the differential equation is that governing the equilibrium of an elastic medium, in which case the integral to be minimised represents the potential energy of the medium. The eigenvalue problem for a given operator can also be transformed into an equivalent problem of minimising a certain functional; this was recognised nearly a century ago, in relation to vibrating systems, by Rayleigh, who enunciated the principle that " the period of a conservative system vibrating in a constrained type about a position of stable equilibrium is stationary in value when the type is normal ", the period being of course a functional depending on the assumed type. Various practical methods for finding approximate solutions to certain problems, including many of the problems of classical mathematical physics, are based on the variational method. They usually depend (e.g. in Ritz's method) on the construction of a " minimising sequence " of functions which under certain conditions converges in some well-defined sense to the function which minimises the appropriate functional, so that the successive terms of the sequence constitute successive approximations to the solution.

The first half of the book under review is theoretical, and gives a lucid account, with a minimum of mathematical sophistication but an adequate degree of precision, of the relevant properties of the operators commonly occurring in mathematical physics, with a careful discussion of the conditions they must satisfy for the variational method to be applicable. The construction of minimising sequences is also treated along with the nature of their convergence (usually a generalisation of convergence in the mean). The illustrations are chosen mainly from the theory of elasticity.

The rest of the book uses more advanced mathematics. In it the results of the first half are re-formulated and generalised in terms of Hilbert Space, using the Lebesgue integral; a concise outline of both of these topics is included, reinforced by references to the standard literature. A subsequent chapter is devoted to the important question of the theoretical estimation of the error of the approximate solution, and there is an illuminating chapter consisting of numerical examples of the methods developed in the text. Variants of the method associated with the names of Trefftz, Dubnov, Galerkin, Kryloff and others including the author himself are treated. A final short chapter discusses finite difference methods, widely used in engineering practice, in which a differential equation is replaced by a set of difference equations. This chapter has no real connection with the rest of the book, but has apparently been retained from an earlier version of the work under the wider title " Direct methods in Mathematical Physics ".

The bibliography will be of limited use to non-Russian readers since references are, perhaps not unexpectedly, almost exclusively to Russian work, of which it is clear that a considerable amount exists, much of it probably little known outside U.S.S.R.

The English version reads well, apart from an occasional Muscovism—such as the use of the dash in place of the copula—which seems to have survived the translation. It is a pity that such an expensive book should be marred by an undistinguished layout and typography which can only be described as slipshod; but these superficial defects should not impair its usefulness. It is a most valuable work which brings together in a systematic treatment a wide range of material not readily accessible elsewhere, and can be studied with great advantage by anyone interested in the theory and practice of approximate solutions in Mathematical Physics. R. SCHLAPP

MEINARDUS, GÜNTER, *Approximation von Funktionen und ihre numerische Behandlung* (Berlin, Springer-Verlag, 1964), 180 pp.

Until recent years, most of the theory of function approximation has remained in original papers, and there have been few, if any, satisfactory textbooks on the subject. The advent of high-speed computers, however, has made it essential that the known facts regarding this topic be gathered together and presented in a manner suitable for

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use by the large number of research workers, who now require function approximation as an essential part of their mathematical equipment. The present author has set out to achieve this and has met with considerable success.

It is felt, however, that the book lacks the practical approach shown in a rival work, *The Approximation of Functions*, Volume 1: Linear theory (1964); Volume 2: Non-Linear theory (to appear) (Addison-Wesley) by J. R. Rice, although on the credit side, the material in the present volume is less heuristic than that in the work by J. R. Rice.

The book is concerned in the main with the approximation of continuous functions along the real axis and in the complex plane. There are two distinct chapters dealing with linear and non-linear approximation respectively.

The material in the first chapter, which covers about two-thirds of the book, is basic in approximation theory and consists in the main of Tchebycheff and polynomial approximation. The second chapter is concerned mainly with the recent research work by the author and others and covers non-linear Tchebycheff, rational, and exponential approximation.

ROTMAN, JOSEPH J., *The Theory of Groups: An Introduction* (Allyn and Bacon, Inc., 1965), xiii+305 pp., \$8.75.

The contents of this enjoyable book are as follows: Chapters 1 and 2: Isomorphism Theorems; 3: Cayley's Theorem and the simplicity of $A_n (n \neq 4)$; 4: Direct products, Basis theorems for finite Abelian groups and applications to modules and matrices, Remak-Krull-Schmidt Theorem; 5: Sylow Theorems; 6: Eight pages on Galois Theory, Solvable groups, Jordan-Hölder Theorem, P. Hall's Theorem on solvable groups of order ab, where $(a, b) = 1$, Nilpotent groups; 7: Automorphism groups, Extensions, Second Cohomology group; 8: Finite fields, Simplicity of the projective unimodular groups $PSL(m, K)$ when $m \geqslant 3$ or when $m = 2$ and K is a finite field of more than three elements, Two non-isomorphic simple groups of the same order; 9: Infinite Abelian groups, Basis theorems for finitely generated Abelian groups; 10: Hom and Ext functors; 11: Subgroup theorem for free groups, Free products with amalgamated subgroups; 12: Turing Machines, Proof of the existence of a finitely presented group with unsolvable word problem.

We have here essentially a simple basic text for beginners, together with a few selected more advanced topics. The exercises, except for some marked with an asterisk, form part of the logical development of the text. This makes the book rather unsuitable for private study, though students working under guidance will find this a good way to learn the subject.

This is above all a lively book with plenty of motivation and discussion. Thus, after the proof of the basis theorem for finite Abelian groups, we read:

> " We now have quite a bit of information about finite Abelian groups, but we still have not answered the basic question: If G and H are finite Abelian groups, when are they isomorphic? Since both G and H are direct sums of cyclic groups, your first guess is that $G \approx H$ if they have the same number of summands of each kind. There are two things wrong with this guess. First of all, since, e.g. $\sigma(6) \approx \sigma(3) \oplus \sigma(2)$, we had better require that G and H have the same number of *primary* summands of each kind. Our second objection is much more serious. How can we count summands at all; to do so would require a unique factorization theorem analogous to the fundamental theorem of arithmetic, where the analog of a prime number is a primary cyclic group. Such an analog does exist; it is called the fundamental theorem of finite Abelian groups, and it is this theorem we now discuss."

The material chosen by the author has been very well presented and the book can be highly recommended.

J. L. BRITTON

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