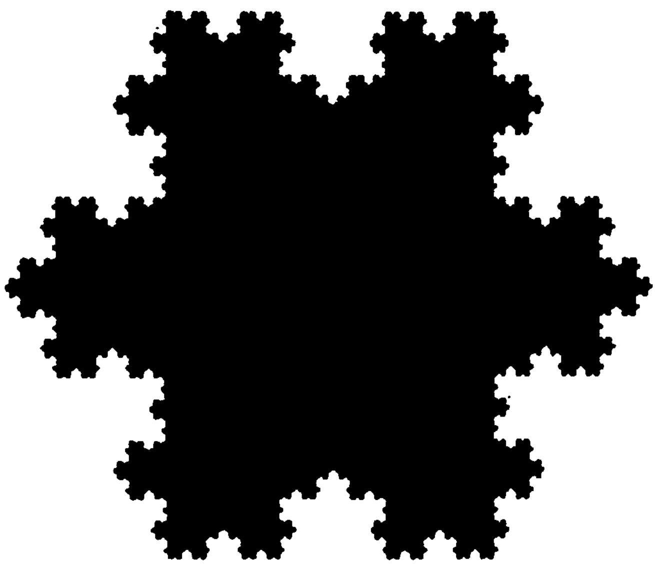

Figure 1

In order to construct this curve, we begin with an equilateral triangle (lying it along one of its lengths). Each side is divided into three equal segments, and each segment is replaced in the middle by two segments that form an equilateral triangle with the first segment. A "star of David" is thus created. Then this same operation is carried out on the twelve sides of the star: each side is divided (lengthwise, $\frac{a}{3}$, into three equal segments, which means replacing this middle segment, and so forth: see figure two). This same operation is carried out on all the lengths, $\frac{a}{9}$, $\frac{a}{27}$, etc. By this means we create a broken line, with numerous sides (one lengthwise) that can be indefinitely multiplied.

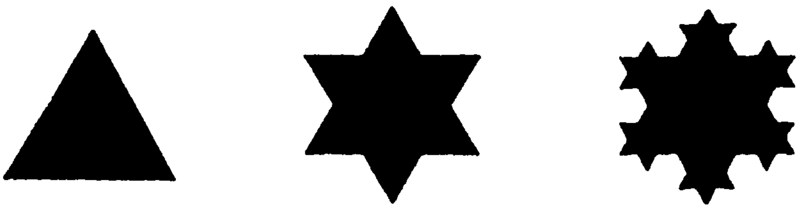

Figure 2

The curve itself does not extend infinitely, but the length between the two points is infinite. Moreover, it possesses the remarkable quality of being continuous yet, at almost every point, lacks a tangent, which is in large measure responsible for making of this form a "curiosity" and even a mathematical "monster."

Many mathematicians have been fascinated by this line (and ones like it) precisely because each particle of it – one could make the particle as small as one wanted – has the same form as the whole (and thus each part contains, in some sense, the whole). Here is the way Césaro describes his fascination:

> It is this similarity between the whole and its parts, even its most infinitesimal ones, that causes us to regard von Koch's curve as truly one of the most marvelous of all lines. If it were endowed with life, it could only be annihilated if it were suppressed at the very beginning, because afterwards it would be eternally reborn from the depths of its triangles, as life is in the universe.[9]

All these examples of theoretically infinite reductions, of parts mirroring the whole, cannot help but remind us of Leibniz, the more so since Mandelbrot himself traces the general idea of scaling back to a letter of des Bosses:[10]

> When I say that there is not a single part of matter that does not contain monads, I can illustrate this by the example of the human body or of any another animal body, of which any single part, solid or liquid, is

> itself made up of other animal and vegetable substances. And I believe this can be said about any part of any living thing, and this to infinity. . . Let me make this comparison: imagine a circle, and inscribe within it three circles, the largest possible, equal among themselves; and in each new circle, as well as in the interval that separates them, once again inscribe three equal circles, as large as possible; and imagine doing this *ad infinitum* (see figure 3).

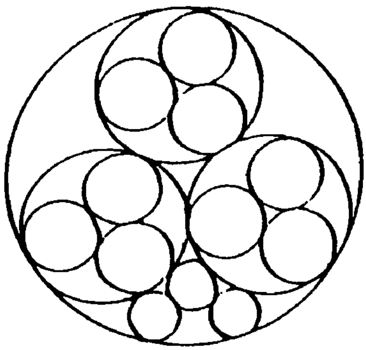

Figure 3

This same type of structure can be found at every level. Indeed it is identical everywhere, except for its size. Nature, Leibniz wrote, is at bottom uniform, although there is some variety between the greatest and the least, and in degrees of perfection. Arlequin, after returning from his travels, noted that it was the same everywhere else as here; the only difference was in size and in the degree of perfection.

It can therefore be said, in a certain sense, that every particle of matter envelops infinity. The same can be said of each one of these "metaphysical points" that Leibniz calls monads. Each monad is characterized (and distinguishes itself from every other monad) by its *perceptions*. But what each monad perceives is the entire universe.[11] Each monad expresses the whole universe, each monad is a living mirror of the universe, each monad is a *concentration* of the universe.

Although each monad expresses, perceives, and reproduces the same universe, each monad's expression of it is different; each monad expresses the universe according to its own point of view.

> And as the same town, looked at from various sides, appears quite different and even seems to take on multiple perspectives; even so, as a result of the infinite number of simple substances, it is as if there were so many different universes, which, nevertheless, are nothing but aspects of a single universe, according to the special point of view of each monad.[12]

Or this:

> for when God turns, so to speak, on all sides and in all fashions, the general system of phenomena that he finds it good to produce in order to manifest his glory . . . the result of each view of the universe, as seen from a different position, is a substance that expresses the universe in conformity to this view, provided God sees fit to render his thought active and to produce the substance.[13]

Each portion of matter contains an infinity of substances, each substance expresses the entire universe. Everything that exists, all that God calls into existence, envelops infinity. And it is this implication of nature's infinity that guarantees that nature will never be perfectly known. If there were atoms, Leibniz said, perfect knowledge of bodies would not be beyond the powers of finite beings. This action of the infinite, as both an index of reality and as a means of determining the limits of human knowledge, comes up in other parts of his system.

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Most notably, it comes up in the discussion of the relationship between freedom, contingency, and necessity. The question of this relationship is one that Leibniz comes back to again and again, and always with the same passion and anxiety. It is because the stakes of the question are large: what must be determined is whether there is a real place for contingency in Leibniz's system, or if his system, in the final analysis, is simply a variation on Spinoza's philosophy (an idea that Leibniz found intolerable). Although Leibniz never tired of affirming that those who accused him of leaving no place in his system for contingency were mistaken, and constantly repeated that there were two fundamentally different types of truth, which he never mistook, – i.e., necessary truths (whose opposite is contradictory) and contingent truths – the least that can be said is that he did not succeed in convincing everyone. This holds true for both his contemporary critics and those who have followed.[14] Let us examine this criticism in more detail.

Leibniz says that a true statement is one for which *praedicatum* *inest subjecto* (the predicate is in the subject), or for which the idea of the predicate is included in the idea of the subject. Therefore, if the statement "the sum of the angles of a triangle is equal to two right angles" is true, it is true because the idea of equality between two right angles and the angles of a triangle is included in the idea of a triangle. (And this statement can be proved by "separating" or "analyzing" the idea of a triangle until the idea of the equality between two right angles and the angles of a triangle explicitly emerges.) In the same way, if the proposition "Caesar crossed the Rubicon" is true, it is because the crossing of the Rubicon is in some way inherent in the idea of Caesar.

For each substance, for each individual, there is a corresponding "complete idea" or notion that, for all eternity, dwells in that "world of possible worlds" that Leibniz calls the understanding of God. God, in examining this idea, can see exactly what will happen to the corresponding substance if he decides to create this substance. Thus, in the idea of Caesar, God discerns that he will cross the Rubicon; in the idea of Alexander, that he will defeat Darius and Porus and that he will die in Babylon.[15] Each created substance expresses its idea and "reveals," over the course of its life, the set of predicates that this substance contains.

But if the predicate is included in the subject of each true statement, then are not all true statements necessarily true? And if everything that is to happen to me is, for all eternity, inscribed in my complete idea, if the order of my actions is but the development, the unfolding of a scenario that I did not write, then am I not deprived of all freedom?

How did Leibniz – or at the very least how he did he try to – resolve this problem? What made him believe that he had protected his system against the perils of determinism? How did he establish the distinction between necessary and contingent propositions?

Let us recall that Leibniz contrasted necessary (or eternal) truths, which are truths whose opposite implies contradiction, with contingent truths (he also speaks here factual truths, and of positive truths). Necessary truths (which include the truths of logic, geometry, metaphysics and, generally speaking, all truths that deal with essences) must be adhered to by God himself (contrary to the God of Descartes); God could not have made a circle whose radii are not all equal or a triangle the sum of whose angles is not equal to two right angles.[16] Contingent truths (which include physical laws and,

https://doi.org/10.1177/039219219304116107 Published online by Cambridge University Press

generally speaking, all truths that "relate to the existence of things and of time") depend on the "free mandates of God," who could have organized the world according to other physical laws (for instance, a world in which all objects fell at a uniform speed) or in which Adam was not a sinner.[17]

But if the nature of all true statements, whether necessary or contingent, is such that the predicate is included in the subject, must it not then be concluded that it is always possible to bring this inclusion to light, that it is always possible to "prove these statements," that is (and this is Leibniz's own definition), to reduce them to identity by substituting the definition for a term, as many times as is necessary? And if any true statement can ultimately be reduced to a statement of identity, then must it not be concluded that any true proposition, whether necessary or contingent, is such that its opposite implies contradiction?

Leibniz pondered this question a long time. When an answer at last came to him, it came from where he might least have expected to find it: in the midst of a mathematical inquiry concerning the nature of infinity.[18]

Although it is indeed the case that in any true statement the predicate is always in the subject, it is essential to distinguish between statements that can in fact be reduced to identity (i.e., necessary statements) and those for which *resolutio procedit in infinitum* (its resolution proceeds infinitely); that is, even if the predicate is in the subject, it is impossible for us to prove this inclusion (which is the case with contingent propositions). Necessary and contingent truths, Leibniz wrote elsewhere,[19] differ from each other in the same way as rational numbers differ from irrational numbers: necessary truths can be reduced to identity, just as commensurable quantities can be reduced to their common measure, while in the case of contingent truths, as in the case of irrational numbers, the reduction can proceed infinitely and will never finish.[20] God alone can apprehend the certainty and reason for contingent truths, since he can encompass the infinite in a single glance. God alone can know them *a priori*. We can only know them *a posteriori*. We can discover the properties of geometrical forms or that the sum of the sides of a triangle is equal to two right angles: God alone can see in the concept of Adam his fall, or in Alexander his death in Babylon. Although we can discover the truths that relate to essences, we can never have complete knowledge of things God brings into existence, since they are enveloped in the infinite. All factual truths, all truths relating to individuals, depend on an infinite series of causes; and God alone can see what is in this series. Since the process of analysis of contingent propositions is never-ending, such analysis can never have a complete and perfect resolution; however, the principle of this truth exists at all times, understood completely only by God – God who, alone, can trace this infinite series back with a single stroke of his spirit.[21] Although it is true, Leibniz wrote elsewhere, that the predicate is included in the subject of any true proposition, only in necessary propositions is it included expressly. In contingent propositions this inclusion is only implicit or "virtual."

This solution, offered in the form of a "distinction" between, on the one hand, perfect, total, divine knowledge, and on the other, imperfect, partial, human knowledge, is reminiscent of a distinction made by contemporary physicists between different types of deterministic systems.[22] Among these systems, there is one particular class that has been the subject of widespread interest over the last few years: this is the class known as chaotic systems, that is, systems that manifest a "special dependence and sensitivity to initial conditions." In brief, these are systems that, although their initial conditions may be as near to identical as imaginable, can evolve in completely different ways (and that therefore entail either a short- or long-term "unpredictability").

Let us define exactly what we mean. As is known, the aim of Newtonian mechanics is to define the temporal course of physical systems. If we know the state of a physical system at a particular instant (that is, if we know the "initial conditions"), Newton's equations can predict (at least in principal: these equations are not always (!) solvable) what the state of this system will be at any other particular instant in the future. Newtonian mechanics are based on a totally deterministic vision of the world: if one knows the state of the universe at some "initial" instant (that is to say, any instant, which can be arbitrarily considered the initial one), it can determine its state at any future time. The most famous formulation of this deterministic vision was given to us by Laplace: "If there existed a mind capable of knowing, at any given moment, all the forces that propel nature and the respective situations of the beings who constitute it, and if this mind was also powerful enough to submit these data to analysis, then it would be capable of subsuming under an identical formula the largest bodies and lightest atoms in the universe; nothing would be uncertain for this

https://doi.org/10.1177/039219219304116107 Published online by Cambridge University Press

mind, and both the past and future would be understood with the same perfect clarity."

Unlike the kind of mind described by Laplace, it is clear that humans never have perfect knowledge of the initial conditions of any system they might be studying. With systems lacking a chaotic character (which is the case with almost all systems encountered in physics textbooks: a pendulum, an harmonic oscillator, a free-falling body, etc.), a slight imprecision concerning initial conditions does not in general prevent a relatively precise prediction of the evolution of the system as such. Indeed, although imprecision at the start does of course have some repercussions on the ultimate accuracy of our prediction about the evolution of a system, this imprecision is not so great that it is multiplied over time, rendering any attempt at prediction futile. If, for example, in trying to determine the trajectory of a ball that I am about to throw from a certain spot, in a certain direction and at certain speed, I make an error of several millimeters in the determination of my initial position, and of several minutes in the angle in which I direct it, the inaccuracy of the trajectory that I will calculate – using Newton's equations – will be slight in comparison with the real trajectory; it will be "slight" to the same order of magnitude as the error made at the beginning.

By contrast, with chaotic systems, even the slightest imprecision in determining initial conditions will make prediction in either the short or long term impossible. With these systems, any slight change in initial conditions produces a change that grows exponentially over time (at least in an approximate sense, and at the beginning); as a consequence, two initially near-identical trajectories will diverge rapidly. Clearly, this fact profoundly limits any predictive capacity, since in virtually any description of initial conditions there is some imprecision; that is, although initial estimations are always "close" to the real initial conditions, they are not identical with them. On the basis of an analysis (and a very idealized one) of the way in which billiard balls work, we can gain a rapid appreciation of how this dependence and sensitivity to initial conditions affects prediction (see figure 4).[23]

It first must be established whether the angle of trajectory of two balls, after they bounce off a round obstacle (we will call the original trajectory α) will be increased as a result of the concussion. To simplify matters, let us assume that this resulting angle equals 2α.

Let us therefore take two trajectories that, initially, form an angle

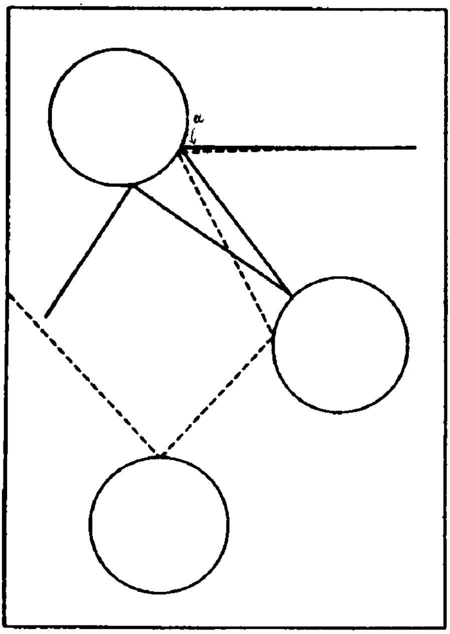

Figure 4

α (one is represented with a straight line, the other with a dotted line: see figure 4). After the first concussion against a round obstacle, their trajectories diverge, forming angle 2α. After the second concussion, they form angle 4α, then 8α, and so on. After each concussion, the angle is doubled. If we then assume that there is one concussion per second, the angle between the trajectories grows at a rate of e^t (that is, grows exponentially over time). Equally, the distance separating the two balls grows exponentially with their trajectories. Thus, if we assume an initial distance of a micron (a thousandth of a millimeter), we will find that after fifteen seconds the distance has grown to the order of several centimeters, which means that the two trajectories are already fundamentally dissimilar: one ball will bang against an obstacle while the other will miss it completely (from this point onward, we can no longer even speak of an exponential growth of the distance separating the two balls, since their trajectory has ceased to have anything in common).[24]

With chaotic systems, the least imprecision, the smallest over-looked decimal, has catastrophic effects on the accuracy of the prediction. Such a system, therefore, can only be said to be determinist for someone capable of knowing all the initial conditions with absolute precision, that is, someone able to see *uno untuitu* the infinite totality of all the decimals. Although we speak of determinist chaos, in truth it is a determinism of which we do not, and will never have, knowledge.[25]

As with Leibniz, we have here a determinism that only has meaning for God. And, inversely, the distinction between chaotic and non-chaotic systems, like the distinction between contingent and necessary truths in Leibniz's system, seems to be important, and indeed to have meaning, only for us.

It should also be noted that the impossibility of predicting the behavior of certain physical systems is directly related to the fact that only a part of the universe is being studied, that is, that the field of investigation is more or less artificially limited. The study of any physical phenomenon must begin with a definition of what is to be studied. Certain parameters must be taken into account and others disregarded. Obviously, the state of each particle in the universe cannot be taken into account – cannot be considered the "initial conditions" – in the description of a system. A physicist about to embark on an experiment in his laboratory generally assumes that the way a leaf falls on another planet in another galaxy will no have no significant impact on the validity of his results. Such a system is defined as an "isolated" system (obviously there is no such thing as a perfectly isolated system, but it is sometimes possible to isolate it sufficiently so that a satisfactory description of the evolution of the system can result).

Yet, as we have seen, the least imprecision in the determination of the initial conditions of a chaotic system makes any long-term prediction about the system impossible. Therefore, with chaotic systems nothing can be disregarded. Any long-term prediction requires that the state of each particle of the universe be taken into account. We cannot, for the purposes of description, cut out and separate one part of the universe from the rest. The beating of a butterfly's wing in Brazil can cause a hurricane in Mexico two weeks later.[26] In order to obtain a satisfactory description of the long-term development of a chaotic system, we would be required constantly to expand the system until it encompassed the entire universe.

All this is once again reminiscent of one of Leibniz's notions. We all know how attentive Leibniz was to the fact that each part of the universe was linked to all others, and that any event echoed throughout the entire system. "Each thing is linked with all others and is affected by them"; everything is connected, and there is no element so absolute or detached from the rest that a competent analysis will not show that it is connected to other things, indeed to *all* other things.

> The universe is a kind of fluid, a single piece, and, like a limitless ocean, all its movements are both preserved and endlessly propagated within it, although imperceptibly, like the circles . . . that are visibly propagated by a stone tossed into water. Although these circles spread out and eventually disappear, their affect continues to be felt and extends infinitely.[27]

Moreover, according to Leibniz, the truths of existence can be neither proven nor predicted *a priori* because each substance is linked to all others, i.e., each envelops the entire world. As Deleuze has written, "the virtual nature of all statements concerning existent things means only that nothing can be included in an existent thing without the entire world being included as well."[28] The free mandates of God, on which contingent truths depend, take into account all the other events of the universe. These laws even take into account all the events of all possible worlds; that is, if God decides to bring the sinner Adam into existence, if he decides to create a world in which Adam will sin, it can only occur after his having envisaged all substances and all the events of all possible worlds in all their details, in order to determine (to calculate) which, among these worlds, is best and therefore worthy of being created.

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Leibniz was not only extremely sensitive to nature's infinite complexity, its infinite variety, but also to the fact that human science would never succeed in exhausting nature's wealth of secrets and that it was futile to expect that we would one day be the masters and possessors of this complexity. Leibniz, whose philosophy of nature was in large measure conceived in opposition to Descartes' mechanistic one, criticized the mechanistic model in terms reminiscent of those leveled against "classical science" by I. Prigogine and I. Stengers. This classical science, they write, "denies complexity and becoming in the name of an eternal and knowable world governed by a limited number of simple and immutable laws"; this science, which can only conceive of nature as being passive and inert, incapable of the slightest novelty, pretends to bring to light, behind the infinite complexity of natural phenomena and their never-ending metamorphoses, a simple and always identical reality.

Leibniz openly faulted Descartes for reducing bodies to their length and denying them all spontaneity or principle of action. Furthermore, he believed that the mathematical concepts on which Descartes hoped to rely, in order to reduce his description of the physical world, could never adequately account for the infinite variety of nature.[29] For Leibniz, it was clear that the aim of thought was not to discover some simple reality behind the apparent complexity of phenomena. Indeed, the real is always more complex than appearances.[30]

Although I do not believe that the Leibnizian corpus contains any exact anticipation of developments in contemporary science, it does seem to me that the spirit animating the labors of some of today's scientists is similar to Leibniz's; that is, they approach nature with the aim of describing (not reducing) its diversity, complexity, and "spontaneity."

Translated from the French by Thomas Epstein.

Notes

1. M. Serres, in *Le système de Leibniz et ses modèles mathématiques* (Paris, P.U.F., 1982), has convincingly demonstrated this communication among the various parts of Leibniz's system. Moreover, the use of the term "intercommunication" [*entr'expression*], which can be applied in general to communication among monads, is in itself significant. One of Serres's theses is that Leibniz's system contains structures similar to the world it describes.

2. It is crucial to differentiate here between this enterprise and those that we have just outlined above. We are not speaking of deducing, explaining, or accounting for Leibniz's philosophy on the basis of his mathematics; mathematics has no particular "priority" within his overall system. Rather it is a matter of "elaborating a system by giving the description of a given area the status of an index value" (hence one can as easily have chosen another area, since Leibniz's system is intrinsically hostile to any kind of linear formulation). Serres's particular choice, therefore, is not based on priority but because, as he writes, "the mathematical art is more transparent, more expressive than other possible indexes." "The advantage of the mathematical art is only heuristic, or pedagogical." It is also because this area is itself constituted into a system, that it is "systematized, as is the whole."

3. Douglas Hofstadter's book *Gödel, Escher, and Bach* could quite easily have been expanded into *Gödel, Escher, Bach and Leibniz*.

4. What I propose to do here in regard to Leibniz's philosophy can be done, in my opinion, with any great philosophy. There is no such thing as a great but outdated philosophy.

5. There would no reason to take up the "translation" of the sphere that comes immediately to mind: infinitesimal calculus. This is because to do so would be no more than to repeat the work of Serres.

6. The question of whether one can legitimately speak here of a "new science" is a hotly debated point. Most of those who work in the field assert that it is a new science and that we are in fact witnessing a change of paradigm. Others are often more circumspect, even if they do not go as far as R. Thom, who has argued that it would be best to stop dreaming about a "new" science, since "it wouldn't be long before this 'new' science joined *nouvelle cuisine*, the 'new right,' the 'new philosophy,' and others, in the common grave of short-lived novelties."

7. *Monadologie*, § 65 and 67. This passage is reminiscent of the famous text in which Pascal describes the two infinities. In fact, Leibniz later recopied and annotated this text, seeing in it "a way into my system."

8. Mandelbrot, B., *Les Objets fractals*, Paris, Flammarion, 1989, p. 154.

9. Quoted by Mandelbrot in *Les Objets Fractals*. The study of curved lines of this type has led mathematicians to acknowledge that dimensionality is not an exclusive notion, and that it is indeed necessary to broaden it, that is, to introduce several different dimensions. Thus – notably – the "fractal dimension" can be defined as that quality possessed by an object that is not necessarily a whole number and that allows us to quantify the degree of irregularity and fragmentation of it as a whole (in the case of Koch's curve, it can be shown that this dimension is equal to $\frac{\log 4}{\log 3} = 1.2618$). The mathematical definition of the fractal thus causes this "fractal dimension" to arise.

10. Letter of 11 March, 1706, translated by Chr. Frémont (*L'Etre et la relation*, Paris, Vrin, 1981, pp. 83–84). This letter was quoted by Mandelbrot during a conference on philosophy and mathematics at the École normale supérieure. The text of this conference has been published in *Penser les mathématiques* (Le Seuil, 1982). Mandelbrot often quotes Leibniz and is of the opinion that Leibniz's thought is the basis for many later developments in the fields of mathematics and physics. He sometimes even has cause to speak of his own *Leibnizmania*.

11. Leibniz defines these perceptions as "representations of the composite, or of that which is outside the simple." That which is outside the simple is the whole universe. These "representations" are purely internal. They are representations of what is outside, but they "do not arrive from outside." Monads do not have windows through which something might enter or leave. The monad derives all its representations from within its own depths. Each substance evolves according to an inner law, each develops the set of its predicates without any form of interaction with different substances. And if there is some correspondence among the transformations of the different substances, it is not because they act upon on another in any way, but rather because, at the moment of creation, God arranged things so that it would be this way for all time.

12. *Monadologie*, § 57.

13. *Discours de Métaphysique*, § 14.

14. Leibniz defines a necessary proposition (a necessarily true proposition) as a proposition whose opposite is contradictory, while a contingent proposition is one that is not necessary. Obviously, one is tempted to apply the very Leibnizian notion of a possible world in order to distinguish between these two types of propositions. B. Mates (*The Philosophy of Leibniz*, Oxford University Press, 1985), who has tried it, affirms that one can say that necessary propositions are those that are true in all possible worlds, while contingent propositions are only true in certain possible worlds (of which ours is one). While acknowledging that Leibniz himself never gave this explicit definition, he writes that it has "always [been] visible in the background."

15. In fact, God reads in each substance not only its past and future but "the entire order of things in the universe." This is because everything harmonizes, and because each monad envelops, in a certain sense, the entire universe. Even more importantly, each state of each substance contains the entire past and future of the

https://doi.org/10.1177/039219219304116107 Published online by Cambridge University Press

universe. If Laplace's devil can determine the past and future of the entire universe by knowing, at any given moment, the state of all the beings who make up the universe, Leibniz's God can read the history of the entire universe in the instantaneous state of a single substance.

16. The least that can be said about this is that Leibniz (no more than Descartes, Pascal, or any other seventeenth-century thinker) did not understand the conventional nature of mathematical axioms. Rather, he believed that these axioms were reducible to statements of identity, and that the task of the mathematician was to bring this reducibility to light, in effect to "prove" the axioms.

17. In truth, the freedom of God is quite limited. Because he is wise, God can only create the best of all possible worlds. God's choice is always determined by what is best, and this holds true for even the smallest details. There is always a reason for the way something is. One must therefore avoid defining the contingent as "that which happens without reason" (a definition that Leibniz himself called "contradictory"). Contingent truths themselves are based on a certain kind of necessity, i.e., "moral necessity, which is the choice made by a wise man worthy of his wisdom."

18. "De libertate" in Foucher de Careil (ed.), *Nouvelles Lettres et Opuscules inédits de Leibniz*, Paris, 1857, pp. 179–180. The solution offered by Leibniz in this text (and which can be found in several other texts) is not the only one he proposed. Those who have commented on this text are not in agreement on the value that should be attached to it.

19. "Specimen inventorum" in C.I. Gerhardt (ed.), *Die Philosophischen Schriften von G.W. Leibniz*, Berlin, 1875–1890, vol. 7, p. 309.

20. Leibniz alludes to the decomposition of a real number into a continuous fraction.

If x is a positive real number, one can assert $x = q + \frac{1}{x_1}$, $x_1 = q_1 + \frac{1}{x_2}$, $x_2 = q_2 + \frac{1}{x_3} \dots$ where $q_1, q_2, q_3, \dots$ are the largest whole numbers contained in $x, x_1, x_2, \dots$ respectively, and therefore write

$$x = q + \cfrac{1}{q_1 + \cfrac{1}{q_2 + \cfrac{1}{q_3 + \dots}}}$$

If x is a rational number $\frac{a}{b}$ (where a and b are whole numbers), the process is finite, i.e, there is an n for which $x_n = q_n$ and the procedure is equivalent to Euclid's algorithm for a division of the type a by b. The last q_i that is not zero gives the "common measure" to a and b. If x is an irrational number the process is never-ending:

(x is the limit of the series $q, q + \frac{1}{q_1}, q + \cfrac{1}{q_1 + \cfrac{1}{q_2}}, q + \cfrac{1}{q_1 + \cfrac{1}{q_2 + \cfrac{1}{q_3}}}, \dots$)

21. Grun, G., *Leibniz. Textes inédits d'après les manuscrits de la Bibliothèque provinciale de Hanovre*, T. I, Paris, P.U.F., 1948.

22. I. Prigogine and I. Stengers, in *La Querelle du déterminisme* (Paris, Gallimard, 1990, p. 250), make explicit reference to Leibniz in this regard.

23. The following model and analysis are based largely on the work of D. Ruelle, in *Hasard et Chaos*, Paris, Éditions Odile Jacob, 1991, p. 56.

24. Obviously, the exponential growth could not continue: after thirty seconds the distance separating the two balls would already have grown to more than a kilometer.

25. We should guard against thinking that the idea of a chaotic system is particularly novel or exceptional. On the contrary, many systems studied by physics exhibit this same dependence and sensitivity to initial conditions. It would not be without

interest to try to determine why it took such a long time to recognize the importance of these systems and why it is just now that we have begun to take them seriously.

26. This is the source of the pretty term "the butterfly effect" that describes this phenomenon. M. Berry has carried out a series of calculations that show the importance of this phenomenon. Thus, for example, it can be shown that if we take two molecules of oxygen at normal pressure and temperature, and subject one of them to the attraction of an electron at a distance of ten to the tenth power light years away, their trajectories (everything else being equal) will diverge completely (will cease to have anything in common) after only fifty collisions.

27. "À l'Électirce Sophie" in C.I. Gerhardt (ed.), *Phil. Schriften*, vol. 7, p. 567. Serres has quite accurately pointed out that alongside Leibniz's studies of falling weights and banging billiard balls (which are often the only elements of his physics that are still remembered), Leibniz developed an entire "physics of propagations," that is, a series of studies devoted to phenomena of diffusion and transmission, to problems of elasticity, acoustics, the mechanics of fluids, and other subjects.

28. Deleuze, G., *Le Pli, Leibniz, et le baroque*, Paris, Éditions de Minuit, 1988, p. 69.

29. Identical points exist only in the imaginary space of geometry; identical instants exist only in Cartesian mechanics. Absolute uniformity and absence of variety exist only in abstractions. I believe that Y. Beleval was correct in asserting a contrast between a kind of Cartesian Platonism and a Leibnizian Aristotelianism. Descartes, like Plato, had a tendency to believe that mathematical entities constituted a higher, purified reality, while Leibniz, like Aristotle, saw only abstraction there.

30. To this we can add that the aim of Leibniz is to conceive of movement as it occurs, and not, as with Descartes, ready-made. Also, as Beleval has pointed out, Leibniz develops, as opposed to Descartes, a philosophy of time (Cartesian philosophy is of eternity) and of becoming, of the description of a world in the process of organization rather than of perpetuation. It seems to me that the authors of *La Nouvelle Alliance* paid relatively little attention to this aspect of Leibniz (which is analogous to their own way of thinking), preferring instead to emphasize what separates him from them (that is, the principle of reason and the affirmation of the equivalence of the total cause with the total effect, which condemns the world to an eternal repetition).