Note on the paper, of Dr Bevan B. Baker, An Extension of Heaviside's Operational Method of Solving Differential Equations.*

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§ 1. Let $f(x)$ and $F(x)$ be polynomials which are supposed to be decomposed into a series of n terms as

$$\frac{f(x)}{F(x)} = \sum_{\nu=1}^{n} \frac{f_\nu(x)}{F_\nu(x)}. \qquad \text{.......................(1)}$$

Further let θ be a distributive operation and $\phi(x)$ be a given function. Then the functional equation

$$F(\theta)\, y(x) = f(\theta)\, \phi(x)$$

has a solution of the form

$$y(x) = \sum_{\nu=1}^{n} y_\nu(x), \qquad \text{.........................(2)}$$

where $y_\nu(x)$ $(\nu = 1, 2, \ldots n)$ is the solution of the equation

$$F_\nu(\theta)\, y_\nu(x) = f_\nu(\theta)\, \phi(x)\dagger$$
$$(\nu = 1, 2, \ldots n).$$

The equation (2) may be written symbolically as follows

$$\frac{f(\theta)}{F(\theta)}\, \phi(x) = \sum_{\nu=1}^{n} \frac{f_\nu(\theta)}{F_\nu(\theta)}\, \phi(x); \qquad (2')$$

and, by a special suitable choice of the decomposition (1) and of the functions ϕ, f, F, this last formula (2′) leads to a series of results both in the differential and in the difference calculus.

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† The demonstration of the equation (2), with certain results belonging to the calculus of differences are given by the author in the paper, "O redukci součtů $\underline{S}\phi(x)\overset{n}{\underset{a}{\nabla}} x$ a $\overset{z}{\underset{\bar{a}}{S}}\ \phi(z)\overset{n}{\underset{\omega}{\Delta}} z$, Časopis, 54, Praha (in the press).

§ 2. For example let the degree of $f(x)$ be less than the degree of $F(x)$, and suppose that the decomposition (1) is into partial fractions. In this case the relation (2) yields

$$\frac{f(\theta)}{F(\theta)}\,\phi(x) = \underset{-}{S}\,\frac{\phi(x)}{\theta - r}\left[\frac{f(r)}{F(r)}\right], \qquad \text{.....................(3)}$$

where $\dfrac{\phi(x)}{\theta - r}$ is the solution of the equation

$$(\theta - r)\,y(x) = \phi(x),$$

and $\underset{-}{S}$ means the sum of residues of the function

$$\frac{\phi(x)}{\theta - r}\cdot\frac{f(r)}{F(r)}$$

with regard to the poles of $F(r)$.

Further let

$$D = \frac{d}{dx},$$

and let us consider the solution of the equation

$$(D - r)\,y(x) = \phi(x)$$

for which $y(0) = 0$. In this case

$$y(x) = \int_0^x e^{r(x-z)}\,\phi(z)\,dz$$

and formula (3) gives

$$\frac{f(D)}{F(D)}\,\phi(x) = \int_0^x \underset{-}{S} e^{r(x-z)}\left[\frac{f(r)}{F(r)}\right]\phi(z)\,dz. \qquad \text{............(4)}$$

To simplify matters let us suppose that $F(r)$ is of degree n and has only simple zeros r_ν. Then if we write

$$A_\nu = \frac{f(r_\nu)}{F(r_\nu)}$$

the equation (4) becomes

$$\frac{f(D)}{F(D)}\phi(x) = \sum_{\nu=1}^{n} A_\nu \int_0^x e^{r_\nu(x-z)}\,\phi(z)\,dz. \qquad \text{..................(5)}$$

By proper choice of $\phi(x)$ in (5) the formulae of Bromwich, Carson, Heaviside and Baker may severally be deduced.

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