On the gyrotropic magnetohydrodynamic turbulence

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1. The reflective non-symmetric (gyrotropic) stochastic magnetic hydrodynamics (MHD) turbulence is investigated by a quantum-field (QF) theory method including a renormalization group (RG), that has been successfully applied to the fully developed HD turbulence [1,2] and to the ordinary (reflective symmetric) MHD turbulence [3].

2. The study of the behaviour of various correlation functions in the inertial (Kolmogorov) interval is the main aim of our research. In terms of QF theory formalism the correlation functions are usually functional averages with the statistical weight $expS$, where S is the action of the QF MHD. In ref.[3] a multiplicative renormalizability of the QF MHD has been proved. It means that all ultraviolet divergences in the diagrams of pertubation theory for correlation functions of velocity and magnetic field has been eliminated. The RG analysis in the first (one-loop) approximation leads to two infrared critical regimes, where the correlation functions have scaling behaviour.

3. In the gyrotropic quantum-field MHD additional ultraviolet divergences occure in the correlation functions, namely in the correlation (response) functions $< \mathbf{v}'\mathbf{b} >$, $< \mathbf{b}'\mathbf{v} >$ on the corresponding initial QF theory with an action

$$
\begin{aligned}
S &= \frac{\mathbf{v}'D^{\mathbf{vv}}\mathbf{v}'}{2} + \frac{\mathbf{b}'D^{\mathbf{bb}}\mathbf{b}'}{2} + \frac{\mathbf{v}'D^{\mathbf{vb}}\mathbf{b}'}{2} + \frac{\mathbf{b}'D^{\mathbf{bv}}\mathbf{v}'}{2} + \mathbf{v}'[-\partial_t\mathbf{v} + \nu\Delta\mathbf{v} \\
&\quad -(\mathbf{v}\nabla)\mathbf{v} + (\mathbf{b}\nabla)\mathbf{b}] + \mathbf{b}'[-\partial_t\mathbf{b} + u\nu\Delta\mathbf{b} - (\mathbf{v}\nabla)\mathbf{b} + (\mathbf{b}\nabla)\mathbf{v}]
\end{aligned}
$$

Here $\mathbf{v}', \mathbf{b}'$ are some auxiliary fields, $D = < \mathbf{FF} >$ is 2×2 matrix of the noise correlators of the external random forces $\mathbf{F}$. They have Gaussian distributions and simulate the stochasticity of the problem - the interaction of velocity pulsations and magnetic fluctuations with mean flows. The magnetic field is measured in Alfvén velocity units, u is the inverse magnetic Prandtl number (PN).

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As a result new terms $w_1\nu\Delta\mathbf{b}$ and $w_2\nu\Delta\mathbf{v}$ must be put into the initial MHD stochastic equations to eliminate the mentioned additional ultraviolet divergences:

$$-\partial_t\mathbf{v} + \nu\Delta\mathbf{v} + w_1\nu\Delta\mathbf{b} - (\nabla\mathbf{v})\mathbf{v} + (\nabla\mathbf{b})\mathbf{b} - \nabla p = \mathbf{F_v}, \qquad (1)$$

$$-\partial_t\mathbf{b} + u\nu\Delta\mathbf{b} + w_2\nu\Delta\mathbf{v} - (\nabla\mathbf{v})\mathbf{b} + (\nabla\mathbf{b})\mathbf{v} = \mathbf{F_b}, \qquad (2)$$

where w_1, w_2 are the new inverse gyrotropic PN. The new terms in Eqs. (1), (2) correspond to:
⋆ the generalization of momentum density flux tensor Π_{ij} in the equation $\partial_t v_i + \partial_j \Pi_{ij} = 0$, which leads to the Navier-Stokes equation, it means to add the pseudotensor $w_1\nu(\nabla_i b_j + \nabla_j b_i)$ to Π_{ij};
⋆ the generalization of the Ohm law in a moving fluid, $\mathbf{j} = \sigma(\mathbf{E} + (1/c)[\mathbf{vb}])$, which leads to the dissipation with the magnetic viscosity $\eta = c^2/4\pi\sigma = u\nu$, it means to extension of $\mathbf{j}$ by the additional term $-(\eta'/c)rot\mathbf{v}$, $\eta' = w_2\nu$.

4. The physical region of allowed values of PN's $\{u,\ w_1,\ w_2\}$ follows from the analysis of the gyrotropic system stability with respect to small pertubations. So $u > -1$, $u \geq w_1 w_2$, $4w_1w_2 \geq -(1-u)^2$. In the gyrotropic MHD turbulence, the solution of RG equations proves the existence of two stable attractive asymptotic regime. For both stable fixed points (the trivial Gaussian point with $u^* = 0$ and the Kolmogorov point with $u^* = 1.393$) $w_1^* = w_2^* = 0$ is valid. It has also been proved by the numerical solution of the corresponding Gell - Mann - Low equations. A very large attractive region for the Kolmogorov fixed point has been demonstrated [4].

5. Note that in the gyrotropic MHD, (as in the ordinary MHD), also rotor-like terms are generated in the correlation functions. These terms lead to instabilities in the theory and to increasing magnetic field fluctuations. The theory is stabilized by the spontaneous occurence of a nonvanishing homogeneous mean magnetic field [5,6].

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