MARGINAL STABILITY AND CHAOS IN THE SOLAR SYSTEM

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1. Introduction

The motion of the planets is one of the best modelized problems in physics, and its study can be practically reduced to the study of the behavior of the solutions of the well known gravitational equations, neglecting all dissipation, and treating the planets as mass points. In fact, the mathematical complexity of this problem, despites its apparent simplicity is daunting and has been a challenge for mathematicians and astronomers since its formulation three centuries ago.

With the advent of computers, numerical integration of the planetary equations appears as a straightforward way to overcome the complexity of the solutions. Moreover, since the work of Poincaré, it was known that the perturbative methods previously used in planetary computations could not provide good approximations of the solutions over infinite time.

But numerical integration of the planetary trajectories, since now, has always been bounded by the available computer technology. The first numerical long time studies of the solar system were limited to the motion of the outer planets, from Jupiter to Pluto (Cohen *et al.*, 1973; Kinoshita and Nakai, 1984). Indeed, the more rapid the orbital movement of a planet, the more difficult it is to numerically integrate its motion. To integrate the orbit of Jupiter, a step-size of 40 days will suffice, while a step-size of 0.5 days is required to integrate the motion of the whole solar system using a conventional multistep integrator.

The project LONGSTOP (Carpino *et al.*, 1987; Nobili *et al.*, 1989) used a CRAY to integrate the system of outer planets over 100 million years. At about the same time, calculations of the same system were carried out at MIT over even longer periods, corresponding to times of 210 and 875

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million years, using a vectorized computer specially designed for the task (Applegate *et al.*, 1986; Sussman and Wisdom, 1988). This latter integration showed that the motion of Pluto is chaotic, with a Lyapunov exponent of 1/20 million years. But since the mass of Pluto is very small (1/130000000 the mass of the Sun), this does not induce macroscopic instabilities in the rest of the solar system, which appeared relatively stable in these numerical studies.

2. Chaos in the Solar System

My approach was different and more in the spirit of the analytical works of Laplace and Le Verrier. Indeed, since these pioneer works, the Bureau des Longitudes has traditionally been the place for development of analytical planetary theories (Brumberg and Chapront, 1973; Bretagnon, 1974; Duriez, 1979). All these studies are based on classical perturbation series; thus, implicitly, they assume that the motion of the celestial bodies are regular and quasiperiodic. The methods used are essentially the same which were used by Le Verrier, with the additional help of the computers. Indeed, such methods can provide very satisfactory approximations of the solutions of the planets over a few thousand years, but they will not be able to give answers to the question of the stability of the solar system over time span comparable to its age. This difficulty which is known since Poincaré is one of the reasons which motivated the previously quoted long time numerical integrations. Nevertheless, it should be stressed that, until 1991, the only numerical integration of a realistic model of the full solar system was the ephemeris DE102 of JPL (Newhall *et al.*, 1983) which spanned only 44 centuries.

A first attempt consisted to extend as far as possible the classical analytical planetary theories, but it was realized quite rapidly that this was hopeless when considering the whole solar system, because of severe convergence problems encountered in the secular system of the inner planets (Laskar, 1984). I thus decided to proceed in two very distinct steps: a first one, purely analytical, consisted in the averaging of the equations of motion over the rapid angles, that is the motion of the planets along their orbits. This process was conducted in a very extensive way, without neglecting any term, up to second order with respect to the masses, and through degree 5 in eccentricity and inclination. The system of equations thus obtained comprises some 150000 terms, but it can be considered as a simplified system, as its main frequencies are now the precessing frequencies of the orbits of the planets, and no longer comprise their orbital periods. The full system can thus be numerically integrated with a very large stepsize of about 500 years. Contributions due to the Moon and to the general relativity are added without difficulty.

This second step, i.e. the numerical integration, is very efficient because of the symmetric shape of the secular system, and was conducted over 200 millions years in just a few hours on a super computer. The main results of this integration was to reveal that the whole solar system, and more particularly the inner solar system (Mercury, Venus, Earth, and Mars), is chaotic, with a Lyapunov exponent of 1/5 million years (Laskar, 1989). An error of 15 meters in the Earth's initial position gives rise to an error of about 150 meters after 10 million years; but this same error grows to 150 million km after 100 million years. It is thus possible to construct ephemerides over a 10 million year period, but it becomes practically impossible to predict the motion of the planets beyond 100 million years.

This chaotic behavior essentially originates in the presence of two secular resonances among the planets: $\theta = 2(g_4 - g_3) - (s_4 - s_3)$, which is related to Mars and the Earth, and $\sigma = (g_1 - g_5) - (s_1 - s_2)$, related to Mercury, Venus, and Jupiter (the g_i are the secular frequencies related to the perihelions of the planets, while the s_i are the secular frequencies of the nodes) (Laskar, 1990). The two corresponding arguments change several times from libration to circulation over 200 million years, which is also a characteristic of chaotic behavior. When these results were published, the only possible comparison was the comparison with the 44 centuries ephemeris DE102, which already allowed to be confident on the results (Laskar, 1986, 1990). At the time, there was no possibility to obtain similar results with direct numerical integration. In fact, partly due to the very rapid advances in computer technology, and in particular to the development of workstations, only two years later, Quinn *et al.* (1991) were able to publish a numerical integration of the full solar system, including the effects of general relativity and the Moon, which spanned 3 million years in the past (completed later on by an integration from -3Myrs to +3Myrs). Comparison with the secular solution of (Laskar, 1990) shows very good quantitative agreement and confirms the existence of secular resonances in the inner solar system (Laskar *et al.*, 1992a). Later on, using a symplectic integrator directly adapted towards planetary computations which allowed them to use a larger stepsize of 7.2 days, Sussman and Wisdom (1992) made an integration of the solar system over 100 million years which confirmed the existence of the secular resonances as well as the value of the Lyapunov exponent of about 1/5 Myrs for the solar system.

3. Planetary evolution over Myr

The planetary eccentricities and inclinations present variations which are clearly visible over a few million of years (Fig .1). Indeed, this was known since Laplace and LeVerrier (for a detailed account see Laskar 1992b), us-

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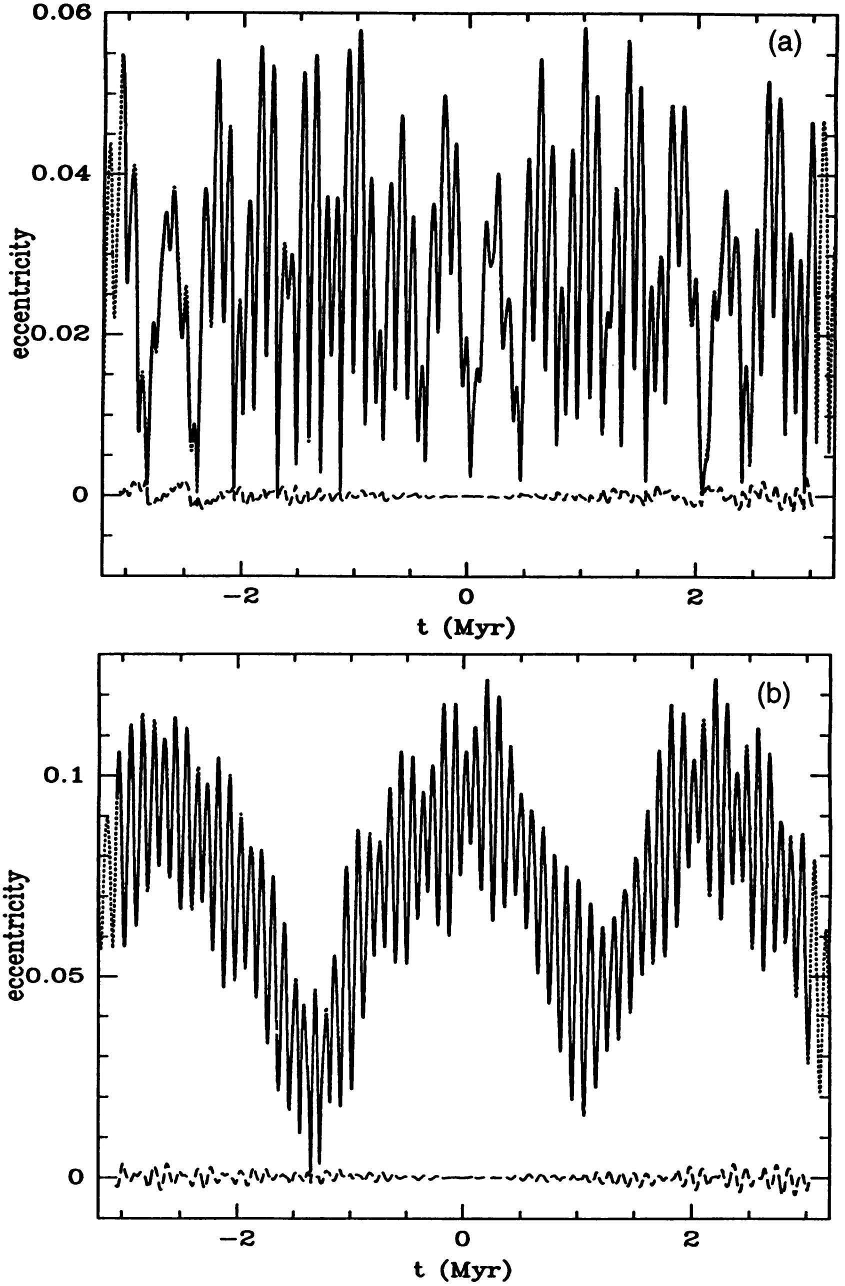

Figure 1. The eccentricity of the Earth (a) and Mars (b) during a 6 Myr timespan centered at the present. The solid line is the numerical solution from Quinn *etal.* (1991) and the dotted line the integration La90 of the secular equations (Laskar, 1990). For clarity, the difference between the two solutions is also plotted (from Laskar, *et al.*, 1992).

ing the secular equations. Over 1 million years, perturbation methods will give a good account of these variations which are mostly due to the linear coupling present in the secular equations. These secular variations involve the precessional periods of the orbits, ranging from 40 000 years to a few million of years. From -200 Myr to +200 Myr, the behavior of the solutions for the outer planets (Jupiter, Saturn, Uranus and Neptune) are very similar to the behavior over the first million years and the motion of these planets appears to be very regular, which was also shown very precisely by mean of frequency analysis (Laskar, 1990).

For the Earth, over such time span, the chaotic effect will induce a lost of predictability for the orbit. The additional change of eccentricity resulting from the chaotic diffusion is moderate and may be estimated to about 0.01 for the Earth (Laskar, 1992a,b). The most perturbed planet is Mercury, the effects of its chaotic dynamics being clearly visible over 400 million years (Laskar, 1992a,b).

It should be stressed that the exponential divergence of the orbits revealed by the computation of the Lyapunov exponent result mostly from the change from libration to circulation of the resonant precession angles, which induce after some time a total indeterminacy of the precessional angles of the orbit, that is its orientation in space. The eccentricity and inclination (which are action-like variables) variations due to the chaotic diffusion is much less rapid, and an important question is to estimate their wandering over the time of the life of the solar system.

4. Planetary evolution on Gyr time scales

If the motion of the solar system were close to quasiperiodic, that is close to a KAM tori, then it could be expected that some bound on the possible diffusion of the orbit over 5 Gyr would result from a Nekhoroshev-like theorem. In fact, as it was shown in (Laskar, 1990), although the system reduced to the outer planets may be considered as close to a KAM tori, the full solar system evolves far from a KAM tori of maximal dimension and diffusion of the action-like variables (eccentricity and inclination) occurs. The natural question is thus to estimate this diffusion. Let us remind that contrarily to two degrees of freedom systems, where the diffusion may be bounded, in such many degrees of freedom system (15 independent degrees of freedom for the secular system), there exist no results on the existence on invariant set which will bound the evolution of the system on infinite time span.

One may be tempted to try to integrate the motion of the solar system over 5 Gyr, that is over its expecting time life. For direct numerical integrations, this can be considered as an interesting challenge as it is still out of

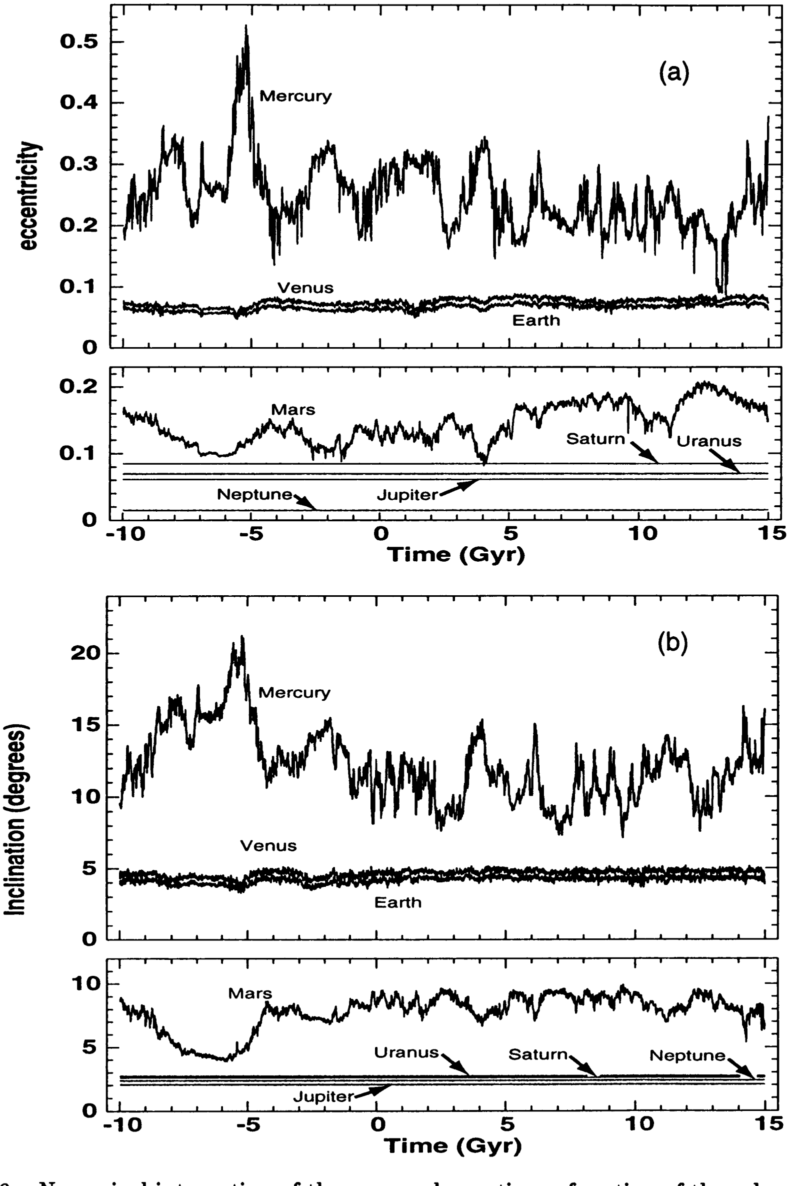

Figure 2. Numerical integration of the averaged equations of motion of the solar system 10 Gyr backward and 15 Gyr forward. For each planet, the maximum value obtained over intervals of 10 Myr for the eccentricity (a) and inclination (in degrees) from the fixed ecliptic J2000 (b) are plotted versus time. For clarity of the figures, Mercury, Venus and the Earth are plotted separately from Mars, Jupiter, Saturn, Uranus and Neptune. The large planets behavior is so regular that all the curves of maximum eccentricity and inclination appear as straight lines. On the contrary, the corresponding curves of the inner planets show very large and irregular variations, which attest to their diffusion in the chaotic zone.(Laskar, 1994)

reach of present computer technology, but it should be stressed, that by no means it can be considered as the description of the evolution of the solar system over 5 Gyr. Indeed, because of the exponential divergence with a Lyapunov time of 5Myr, after about 100 Myr the computed solution will be very different from the real solution followed by the actual solar system. Such a solution still present some interest, as it gives one of the possible behavior of the solar system, but it is much more important to obtain some description of the chaotic zone where the solar system evolves. In particular, it is more interesting to estimate the speed of diffusion in this chaotic zone. For such a goal, an integration of the solar system over 5 Gyr can be used, but will not be sufficient. Quite surprisingly, we can use integrations over even longer time span, which will act as scout exploring this chaotic zone. We can also send multiple of these explorers with very close initial conditions, in order to reach a larger portion of the phase space which can be attained by the solar system in 5 Gyr.

In order to achieve this task, it becomes quite obvious that we need to be able to integrate very rapidly the motion of the solar system, and the secular system of equations was even more simplified (Laskar, 1994), retaining only 50 000 terms and conserving the symmetries of the equations. Doing that, only about 6000 terms really need to be computed during the evaluation of the second hand member of the equations and the computations could be achieved on an IBM RS6000/370 workstation at a rate of about 1 day of CPU time per Gyr, without any loss in the precision.

As we want to understand the dynamics of this secular system, it is actually necessary to make the integration with great accuracy. The secular system is an approximation of the real equations of motion, but by understanding completely the global dynamical behavior of this system, we will obtain a lot of information on the original system.

Some first integrations were conduced over 25 Gyr (-10Gyr to + 15 Gyr) (Fig.2). It may seem strange to try to track the orbit of the solar system over such an extended time, longer than the age of the solar system, but one should understand that it is done in order to explore the chaotic zone where the solar system evolves and, after 100 Myr, can give only an indication of what can happen. On the other hand, if there is a sudden increase of eccentricity for one planet after 10 Gyr, this still tells us that such an event could probably also occur over a much shorter time, for example in less than 5 Gyr. In the same way, what happens in negative time can happen as well in positive time.

In order to follow the diffusion of the orbits in the chaotic zone, one needs quantities which behave like action variables, that is quantities which will be constant for a regular (quasiperiodic) solution of the system. Such quantities are given here by the maximum eccentricity and inclination attained by each planet during intervals of 10 Myr (Fig. 2).

The behavior of the large planets is so regular that all the corresponding curves appear as straight lines (Fig. 2). On the contrary, the maxima of eccentricity and inclination of the inner planets show very large and irregular variations, which attest to their diffusion in the chaotic zone. The diffusion of the eccentricity of the Earth and Venus is moderate, but still amounts to about 0.02 for both planets. The diffusion of the eccentricity of Mars is large and reaches more than 0.12, leading to values higher than 0.2 for the eccentricity of Mars. For Mercury, the chaotic zone is so large (more than 0.4) that it reaches values larger than 0.5 at some time. The behavior of the inclination is very similar.

Strong correlations between the different curves appear in figure 2. Indeed, as the solar system wanders in the chaotic zone, it is dominated by the linear coupling among the proper modes of the averaged equations (Laskar, 1990), which induces a very similar behavior for the maximum eccentricity and inclination of Venus and the Earth. This coupling is also noticeable in the solution of Mars. On the other hand, an angular momentum integral exists in the averaged equations and explains why when Mercury's eccentricity and inclination increase, the similar quantities for Venus, the Earth and Mars decrease. Thus it appears that, despite the small values of the inner planets' masses, the conservation of angular momentum plays a decisive role in limiting their excursions in the chaotic zone.

5. Escaping planets

At some time, Mercury suffered a large increase in eccentricity (Fig. 2) rising up to 0.5. But this is not sufficient to cross the orbit of Venus. The question then arises whether it is possible for Mercury to escape from the solar system in a time comparable to its age. A first attempt to answer this was made by slightly changing the initial conditions for the planets. Indeed, because of the chaotic behavior, very small changes in the initial conditions lead to completely different solutions after 100 Myr. Using this, I decided to change only one coordinate in the position of the Earth, amounting to a physical change of about 150 meters (10^{-9} in eccentricity). The full system was integrated with several of these modified solutions, but this led to similar (although different) solutions. In fact, it should not be too easy to get rid of Mercury, otherwise it would be difficult to explain its presence in the solar system.

I thus decided to guide Mercury somewhat towards the exit. A first experiment was done for negative time: for 2 Gyr, the solution is left unchanged, then, 4 different solutions are computed for 500 Myr, in each of which the position of the Earth is shifted by 150 meters, in a different direction (due to the exponential divergence, this corresponds to a change smaller than Planck's length in the original initial conditions).

The solution which leads to the maximum value of Mercury's eccentricity is retained up to the nearest entire Myr, and is started again. In 18 of such steps, Mercury attains eccentricity values close to 1 at about −6 Gyr (Fig 2) when the solution enters a zone of greater chaos, with Lyapunov time $\approx$ 1 Myr, giving rise to much stronger variations of the orbital elements of the inner planets. A second solution was also computed in positive time, with changes in initial condition of only 15 meters instead of 150 meters. As anticipated, this led to a similar increase in Mercury's eccentricity, this time in only 13 steps and about 3.5 Gyr (Fig 2).

While the eccentricity increases, the inclination of Mercury can change very much but the computation of the relative positions of the intersection of the orbits of Mercury and Venus with their line of nodes demonstrated that the orbits effectively intersect at about 3.5 Gyr. At this time, the two planets can experience a close encounter which can lead to the escape of Mercury or to collision.

For very high eccentricity of Mercury, the model used here no longer gives a very good approximation to the motion of Mercury, but it is very important to know that in this approximation, the chaotic zone allows the escape of a planet from the solar system in a time smaller than the expected life of the solar system, due to diffusion in the chaotic zone. Even more, in this averaged system, the degrees of freedom corresponding to semi major axes and mean longitudes are removed, but in the real system the addition of these extra degrees of freedom will probably lead to even stronger chaotic behavior, as in general, addition of degrees of freedom increases the stochasticity of the motion.

Similar computations were made for Mars and the Earth, but did not lead up to now to an escaping solution. For the Earth, the maximum eccentricity reached after 5 Gyr is about 0.1, while for Mars, the eccentricity attained 0.25 after 5 Gyr. With such a high eccentricity, Mars comes very close to the Earth, and it may be possible to find some escaping solution for Mars when considering the complete equations, but this probably needs the next generations of computers.

6. Marginal stability of the solar system

The existence of an escaping orbit for Mercury does not mean that this escape is very likely to occur. In fact, the solution computed here which lead to an escape was very carefully tailored, by selecting at each step one solution among 4 equivalent ones. The result is the existence for an escaping orbit, but does not tell us the probability for this escape to occur.

The computation of an estimate of this probability would require to take into account the full equations in order to be accurate. From the present computation, it can be thought that this probability is small, but not null, which is compatible with the present existence of Mercury.

Even without speaking of escaping orbits, the very large diffusion of the inner planets orbits is very striking. Even after the discovery of the chaotic behavior of the solar system, and despite the results of (Laskar, 1990), many assumed that the chaotic diffusion in the solar system was very small. Here is clearly demonstrated that for the inner planets, it is not the case. More, for the inner planets, the excursion of the eccentricity and inclination variables seems to be essentially constrained by the angular momentum conservation. This is quite surprising, when considering that the essential part of the angular momentum comes from the outer planets. In fact, the outer planets system is very regular, and practically no diffusion will take place among the degrees of freedom related to the outer planets. Thus, exchanges of angular momentum among the proper modes related to the inner planets result from the chaotic diffusion. This explains that when the maximum eccentricity of Mercury increases, the maximum eccentricity of Venus, the Earth and Mars decreases. One can also notice that the eccentricity curves of Venus and the Earth are very similar. This is due to the strong linear coupling between the proper modes of these two planets.

On figure 2, it is evident that the less massive planets are subject to the largest variation of eccentricity. This becomes obvious when considering that these variations are essentially bounded by the angular momentum conservation, which for each planets is proportional to $m\sqrt{a}$, where m is the mass of the planet, and a its semi major axis.

If, for each planet, we consider the maximum diffusion of the eccentricity over 5 Gyr (Fig. 3) we find that Mercury's eccentricity can go sufficiently high to allow Mercury's orbit to cross the orbit of Venus, Venus and the Earth's eccentricity can go up to 0.1, and Mars as high as 0.25. Apart from some small place in between Venus and the Earth, or the Earth and Mars, all the inner solar system is swept by the planetary orbits, and the small planets (Mercury and Mars) are the planets which present the largest excursions. Practically, we can conclude that the inner solar system is full. That is there is no room for any extra planet. Indeed, even if there are some place which seems not to be possibly reached in 5 Gyr, the additional planet orbit will present some eccentricity variations, and thus most probably will intersect with one of the already existing orbits. If we add a large planet, of the size of the Earth or Venus, its orbital elements will not vary much but it will induce strong short periods perturbations. On the contrary, a small object will suffer large orbital variations, as it will not be much constrained

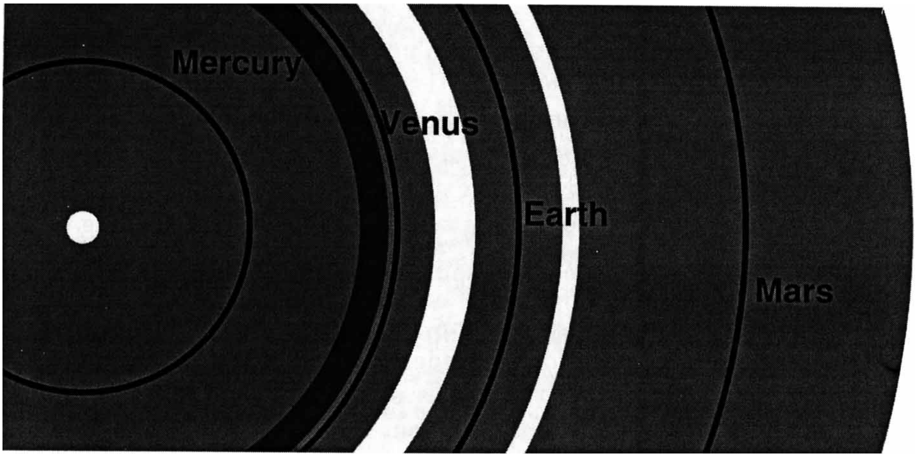

Figure 3. Estimates of the zones possibly occupied by the inner planets of the solar system over 5 Gyr. The circular orbits correspond to the bold lines, and the zones visited by each planet resulting from the possible increase of eccentricity are the shaded zones. In the case of Mercury and Venus, these shaded zones overlap. Mars can go as far as 1.9 AU, which roughly corresponds to the inner limit of the asteroid belt (Laskar, 1995).

by the angular momentum conservation. In this case, encounters with the already existing planets is very probable. It could be said that the variations which are plotted in fig 3 are the maximum variations possible over 5 Gyr, and not the most probable variations. This is true, but the addition of an extra planet will most probably increase very much the diffusion by increasing the numbers of degrees of freedom, and these maximum possible variations can probably be considered as the probable variations over 5 Gyr in the eventuality of the addition of an extra planet in the inner solar system. It becomes thus interesting to speak of marginal stability when considering the solar system. Maybe there was some extra planet at the early stage of formation of the solar system, and in particular in the inner solar system, but this lead to so much instability that one of the planets (probably among the smallest ones, of the size of Mercury or Mars) suffers a close encounter, or a collision with the other ones. This lead eventually to the escape of this planet and the remaining system gets more stable. In this case, at each stage, the system should have a time of stability comparable with its age, which is roughly what is achieved now, when ones finds that escape of one of the planets (Mercury) can occur within 5Gyr.

7. Conclusions

The analysis of the possible diffusion of the planetary orbits over 5 Gyr gives new insight on several question on the formation and evolution of the solar system. First of all, the existence of an escaping orbit for Mercury demonstrates that the the solar system in not stable, even when considering the strongest meaning of this word. On the contrary, although the solar system is not stable, it can be considered as marginally stable, that is. strong instabilities (collision or escape) can only occur on a time scale comparable to its age, that is about 5 Gyr. Some extra inner planets may have existed, but their existence gave rise to a much more unstable system, leading to the escape or collision of one of the planets. The organization of the inner planetary system is thus most probably due to its long run orbital evolution, and not uniquely to its rapid (less than 100 million years) formation process. This result is important for the understanding of the formation of the solar system, as it tells us that the solar system at the end of its formation process may have been significantly different from the present one, and has then evolved towards the present configuration because of the gravitational instabilities. It should be said that the outer system is very stable, but the long time recent numerical integrations (Gladman and Duncan, 1990; Holman and Wisdom, 1993; Levison and Duncan, 1993) also demonstrate that the outer solar system is full, that is most of the objects introduced in this system will escape on time scale much shorter than 5 Gyr. Apart from some special locations, stable zones only begins at about 40 AU, where some objects were recently founded. The inner solar system is also full, from the 0 AU to about 2 AU, which coincide with the beginning of the asteroidal belt. It should be interesting to investigate this point further using simulations with the addition of an extra planets, but many features have already been deduced here from the present computations. In particular, in the repartition of the inner planets, which can be thought as the so called Titius-Bode law, it is very striking that the spacing between the planets does not seem to be related to their masses. Indeed, when considering the most direct perturbation, that is the short-period perturbations, the zone depleted by a planet should increase with its mass, due to the overlap of the mean motion resonances. This seems to be primordial for the outer planets system (although more complicated combination of resonance may be present involving also secular resonances, as for the asteroid dynamics), but does not work for the inner planets system, where the short-period perturbation are not very important. In this case, as was presented above, the smallest a planet is, the largest will be its diffusion due to the chaotic behavior of the secular system. There will thus be an equilibrium between the short term perturbations which increases with the mass of the planet, and the long time diffusion of the orbital eccentricity and inclination, which is larger for the small planets. These competing effects could end up with an apparent rapartition which does not depend any longer on the masses. In any case, the marginal stability of the solar system revealed by the analysis of its long time behavior over 5 Gyr is an indication that its present organization results from its dynamical evolution.

In particular, one may be now tempted to answer to the question of what will be a generic planetary system ?

Considering the present results on our solar system, I would think that a generic planetary system will always be in a state of marginal stability, resulting from its gravitational interactions. If the formation process is such that there exists some large outer planets and some small inner planets, after 5 Gyr, the inner planets will therefore be subject to some instabilities similar to the present ones, and thus so will be their obliquities (Laskar and Robutel, 1993), with all the climate implications (Laskar, 1993; Laskar *et al.*1993). In particular, a planetary system with only one or two planets should be excluded, or, if it does exist, would be crowded with asteroids everywhere which would be the original remaining planetesimals, not ejected by planetary perturbations.

Most of the results presented here rely on the analysis of the secular equations of the solar system and not on the complete equations. This was the price to pay for allowing a more global approach on the problem of the stability and long time evolution of the solar system. It is quite obvious that some integrations of the full equations are still needed, but it is doubtful that these future integrations will change much the global landscape of the dynamics of the solar system portrayed here.

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