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SYMPOSIA PAPER

Against Quantitative Primitivism

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Abstract

The problem of quantity is the problem of identifying what about the physical world explains why it can be so well represented with mathematical entities. I introduce "quantitative primitivism," the dominant position in the literature, which offers only a partial solution to the problem of quantity. I argue that a reductive account of quantitativeness provides a full solution to the problem and describe two reductive accounts in the literature. I discuss some of the unique metaphysical consequences of reductive accounts of quantity, including a novel dissolution to the long-standing absolutist–comparativist debate.

1. Introduction

In this article, I introduce a novel approach to a problem that, in the metaphysics of quantity literature, is often thought to admit of only a partial solution. The problem, sometimes called the "problem of quantity," is central to the metaphysics of quantity. The dominant position in the literature is that the best we can hope for is a partial solution to this problem. I'm going to argue against this view, which I call *quantitative primitivism*.

We can seek a full solution to the problem of quantity, what I call a "fully reductive account of quantitativeness," but this has interesting downstream effects on our metaphysics of quantity. In what follows, I introduce the problem of quantity and a partial solution offered by the quantitative primitivist, and I explain why the primitivist's solution is not enough to solve the problem of quantity adequately. In section 2, I present two examples in the literature of fully reductive accounts of quantity that do solve the problem fully.

In section 3, I discuss the most striking limitation reductive theories of the quantitative face, which is that they are invariably limited in scope relative to their primitivist competitors. I argue that this limitation in scope is the natural consequence of the fact that fully reductive accounts provide better explanations of quantitative structure in terms of the underlying nonquantitative physics. Section 4 sketches some other notable benefits of a move away from quantitative primitivism. The first is that it renders the absolutist–comparativist debate largely moot, and the second is that fully reductive accounts of quantity are better able to handle quantities whose quantitative structure is contingent.

1.1. Mathematical representations and scientific explanation

We represent quantitative properties and relations, in science and our everyday practice, using mathematical entities like numbers and vectors. Often genuine scientific explanations depend on obtaining features that we (sometimes only ever) represent mathematically. We then appeal to the arithmetical relations between those numbers to explain certain physical facts, for instance, I cannot reach the iced coffee on the table because the shortest path between it and me is 3 feet long, whereas my arm is only 2.2 feet long, and $2.2 < 3$. Let's look at this explanation in more detail.

We can explain "I cannot reach the iced coffee on the table" with three sentences:

1. The shortest distance between my torso and the iced coffee is 3 feet.
2. My arm is 2.2 feet long.
3. $2.2 < 3$.

Sentences 1 and 2 describe physical facts about the state of my arm and the region between myself and the coffee. In contrast, sentence 3 is an *entirely* arithmetical statement. It refers to the *numbers* 2.2 and 3 and makes use of the *arithmetical less-than relation* "$<$."

1.2. The problem of quantity

This leads us to the problem of quantity. While our mathematical representations play a key role in these explanations, it seems clear that the arithmetical "$<$" relation, the "$+$" and "$*$" operations, and the numbers to which they connect are not *really* part of the physical explanations of these events. A different representational system would assign the numbers 67 and 91.4 to those same two objects, and the same physical explanation could be expressed using those numbers. Our mathematical representations represent similarly structured but otherwise nonmathematical aspects of the physical world. And it is *those* facts that are explanatorily relevant, not the numbers we use to represent them.

A theory of "quantitative structure" is an account of those underlying physical facts that our mathematical notions represent. Some part of the physical world "is quantitative" insofar as it has this kind of structure. The problem of quantity is the problem of explaining what quantitative structure *consists in*, that is, what makes a given property, relation, or law *quantitative*.

1.3. One bad solution

Let's see how one might solve the problem of quantity. Here's a simple toy solution that provides a "fully physical" backing for all our mathematical talk, but at the cost of being utterly absurd.

Step 1. Introduce a class of physical entities, call them $\text{Numbers}_{\text{PHYS}}$, that admit of operators like $+_{\text{PHYS}}$, $*_{\text{PHYS}}$, and $\sqrt{}_{\text{PHYS}}$ and relations like $<_{\text{PHYS}}$. These entities behave just like their abstract namesakes but are stipulated to be physical.

Step 2. Replace the numbers in the iced coffee explanation with $\text{Numbers}_{\text{PHYS}}$.

On this view, sentences 1 and 2 of the iced coffee explanation express relationships between bodies, regions of space, and certain $\text{Numbers}_{\text{PHYS}}$, which are all physical entities. Recall the purely mathematical sentence 3: "$2.2 < 3$." That's as far from a physical state of affairs as you can get! But, with the magic of $\text{Numbers}_{\text{PHYS}}$, this sentence is to be replaced with "$2.2_{\text{PHYS}} <_{\text{PHYS}} 3_{\text{PHYS}}$" (where "$2.2_{\text{PHYS}}$" and "$3_{\text{PHYS}}$" are $\text{Numbers}_{\text{PHYS}}$). Now we have two $\text{Numbers}_{\text{PHYS}}$ instead of two numbers, and we've replaced the arithmetical less-than relation "$<$" with the *physical* less-than relation "$<$." This explanation is now entirely physical. All appeal to abstract mathematical entities has been removed.

Problem solved, right? Of course not! Positing a bunch of $\text{Numbers}_{\text{PHYS}}$ does nothing to explain why length is quantitative. This "solution" fails because it amounts only to slapping a sticker that says "physical" on top of the mathematical bits of our original explanation. All of the mysteriousness of the explanation is still there.

1.4. Quantitative primitivism

Let us turn to a more widely accepted solution in the metaphysics of quantity literature, which I will call *quantitative primitivism* (or Q-P). Q-P is the view that all *quantitative structure*, the features of the physical world that explain how it plays the roles just discussed, is either *primitive* or (at least partially) derived from some *other* quantitative structure that *is* taken as primitive.

Quantitative primitivism *isn't* the view that particular *quantities* are primitive. Whether mass, or distance, or charge, or velocity is metaphysically fundamental or grounded in some other physical ontology is a separate question. What matters is its quantitativeness. That is, Q-P is the view that a quantity's *distinctive structural features* cannot be wholly reduced to something "nonquantitative."

Hölder (1901) is the original proponent of this sort of view, and many modern theorists, such as Mundy (1987) and Eddon (2013) (and, on one interpretation, Arntzenius [2012]), follow his lead. Others, like Krantz et al. (1971) and Field (1980, 1984), are very strongly influenced by Hölder's (1901) formal advancements. These views all accept that the proper account of quantities requires that we posit at least some *primitive* quantitative structure (of the first or second order).[1] And, if we adopt a metaphysically heavy-duty interpretation of Krantz et al. (1971) and Field (1980, 1984), these views, too, can be understood as accepting a form of primitivism.

1.5. Primitivism and the problem of quantity

According to a primitivist account of the quantitative, some quantitative structure is posited as part of the fundamental physical facts. However, other things (ratios, proportions, other higher-complexity structural relationships) are given a reductive definition in terms of the fundamental quantitative structure. As such, a primitivist account of quantitativeness provides a mixed or partial solution to the problem of

[1] Mundy (1987) and Eddon (2013) posit primitive second-order relations, whereas accounts based on Hölder (1901) or Krantz et al. (1971) posit primitive ordering relations between, and concatenation operations on, physical objects.

quantity. The problem is that this means we get a view that "solves" the problem by positing something dangerously close to $\text{Numbers}_{\text{PHYS}}$!

Consider, for instance, the account of Mundy (1987), who posits primitive *second-order* relations of "ordering" and "summation," which relate fundamental mass universal *properties*. Mundy's definition of "less massive than" is as follows:

> x is less massive than $y =_{df}$ there exist mass universals U_1 and U_2 such that $U_1(x)$ and $U_2(y)$ and $U_1[<]U_2$ (where $[<]$ is the primitive second-order ordering relation).

But Mundy's (1987) $[<]$ is just $<_{\text{PHYS}}$ all over again! It's a primitive two-place relation that we have added to our metaphysics of the physical world to be a physical stand-in for our mathematical notion.

Of course, Mundy (1987) isn't committed to the *entirety* of $\text{Numbers}_{\text{PHYS}}$, and so he avoids the complexity of taking all the $\text{Numbers}_{\text{PHYS}}$, and the relations and operations over them, to be metaphysically fundamental. Most of the quantitative structure of mass will be defined by or grounded in a much smaller fundamental substructure. In Mundy's case, these structures are just the $[<]$ and a three-place mass summation relation, $[*]$. But this isn't much consolation. If you think that the *only* bad thing about $\text{Numbers}_{\text{PHYS}}$ is that the system is too *complex* or *unparsimonious* (because it has to posit infinitely many distinct $\text{Numbers}_{\text{PHYS}}$ and the many relations and operations over them), you've gotten something very wrong. The explanatory failure of $\text{Numbers}_{\text{PHYS}}$ runs much deeper than that.

1.6. We should want more

The prevailing impression is that some amount of quantitative primitivism is unavoidable, that we must pick *some* primitive quantitative structure no matter what. On this way of thinking, the task of a metaphysics of quantity is just a matter of coming up with a short and elegant list of primitive quantitative relations (together with axioms governing those relations) from which the rest of that quantity's structure can be derived.

There is a lot to like about these kinds of primitivist accounts! They manage to reduce and explain lots of quantitativeness! However, these explanations invariably end up defining quantitativeness either in terms of other quantitativeness or not at all (i.e., they take it on as an unexplained primitive). This is a problem if you think that quantitativeness *itself* demands explanation in terms of the underlying (nonquantitative) physical facts.

Any theory that posits some quantitative structure as an unexplained fundamental physical positing is just a (potentially very fancy) nonsolution to the problem of quantity. If you find the problem motivating, and I think you should, then you should want a complete solution. Of course, if it turns out that some amount of primitivism about the quantitative really is unavoidable, we may have to settle for a primitivist solution.

I maintain, however, that primitivism is not unavoidable. We should reject primitivism and adopt the much more difficult project of full reduction of quantitative structure. In the next section, I sketch how this can be done using two case studies of fully reductive accounts of quantitativeness that exist in the literature.

2. What a reductive account of quantitativeness looks like

I present three reductive accounts of quantitative structure: one toy example and two case studies.

The toy account, call it the "one true mass" view, is a great illustration of the benefits of a reductive explanation of the quantitative, but it has the disadvantage of being almost certainly false about the actual world. On this account, there's only one mass property, call it "M." All massive fundamental particles possess M, and all composite material objects are composed of such fundamental particles. On the one true mass view, all quantitative facts about mass are just a matter of counting up of massive particles. To be *more massive than* something is to be composed of more massive particles (each of which contains the one true mass). For *x*'s mass to be the *sum* of *y*'s and *z*'s masses is just for the number of massive particles that compose X to be the arithmetical sum of the numbers of massive particles that are part *y* and part *z*, respectively. All mass *ratios* are, similarly, just arithmetical ratios between the cardinal numbers that correspond to *how many* massive particles compose each of a given pair of objects.

The one true mass view is almost certainly false of the actual world. The standard model of particle physics allows for fundamental particles whose masses differ (the electron is more massive than the muon, for instance), and some physical theories don't countenance (fundamental) particles at all. But, if it were true, it would eliminate the mystery of mass's quantitativeness. We use numbers to talk about mass facts because mass facts depend only on how many massive particles you have as parts and "how many" facts are well represented by cardinal numbers!

In the remainder of this section, I discuss two reductive accounts that have appeared in the literature and look at how they manage to ground quantitativeness in the underlying structures of the physical world. I argue that these accounts offer substantive explanations of what it is for these physical quantities to have that structure.

2.1. The mereological–reductive account

The mereological–reductive (M-R) account of quantitative structure was introduced in Perry (Forthcoming). The M-R account applies to quantities like length and volume, whose mereological (parthood) structure is closely tied to the quantitative structure that we ascribe to them.

To give an example, this view defines "*x* is less voluminous than *y*" as "*x* has the same volume as some part of *y and x* and *y* do *not* have the same length."

Primitivist accounts of quantitative structure can capture the intuitive connection between these two statements only if they posit additional bridge laws to connect their primitive quantitative relations to the mereological ones. The M-R account avoids this by taking the connection to be *definitional*.

According to the M-R account, volume is an ordinary determinable with a class of length determinants that have *no fundamental structure*, along with certain fundamental principles about how those determinants are distributed in the world. These principles refer only to mereological relationships and to whether two entities instantiate the same determinants (a purely qualitative notion). It is through those principles that the M-R account gives a definition—in terms of parthood and the sharing of determinate length properties—for *all* the relations that constitute length's quantitative structure. That is, the M-R account *does not rely on primitive quantitative structure*.

2.2. Field's distances without numbers

Another genuinely reductive account of quantitativeness is Field's (1980) account of spatial (or spatiotemporal) distance. For example, Field defines that "the distance from x to y is twice that from z to w," where x, y, z, and w are spatiotemporal points as

$$\exists u(u \text{ is a point } \wedge \ u \text{ is between } x \text{ and } y \ \wedge \ xu\textsc{Cong}uy \ \wedge \ uy\textsc{Cong}zw)$$

We may interpret "$xy\textsc{Cong}zw$" as saying either that x and y stand in some two-place distance relation or that z and w stand in the *very same* relation.[2]

Betweenness is a geometrical relation that holds based on how bodies or points are distributed in space. It describes, not the quantitative structure of distance, but the spatial configuration of individuals. For this reason, it manages to qualify as genuinely reductive. Field's (1980) account requires substantival space or space-time for there to be sufficiently many points in the right configurations (i.e., all possible ones) so that the existentials from the definitions are satisfied. According to Field, the structure of spatial configurations in a substantival space is what grounds the quantitative structure of the distance relation.

3. Reductive accounts and limited scope

All three accounts described in the previous section suffer from some limits in their scope of application compared to their primitivist counterparts. Primitive positings are as adaptable as their axioms allow them to be, and it's easy to generalize an account that makes use of, for example, primitive ordering relations to apply to any quantity that's ordered. A primitivist account of one quantity typically extends to all other quantities that can be represented with the same mathematical structure. In contrast, a reductive account's definitions of quantitative relations can be satisfied only by special subclasses of quantities.

I'll explain how this limitation arises in our two case studies. However, I will argue that this isn't as much of a disadvantage as it seems.

3.1. Limitations in the M-R account

The M-R account applies to quantities like length and volume, which are additive scalars. However, the account cannot be extended to *all* scalar quantities, or even to all additive quantities.

The reason is that the definitions given by this account have physical upshots that a quantity must satisfy for the theory to apply to it. These accounts put demands on *what kinds of parts* a lengthy or voluminous object or region must have, and there are scalar quantities that don't satisfy those demands.

For two voluminous spatial regions, if x is smaller (less voluminous) than y, this implies the existence of a part of y that has the same volume as x. The same does not

[2] An alternative reading would make the Cong relation a primitive four-place quantitative relation. We will not use this one, for obvious reasons.

hold for something like mass, velocity, or temperature. That *x* is less massive than *y* (or has a lower temperature) does not guarantee that *y* has a proper part that has the same mass (temperature) as *x*. This is stronger than mere additivity; rather, the M-R account applies only to quantities that Perry (2015) calls "properly extensive."

The M-R account extends to other properly extensive quantities, such as area, temporal duration, and the invariant relativistic interval. But quantities like mass, charge, velocity, and temperature, which are not properly extensive, will not have the right relationship to mereology to satisfy its definitions.

3.2. Limitiations in Field's scope

Likewise, though Field's (1980) formal account of distance can extend to monadic scalar quantities like length, mass, volume, and temporal duration, it does so at the expense of its reductiveness. This is because Field's account is reductive precisely because it reduces distance's quantitative structure to the *configuration* of points in a substantival physical space.

Field (1980) extends his account to other scalar quantities by replacing the spatiotemporal "betweenness" and "congruence" relations with "SC-betweenness" and "SC-congruence" ("*SC*" for scalar). This change is significant. It replaces the geometrical betweenness and congruence with *primitive quantitative relations*!

Field's (1980) definition of "*x* is twice as different in **mass** from *y* as *z* is from *w*" is $\exists u$(*u* is a massive body $\wedge$ *u* is SC-between *x* and *y* $\wedge$ *xuCuy* $\wedge$ *uyCzw*). This is not a reductive definition, because SC-betweenness and SC-congruence are *primitive quantitative relations*.

SC-betweenness can only be interpreted as saying that, for example, SC*Byxz* means *x*'s mass is greater than *y*'s but less than *z*'s. Likewise, the four-place SC-congruence in that definition is a quantitative relation of *comparative mass*, and so *xyCzw* means that the *difference in mass* between *x* and *y* is the same as the *difference in mass* between *z* and *w*. So Field's (1980) view can be extended to other quantities only by sacrificing its reductiveness.

3.2.1. Aside: Can we fix Field on mass?

Field's (1980) view cannot remain reductive and apply to other scalars because of the unique role that geometrical/configurational *betweenness* plays in reducing spatial distance's quantitative structure. I've argued that introducing SC-betweenness amounts to adding in primitive quantitative structure.

Arntzenius (2012) has defended a view that looks to extend the Field (1980) project to quantities like mass in a way that avoids primitivism. He eschews SC-betweenness and instead uses the same spatiotemporal betweenness and congruence relations used in Field's account of distance to ground the structure of other quantities. He achieves this by positing additional *substantival physical spaces* beyond our ordinary physical space-time.

On this view, a body will have a position in space (or a world-line in space-time) as well as a position in *mass-space*, which is a one-dimensional physical space. Because this is just another physical space, we can use the same betweenness and congruence relations to define a version of *spatial distance* for points in that space! If a body's position in mass-space is just its mass, then the distance structure on this one-dimensional substantival physical space *just is* that of which the quantitative structure of mass consists.

The problem with this account is that the label "just another substantival physical space" is doing a ton of work and so should have a metaphysical upshot. This could take a variety of forms, but, at a bare minimum, an object should be spatially located in the same place as its parts.

If a composite is located anywhere in ordinary physical space, it should be located wherever all of its parts are. It's not merely that it's easy to figure out where the whole is, given the positions of the parts. If I tell you the locations of all its parts, I've *already told you* where the whole is located. However, the position of a composite in Arntzenius's (2012) mass space is (as a rule) *never* going to be the same as the positions of its parts. So, for instance, if a large dachshund's back half weighs four kilograms and its front half weighs five kilograms, then the position in mass-space of the whole dog will be far away from its two halves (all the way at far-flung nine kilograms).

Of course, if mass-"space" is a *metaphorical* "property space"—a way to conceptualize a large, structurally complex class of properties—then there's no such requirement. A composite does not need to share the *properties* of its parts. As such, we can fix the preceding problem by considering "mass-space" a mere representation of the property-space of mass. This fix, however, will also turn the Arntzenius (2012) view into a primitivist account of quantitativeness, on par with Mundy (1987).

3.3. This "downside" is good, actually

Reductive accounts of quantity are often limited in scope. I've argued that this is because their analyses of quantitative concepts must link up to the underlying physics of the quantity, which is not always well behaved. As such, a reductive theory that applies to all quantities would have to be very disunified/disjunctive (it would be more of a patchwork of many reductive accounts of specific quantities). I contend that this is not a true drawback.

Compare Mundy (1987). Mundy's primitivist account applies to all scalars (i.e., all quantities with a structure we'd represent using the nonnegative real numbers). But, I object, Mundy's view achieves such wide scope by papering over *deep physical and metaphysical differences* between the quantities it covers. Mass and length (for example) have *vastly* different underlying *physical* structures and traffic with the rest of the physical world in very different ways! So why would their fundamental metaphysics ignore these differences? To unify all the scalars under one fundamental metaphysics is to treat "being most perspicuously represented in our science with the nonnegative real numbers" as if it carves at the joints of reality.

The M-R account, by contrast, is limited to quantities with a specific kind of relationship to mereology precisely because it appeals to *that very relationship to mereology* to explain their quantitative structure! Likewise, Field's (1984) account of distance in terms of geometrical/configurational betweenness tells us something genuinely interesting about the nature of distance. Field argues that substantivalist theories of space, and not relationalist ones, are able to account for the structure of spatial quantities because only the former posit enough points that exhibit the right patterns of betweenness to satisfy Field's definitions.

4. Other benefits of reductive accounts

Two other interesting characteristiscs of reductive accounts of the quantitative is how they interact with the absolutism–comparativism debate and how they are able to capture the ways that quantitative structure might be contingent.

4.1. Dissolution of the absolutist–comparativist debate

An unusual outcome of a reductive theory of quantitative structure is that it renders much of the absolutist–comparativist debate moot. This is because the absolutist–comparativist debate is, ultimately, a dispute about *which* quantitative structure we should take as primitive (see Dasgupta 2013).

Absolutism and comparativism can be understood as follows:

> **Absolutism.** The primitive quantitative structure consists of properties or relations instantiated by physical bodies, regions, and so on, as well as primitive structuring relations between those properties or relations.
>
> **Comparativism.** The primitive quantitative structure comprises *comparative* quantitative relations between physical bodies, regions, and so on (Where "comparative" means ones that characterize relative differences in that quantity between those bodies, regions, and so on).

The absolutist–comparativist debate is, almost entirely, an internal debate among primitivists about the quantitative. Theories that do not posit fundamental quantitative structure will be neither absolutist nor comparativist.

What does this mean for the reductionist? The questions that motivate the absolutist and comparativist camps will have to be evaluated based on the *physical role* of the quantity in question and to what physical structures our mathematical representations correspond. It will depend on this reduction whether we can make sense of talk of "doubling all the masses" (pick your favorite comparativist bugbear). Whether a doubled-mass world, or a doubled-length world, or what have you is possible (and is a different possibility than the actual world) will depend on the underlying physical facts about that quantity.

4.1.1. Aside: What do I mean by "comparative"?

I described comparativism as the view that takes as primitive *comparative* quantitative relations. Why not just say the simpler thing: that "any theory that posits only primitive relations is comparativist"?

Here's why. Some physical quantities—spatial and temporal distance, quantum entanglement relations (perhaps), and so on—are relations. Our definitions should allow for **absolutist** theories of those quantities.

A theory that posits a class of primitive distance relations between points, and some Mundy-style primitive second-order relations between those relations, is an absolutist one. In contrast, a theory that posits a primitive four-place congruence relationship between points (like the primitivist reading of Field's CONG in note 2), but no primitive distance relations, is a comparativist one.

One key marker of an absolutist view is that the absolutist admits the possibility of "doubled worlds." On this understanding, absolutist accounts of distance admit of possibilities where all the distances doubled, whereas the comparativist accounts of distance do not.

4.2. Flexibility in structure

There's an interesting trade-off for the primitivist that relates to the discussions in section 3. The same thing that makes primitivist accounts able to apply with such wide scope—that their theories bottom out in primitive quantitative structure—also restricts their flexibility when trying to represent quantities whose structure is *contingent*.

Here's what I mean. We might want to say that, for a given quantity, it is contingent whether its quantitative structure is best represented by the nonnegative reals or by the nonnegative integers (or even by the nonnegative rationals). At least in the case of discreteness, it is not a settled question whether some quantities are really discrete at a small enough scale.

However, the primitivist will typically have great trouble giving an account of quantity that can accommodate these possibilities. This is because (for the primitivist) the structural features of the quantities are "baked in" to the fundamental facts. A reductive account of quantitativenes, by contrast, is much more able to account for the possibility of a quantity having a different structure, so long as that difference is reflected in the underlying (nonquantitative) physical facts.

For instance, the M-R account of volume and Field's (1984) account of distance are in a good position to explain how these quantities could have had different quantitative structure (e.g., could have been *discrete*); that is, they will imply that their respective quantities have discrete quantitative structure, *so long as* that difference is reflected in the mereological/geometrical structure of space-time (i.e., that *space* is discrete).

Here's how that would work in a world in which points of space are distributed in something like a discrete lattice. The Fieldian definition of "the distance from x to y is twice that from z to w," presented in section 2.2, will still be applicable in such a world. Because there are fewer constituents of space (countably rather than uncountably infinite points), fewer 4-tuples of points will that satisfy that definition. However, the definition itself will still apply successfully to those points and will still capture what it means for one pair of points in this lattice to be twice as far apart as some other pair.

5. Conclusions

I've argued that the received position in the metaphysics of quantity, according to which some quantitative structure must be taken as primitive, should be rejected. Only a fully reductive account of quantitativeness can offer a complete solution to the problem of quantity. I presented two such accounts. The M-R account, as well as (one interpretation of) Field (1980), gives a wholly reductive account of the quantitative structure of a certain class of quantities.

Reductive accounts of quantitative structure have advantages and disadvantages. These accounts are inevitably limited in scope, but I've argued that this limitation is not evidence against reductive theories of quantity. A unified account is valuable only to the extent that it does not gloss over metaphysically important distinctions.

I contend that these reductive accounts are limited in scope because they offer explanations of *what it is* about those specific quantities that grounds their physical quantitative structure.

I've also outlined some other benefits of a move away from quantitative primitivism. One is that the absolutist–comparativist debate seems of central importance only if one is already committed to primitivism about the quantitative. Reductive theories of quantitativeness answer "neither" when asked which of the two kinds of quantitative structure we should take as fundamental. In addition, reductive accounts of quantity, because they do not build quantitative structure into their fundamental bases, are much better suited to representing quantities whose specific structures are contingent.

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