'Practical Rhumb Line Calculations on the Spheroid'

Roy Williams *writes*

I read with interest the paper by G. G. Bennett in the January issue of the *Journal*[1] and, although there is no fault with this work that I can see, I feel it necessary to make one or two comments.

The first is that I cannot understand why he chooses to use an approximate equation for computing departure (even though it gives good results) when an exact equation is well known. This equation is

$$\text{Departure} = \text{D'Long} \times \left(\frac{\text{Meridian distance}}{\text{Difference of meridional parts}}\right)$$

where meridian distance (MD) and difference of meridional parts (DMP) must be measured in the same units and where, consequently, departure will be measured in these units also. This equation is very stable and gives good results when the difference of latitude, $\Delta\phi$, is as little as 1 minute of arc and using only the accuracy available on a hand calculator. Indeed, by considering the analytical expressions which define meridian distance and DMP, it can be seen that the ratio (MD/DMP) approaches a precise limit as $\Delta\phi \to 0$ which is

$$\lim_{\Delta\phi \to 0}\left(\frac{\text{MD}}{\text{DMP}}\right) = \frac{\cos\phi}{\sqrt{(1 - e^2 \sin^2\phi)}}$$

and this leads to the formula for the distance along a parallel where the geographical latitude is ϕ.

I feel that the theory behind 'middle latitude sailing' was never widely understood by navigators and the use of mean latitude in place of middle latitude was a widespread practice. The very use of MEAN LAT in Bennett's formula is, therefore, in my opinion, risky. Although he uses it with an appropriate correction term, which takes in not only the correction to mean latitude for middle latitude but also includes the correction factor which will express his distances in nautical miles of 1852 metres, a less meticulous practitioner will, I fear, revert to the truncated version of his formula.

The second point I wish to make is that I note that Bennett expresses his distances in nautical miles. However, if we consider normal navigational practice, we find that the recommended great circle calculation gives us an answer in units of the length of 1 minute of arc of a great circle – the geographical mile. In nautical tables, Meridional Parts are tabulated in units of the length of 1 minute of arc of the equator – the geographical mile. When a navigator 'picks off' his distance on a Mercator chart, this distance is also in units of the length of 1 minute of arc of the equator. Most navigators, I feel sure, use these above measurements as they stand as their measures of distance and still refer to these distances in 'miles'. Who is correct?

I think that we suffer, quite generally, from a lack of rigour in the application of navigational methods even though the individual errors in so doing are not serious. It is caused partly by the fact that the Earth is so nearly a sphere and can be treated so with reasonable safety, and partly by the fact that we have defined different units called 'miles' which are far too close in magnitude so that they coalesce in the minds of the practitioners. One way around this, I suggest, is for navigators to express their distances in kilometres. We changed from fathoms to metres with very little fuss, so why not change from 'miles' to kilometres in the same way and join with the rest of the scientific community?

REFERENCE

[1] Bennett, G. G. (1996). Practical rhumb line calculations on the spheroid. This *Journal*, **49**, 112.

KEY WORDS

1. Rhumb lines. 2. Loxodromes. 3. Geodesy.

Dr. George Bennett *replies*

I feel that the comments of Dr. Roy Williams should be put in perspective as far as the subject matter of my paper is concerned.

(1) I have not given an equation for computing departure (horizontal component in the lower triangle of the diagram) because it is unnecessary. My equations $\tan C = \Delta\lambda/\Delta M$ and $S = \Delta m/\cos C$ are rigorous and do not require the computation of the intermediate quantity departure to effect a solution. The advantage of using formula (iv) is that it takes into account distances both on or lying close to the parallel. There is no risk involved. The prudent programmer, where appropriate, will include formula (iv) and the person computing with tables will use either formulae (vi) or (vii). If a practitioner wants to deviate from the simple routines that I have given, then naturally errors may arise.

(2) For the problems under discussion, distances, in my view, should be expressed in terms of a generally accepted standard of length, which is the international nautical mile (1852 m). In my opinion great circle calculations on the sphere should be confined to route planning, where the difference between minutes on the sphere or spheroid is immaterial. With graphical solutions on a chart it would be almost impossible to differentiate between results obtained using the various versions of 'mile'. However, if the major marine agencies were to agree to another unit, for example the kilometre, then an additional scale would need to be given on charts.

(3) Please note the error in formula (5) where the right hand side of the equation should be multiplied by S.

(4) Tables for the solution of problems associated with rhumb line courses and distances using WGS 1984 are available from: CN Systems, 27 Cabramatta Rd, Mosman, Australia, 2088. Price A$12 (inc. P & P)

Many worked examples are given which cover both terrestrial hemispheres and east and west longitudes.