EMANUELE FRITTAION, ***Reverse Mathematics and Partial Orders***, University of Udine, Italy, 2014. Supervised by Alberto Marcone. MSC: 03F35, 03B30, 06A07. Keywords: reverse mathematics, partial orders.

Abstract

The thesis investigates the reverse mathematics of several theorems about partial orders with emphasis on scattered (no copy of the rationals) and FAC (no infinite antichains) partial orders. The thesis consists of 7 chapters.

Chapter 1 introduces some reverse mathematics and recursion theoretic results that are used throughout the thesis.

Chapter 2 investigates a theorem due to Bonnet which characterizes FAC partial orders in terms of initial intervals. Although the classical proof of Bonnet's theorem is based on a result by Erdös and Tarski on strong antichains, it is shown that one direction of Bonnet's theorem has indeed the same reverse mathematics strength of Erdös and Tarski theorem: in fact they are both equivalent to ACA_0 over RCA_0. The other direction of Bonnet's theorem is shown to lie below WKL_0 and strictly above RCA_0.

Chapter 3 studies four classically equivalent definitions of scattered FAC partial orders and provides a reverse mathematics analysis similar to that for well-partial orders given by Cholak, Marcone, and Solomon, *Reverse mathematics and the equivalence of definitions for well and better quasi-orders*, J. Symbolic Logic, 69(3):683–712, 2004. The analysis leads to consider a partition theorem on the rationals due to Erdös and Rado, which turns out to be of reverse mathematics strength between RT^2_2 and ACA_0.

Chapter 4 analyzes another characterization theorem for scattered FAC partial orders due to Bonnet. This theorem asserts that a countable partial order is scattered and FAC if and only if there are countably many initial intervals. It is shown that one direction (left to right) is equivalent to ATR_0 over ACA_0, while the other is provable in WKL_0 but not in RCA_0.

Chapters 5 and 6 investigate several results about scattered partial orders. The well-known Hausdorff's classification theorem for scattered linear orders is shown to be equivalent to ATR_0 over ACA_0. Two generalizations of Hausdorff's theorem, for scattered FAC partial orders and for countable FAC partial orders, are studied and shown to be provable in Π^1_2-CA_0.

Chapter 7 studies the notion of linearizability by showing that for certain order types τ (namely, $\omega, \omega^*, \omega + \omega^*$ and ζ) and for a certain notion of τ-likeness, the statement "every τ-like partial order has a τ-like linear extension" is equivalent to either $\mathsf{B}\Sigma^0_2$ or to ACA_0 over RCA_0.

Abstract prepared by Emanuele Frittaion
E-mail: emanuelefrittaion@gmail.com
URL: http://hdl.handle.net/10990/458

JAMES FIROZE APPLEBY, ***Choice Sequences and Knowledge States: Extending the Notion of Finite Information to Produce a Clearer Foundation for Intuitionistic Analysis***, Keele University, UK, 2017. Supervised by Peter Fletcher. MSC: 00A30, 03A05, 03F55, 03B60. Keywords: intuitionism, intuitionistic logic, choice sequences, lawless sequences, lawlike sequences, foundations of analysis, foundations of mathematics, foundations of intuitionistic analysis.

Abstract

There are currently four major formal foundational systems for intuitionistic analysis: *LS*, *CS* (both in [1]), *FIM* (given in [2]), and the derivable *FIRM-INT* (given in [3] but not named). All these systems rely on different universes of choice sequences and different conceptions of what a choice sequence is. There is a strong common ground between these systems as they use the same very restrictive notion of finite information when dealing with these choice sequences—the notion of restricting ourselves to initial segments. This text

extends the notion of a choice sequence given in [4] and uses it to construct a generalised system capable of expressing results about intensional properties of choice sequences. This is achieved by constructing a language capable of representing intensional first order restrictions on choice sequences (the language of knowledge states) and their relations to other sequences. This extended system allows us to formulate a notion of lawlessness that evades a series of paradoxes highlighted in [4], allows us to prove a generalised form of open data and offers additional clarity to other key areas of the theory. When a certain set of restrictions are applied to this extended theory (extensionality and a second order restriction on knowledge states) we obtain a system suitable for the foundation of analysis.

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Abstract prepared by James Firoze Appleby
E-mail: j.f.appleby.work@gmail.com
URL: http://eprints.keele.ac.uk/4359

Diana Carolina Montoya, ***Some Cardinal Invariants of the Generalized Baire Spaces***, Universität Wien, Austria, 2017. Supervised by Sy-David Friedman. MSC: 03E17, 03E35, 03E55. Keywords: cardinal invariants, consistency results, forcing, large cardinals.

Abstract

The central theme of the research in this dissertation is the well-known *Cardinal invariants of the continuum*. This thesis consists of two main parts which present the results obtained in joint work with (alphabetically): Jörg Brendle, Andrew Brooke-Taylor, Vera Fischer, Sy-David Friedman, and Diego Mejía.

The first part focuses on the generalization of the classical cardinal invariants of the continuum to the generalized Baire spaces κ^κ, when κ is a regular uncountable cardinal. First, we present a generalization of some of the cardinals in Cichoń's diagram to this context and some of the *ZFC* relationships that are provable between them. Furthermore, we study their values in some generic extensions corresponding to $\lhd\kappa$-support and κ-support iterations of generalized classical forcing notions. We point out the similarities and differences with the classical case and explain the limitations of the classical methods when aiming for such generalizations. Second, we study a specific model where the ultrafilter number at κ is small, 2^κ is large and in which a larger family of cardinal invariants can be decided and proven to be $< 2^\kappa$.

The second part focuses exclusively on the countable case: We present a generalization of the method of matrix iterations to find models where various constellations in Cichoń's diagram can be obtained and the value of the almost disjointness number can be decided. The method allows us also to find a generic extension where seven cardinals in Cichoń's diagram can be separated.

Abstract directly taken from the thesis
E-mail: dcmontoyaa@gmail.com
URL: http://www.logic.univie.ac.at/∼montoyd8/Main.pdf