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ARTICLE

Endogenous innovation scale and patent policy in a monetary schumpeterian growth model

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Abstract

This paper develops a monetary R&D-driven endogenous growth model featuring endogenous innovation scales and the price-marginal cost markup. To endogenize the step size of quality improvement, we propose a tradeoff mechanism between the risk of innovation failure and the benefit of innovation success in R&D firms. Several findings emerge from the analysis. First, a rise in the nominal interest rate decreases economic growth; however, its relationship with social welfare is ambiguous. Second, either strengthening patent protection or raising the professional knowledge of R&D firms leads to an ambiguous effect on economic growth. Third, the Friedman rule of a zero nominal interest rate fails to be optimal in view of the social welfare maximum. Finally, our numerical analysis indicates that the extent of patent protection and the level of an R&D firm's professional knowledge play a crucial role in determining the optimal interest rate.

Keywords: Intellectual property rights; economic growth; endogenous innovation scales; endogenous markups; inflation

JEL Classification: O30; O40; E41; L11

1. Introduction

This paper develops a Schumpeterian monetary growth model featuring an endogenous innovation scale and professional knowledge within R&D firms, and then uses the model to explore the long-run effects of monetary policy and patent protection policy on economic growth and social welfare. The reasons for selecting such macroeconomic policies and model features are based on the following empirical observations:

First, by using cross-industry, cross-country panel data for 15 industrial OECD countries covering the period 1995–2005, Aghion et al. (2012) find that, by virtue of credit and liquidity constraints, countercyclical monetary policies are powerful in raising R&D investment and long-run growth. Building on the Aghion et al. (2012) finding, this paper is motivated to develop a monetary R&D-driven growth model and discuss the efficacy of monetary policy in relation to R&D investment and economic growth.

Second, a significant body of empirical studies, including works by Scherer (1982), Griliches (1992), Hall (1996), Jones and Williams (1998), and Hall and Lerner (2010), suggest that R&D investment in most countries has deviated from the socially optimal level. Patent protection policy is commonly viewed as a crucial and effective tool for the government to remedy this market failure in R&D investment (on this refer to Chu (2022) for a survey). This motivates us to explore the effects of patent protection policy on innovation and economic growth.

Third, empirical findings from Shenhar (1993) and Robertson and Gatignon (1998) indicate that R&D firms freely adjust the magnitude of quality improvement based on the circumstances they encounter, and this adjustment in the innovation scale is crucially related to their professional knowledge. These empirical findings are inconsistent with the theoretical setting in the existing literature on Schumpeterian quality-ladder models, which unanimously assume that the step size of quality improvement is exogenous. To reflect these empirical findings, this paper is motivated to build up a Schumpeterian quality-ladder model characterized by an endogenous innovation scale and professional knowledge within R&D firms. We then employ this model to examine how the endogenous innovation scale and the professional knowledge level affect economic growth and social welfare.

Compared to the existing literature on monetary policy and R&D-driven growth, the main salient feature of our analysis is that the R&D firm is inclined to choose the innovation scale freely in order to achieve profit maximization. To be more specific, existing studies on the monetary Schumpeterian growth model, such as Chu and Cozzi, 2014 and Chu et al. 2015, generally assume that the innovation scale of the R&D firm is exogenous. This simplified assumption implies that the R&D firm cannot choose the magnitude of quality improvement in innovation to maximize its profits. However, this assumption does not fit the empirical observations, such as those in Shenhar (1993), Robertson and Gatignon (1998) and Muratori (2020), with the results of the analysis being insufficiently complete to describe the R&D firm's behavior.

To reflect the empirical observations, we relax the assumption regarding the exogenous innovation scale for the R&D firm and focus on examining how the endogenous innovation scale will affect the linkages among monetary policy, economic growth, and social welfare. To rationalize the endogenous innovation scale, we emphasize that the R&D firm will choose the magnitude of the innovation scale so as to maximize its profit. More specifically, a rise in the size of the innovation scale exerts two conflicting effects on the expected profit of the R&D firm from innovation. On the one hand, it increases the markup of monopolistic intermediate goods, and therefore causes a rise in the expected profit of the R&D firm from innovation. On the other hand, it leads the R&D firm to face a higher risk of innovation failure, and hence causes a decline in the expected profit of the R&D firm from innovation. The R&D firm thus selects the optimal innovation scale at the level where these two conflicting effects are balanced.

Apart from providing a positive analysis on how the endogenous innovation scale mechanism affects the linkage between monetary policy and economic growth, this paper also presents a normative analysis regarding how the government sets its optimal monetary policy from the viewpoint of welfare maximization. In his pioneering article, Friedman (1969) proposes that, to lead the economy in the direction of increased efficiency, the optimal money growth targeting should be set in such a way that the nominal interest rate goes to zero. The result of a zero nominal interest rate is now well known as the Friedman rule in the literature. This paper will analyze whether the Friedman rule is valid when the innovation scale is endogenously determined by the R&D firm.

The main findings and contributions of this paper are as follows. First, in response to stronger patent protection, the R&D firm is inclined to choose a smaller innovation scale (i.e. a lower innovation challenge project). Intuitively, a stronger patent protection policy implies that once R&D firms achieve successful innovation, they will secure a more advantageous monopoly position and anticipate greater future profits. Consequently, to mitigate the risk of innovation failure and secure future monopolistic profits with a higher likelihood, R&D firms are inclined to reduce their incentives to pursue more ambitious innovation projects. With this endogenous step size of improvement, we find that strengthening patent protection is ambiguously related to economic growth. More specifically, as documented by Li (2001), strengthening patent protection leads to a rise in the patent value, which attracts the R&D firm to hire more labor in R&D. Accordingly, it will stimulate economic growth. However, compared with previous studies on the exogenous innovation scale, with the additional channel of an endogenous innovation scale, strengthening patent protection tends to lead the R&D firm to choose a smaller innovation scale, which will reduce the step size of quality improvement. This gives rise to an additional effect that harms economic growth. As a result, when the channel through which the innovation scale that is endogenously determined by the R&D firm is brought into the picture, strengthening patent protection tends to generate an ambiguous overall effect on economic growth. This finding not only significantly diverges from the existing literature, which predominantly focuses on an exogenous innovation scale, but also provides a plausible explanation for the empirically ambiguous results regarding the relationship between the extent of patent protection and economic growth, as suggested by Thompson and Rushing (1999) and Falvey et al. (2006).

The second finding of this paper is that the R&D firm is inclined to choose a larger innovation scale when it has a higher level of professional knowledge. This finding is quite intuitive. When the R&D firm has a higher level of professional knowledge, it will face a lower risk of innovation failure. As a result, the R&D firm is motivated to choose a larger size of innovation scale to achieve profit maximization, which is similar to recent empirical research by Hsu et al. (2021).[1] Armed with this endogenous innovation scale, the R&D firm that possesses a higher level of professional knowledge is not necessarily associated with higher economic growth. Intuitively, a higher level of professional knowledge motivates the R&D firm to choose a larger innovation scale to maximize its profit. This tends to raise the risk of innovation failure, and thus is detrimental to economic growth. Nevertheless, the R&D firm that chooses a larger innovation scale is also characterized by a larger improvement in technology, which is in turn associated with a higher patent value. This would be beneficial to economic growth. Thus, with these two conflicting effects on economic growth, the relationship between the level of the R&D firm's professional knowledge and economic growth is ambiguous. Previous studies, that primarily focused on the exogenous innovation scale, have overlooked the discussion concerning how the R&D firm's professional knowledge interacts with its innovation behavior, thereby neglecting the linkages among the R&D firm's professional knowledge, innovation investment, and economic growth. Compared to the existing literature, our study endogenizes the innovation scale by exploring the impact of the R&D firm's professional knowledge on its innovative behavior, thereby affecting economic growth. This provides a possible vehicle to establish the empirical link between the R&D firm's professional knowledge and its innovative behavior, as put forth by Hsu et al. (2021).

The third finding is related to the social welfare effect of monetary policy implemented in the form of nominal interest rate targeting. Our analysis reveals that investment in R&D involves two primary market distortions, which significantly influence the government's decision in determining the optimal nominal interest rate. The first externality arises from the fact that the R&D firm does not take into account the benefits of R&D in final good production when determining its R&D investment, resulting in under-investment in R&D compared to the social optimum. The second externality occurs when existing intermediate goods firms are replaced by new entrants following their successful innovation. However, the loss of monopolistic profit for existing intermediate goods firms is not considered in each R&D firm's profit-maximization decision, thus leading to over-investment in R&D compared to the social optimum.[2] With these two conflicting effects on social welfare, to maximize social welfare, the government will choose the optimal nominal interest rate at the positive level where these two conflicting effects are balanced. This leads to the outcome that the Friedman rule of a zero nominal interest rate fails to be optimal. Moreover, in going beyond the existing studies, we highlight a key role of the endogenous innovation scale on these two conflicting effects. With this, we are able to further examine how the optimal interest rate is related to the endogeneity of the innovation scale.

This paper also provides a quantitative assessment by resorting to a numerical analysis, from which two main findings emerge. First, the monetary authority should choose a higher optimal nominal interest rate to correct the distortions from strengthening patent protection. Second, in response to a lower level of the R&D firm's professional knowledge, the monetary authority should set a higher optimal nominal interest rate as a remedy.

2. Related Literature

One of the most salient features of our paper is the setting of an endogenous innovation scale, which leads to the endogenous markup of monopolistic intermediate goods. The step size of quality improvement is usually specified to be exogenous in existing studies. However, this specification regarding the scale of quality improvement does not fit realistic observations. For example, Shenhar (1993) and Robertson and Gatignon (1998) indicate that innovation plans could be classified into four types in terms of their innovation risks, which are low technological uncertainty, medium technological uncertainty, high technological uncertainty, and super high technological uncertainty. The main feature of this classification is that higher risk is associated with a higher return from innovation. Therefore, R&D firms will choose different types of innovation plan according to their capacity. In addition, by using United States Patents and Trademark Office data, Muratori (2020) finds that the quality of entrants' innovation increases over time during the period between 1980 and 2000. Equipped with the Shenhar (1993), Robertson and Gatignon (1998), and Muratori (2020) observation, this paper sets up an R&D-based model that is able to reflect the R&D firm's optimal decision regarding the scale of quality improvement.

There are only a few theoretical studies attempting to deal with the issue of the endogenous innovation scale in the R&D-based growth model. Among these studies, Chu and Pan (2013) introduce the profit-division rule between incumbents and new entrants in a Schumpeterian growth model and examine the effects of blocking patents (leading patent breadth) on economic growth and social welfare. Their analysis assumes that the new entrant will infringe the patent of the incumbent, and hence should transfer a share of its profit to the incumbent. In line with Chu and Pan (2013), Lu (2022) and Lu et al. (2024) also endogenize the step size of quality improvement by resorting to the presence of blocking patents. In a recent article, Hu et al. (2021) provide another mechanism for the endogenous step size, that is, R&D firms can increase patent value by hiring more research labor to improve the quality increment. In departing from Chu and Pan (2013), Hu et al. (2021), Lu (2022), and Lu et al. (2024), this paper instead develops the Schumpeterian growth model featuring the lagging patent breadth (i.e. patent breadth against imitation), and proposes an alternatively plausible mechanism to endogenize the step size of quality improvement. To be more specific, based on the empirical finding in Shenhar (1993) and Robertson and Gatignon (1998), this paper endogenizes the step size of innovation by way of the mechanism through which the size of the innovation scale is crucially related to the risk of innovation failure.

Our paper is also related to earlier studies that examine the patent protection-economic growth nexus in the R&D-based growth model. Some empirical studies point to a non-monotone relationship between patent protection and economic growth. Within the literature, Thompson and Rushing (1999) find that strengthening patent protection will stimulate economic growth only for advanced countries, while it has an insignificant correlation with economic growth for developing countries. Falvey et al. (2006) find that intellectual property rights (IPR) protection is positively related to growth for low- and high-income countries, but not for middle-income countries. In addition, some theoretical studies point out that the strengthening of IPR protection may impede innovation or growth, such as in Goh and Olivier (2002), Horii and Iwaisako (2007), Iwaisako and Futagami (2013), Pan et al. (2018), and Chen (2021). However, these studies remain silent on how patent protection affects growth through the endogenous adjustment in the innovation scale. Our study aims to fill this gap and shows that the endogenous innovation scale is a plausible channel for the emergence of the inverse U-shaped relationship between patent protection and economic growth.

In addition, our paper is related to previous theoretical studies that examine monetary policy and social welfare in the innovation-led growth model. To analyze the effects of monetary policy, we introduce money demand via cash-in-advance (CIA) constraints on R&D investment and consumption in the Schumpeterian growth model, which is in line with the following empirical findings. Hall (1992), Himmelberg and Petersen (1994), Opler (1999), and Brown and Petersen (2009) show that cash flows are positively and significantly related to R&D investment in US firms. Hall et al. (1999) and Brown and Petersen (2011) further indicate that the sensitivity of R&D investment-cash flows is stronger than that for physical investment. Moreover, Bates et al. (2009) show a sharp increase in the average cash-to-assets ratio for US industrial firms during 1980-2006 mainly because of increased R&D expenditures. A recent study by Brown and Petersen (2015) points out that firms with positive R&D investments tend to expend their cash reserves on buffering R&D instead of on protecting fixed investment. The empirical evidence mentioned above reveals that R&D firms will finance their R&D investment via cash holdings as a response to financial frictions. This paper therefore employs a CIA constraint on R&D, in line with Chu and Cozzi (2014), to capture the cash requirements for R&D investment.

Theoretical underpinning studies that analyze the effect of monetary policy on growth and social welfare in the R&D-based growth model have recently been developed, such as Chu and Lai (2013), Chu and Cozzi (2014), Chu et al. (2015), Zheng et al. (2021), and Huang et al. (2021). Perhaps for analytical convenience, these studies unanimously specify that the step size of innovation is constant and, as pointed out previously, the specification of an exogenous innovation scale does not fit the empirical evidence. This paper thus contributes to this strand of the literature by highlighting the importance of endogenous adjustment in the innovation scale, and then shows that the interest rate and social welfare exhibit an inverse U-shaped relationship with an endogenous innovation scale.

The remainder of this paper is organized as follows. Following the review of the related literature in Section 2, Section 3 develops a monetary Schumpeterian growth model with an endogenous innovation scale. Section 4 derives the macroeconomic equilibrium, and examines the effects of the endogenous innovation scale and monetary policy on labor allocation. Section 5 analytically examines the growth effect of the endogenous innovation scale and monetary policy. Section 6 deals with the welfare analysis and provides a numerical analysis. Finally, Section 7 concludes.

3. The model

We consider a Schumpeterian growth model in which growth is driven by innovation that improves the quality of intermediate goods (see, e.g. Grossman and Helpman (1991)). To introduce money demand, following Chu and Cozzi (2014), we impose a CIA constraint on the firm's R&D investment and consumption. The major departure from existing studies is that, in our framework, the R&D firm is allowed to choose a suitable innovation scale (innovation project) after balancing its risks and benefits. To be more specific, we introduce a variable to capture the risk of the different innovation scales. R&D firms that select a larger innovation scale (a high innovation challenge project) will bear a higher risk of innovation.[3] In what follows, we will in turn describe the economy's structure.

3.1. The household

The representative household has N_t members, and the members grow over time at the exogenous rate $n > 0$. By the law of motion, we can write $\dot{N}_t = nN_t$. The representative household derives utility from the consumption of final goods and leisure, and its lifetime utility function can be expressed as follows:

$$U = \int_0^\infty e^{-\rho t} \left[\ln c_t + \theta \ln \left(1 - l_t\right)\right] dt, \tag{1}$$

where c_t is the consumption of final goods per member of a household at time t, and l_t is the supply of labor per member of a household at time t. The parameters $\rho > 0$ and $\theta > 0$ denote the subjective time preference and leisure preference, respectively. The household maximizes lifetime utility (1) subject to the following budget constraint:

$$\dot{a}_t + \dot{m}_t = (r_t - n)\, a_t + w_t l_t + \tau_t - c_t - (\pi_t + n)\, m_t + i_t b_t, \tag{2}$$

where a_t denotes the real value of assets (in the form of equity issued by intermediate goods firms) owned by each member of the household, m_t is real money balances held by each member of the household, w_t is the real wage rate,[4] and τ_t is the lump-sum transfer. r_t is the real interest rate, π_t is the inflation rate, i_t is the nominal interest rate, and b_t is the real money balances that each member of the household lends to R&D firms to finance their R&D investment. According to the Fisher equation, the nominal interest rate can be expressed as $i_t = r_t + \pi_t$. Each member of the household holds real money balances m_t which are used partly to consume final goods and partly to lend to R&D firms. The cash-in-advance constraint takes the following form: $\xi c_t + b_t \leq m_t$, where $\xi > 0$ is the fraction of consumption subject to the CIA constraint.

Each member of the household maximizes equation (1) subject to equation (2) and $\xi c_t + b_t \leq m_t$, which is binding in equilibrium. The optimum conditions for consumption and labor supply are, respectively, given by:

$$\frac{1}{c_t} - \lambda_t\, (1 + i_t \xi) = 0, \tag{3}$$

$$w_t \left(1 - l_t\right) = \theta\, c_t\, (1 + i_t \xi)\,, \tag{4}$$

where λ_t is the shadow value of the real wealth (the sum of a_t and m_t) owned by each member of the household. Moreover, the Euler equation for the dynamic optimization of consumption behavior is given by:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho - n. \tag{5}$$

3.2. *Final goods*

The final goods are produced by competitive firms using a unit continuum of intermediate goods industries indexed by $j \in [0, 1]$, according to a standard Cobb-Douglas aggregator. The production function for the final goods is given by:

$$y_t = \exp\left(\int_0^1 \ln x_t\left(j\right) dj\right), \tag{6}$$

where $x_t(j)$ is the quantity of intermediate good j.

The profit-maximization problem for the final goods firm implies the following conditional demand function for intermediate good j:

$$x_t\left(j\right) = \frac{y_t}{p_{x,t}\left(j\right)}, \tag{7}$$

where $p_{x,t}(j)$ is the price of $x_t(j)$.

3.3. *Intermediate goods*

There is a unit continuum of intermediate goods industries indexed by $j \in [0, 1]$. Each intermediate good firm is a temporary quality leader in industry j. Thus, it produces the highest-quality intermediate good and enjoys a monopoly position until the next higher-quality innovation takes place. In line with Grossman and Helpman (1991) and Chu and Cozzi (2014), we assume that labor is the only factor involved in the production of intermediate goods. The production function for each intermediate good firm is given by:

$$x_t\left(j\right) = z^{q_t(j)}L_{x,t}\left(j\right); \quad j \in [0, 1], \tag{8}$$

where $L_{x,t}(j)$ denotes the labor input required to produce the intermediate good in industry j at time t, $z > 1$ is the step size of the quality improvement,[5] and $q_t(j)$ is the number of the quality improvements in an industry j during the time interval between 0 and t. Notice that, in departing from existing studies in which innovation comes from quality improvement, in this paper the step size of quality improvement z is an endogenous variable. It can be treated as the extent of the innovation chosen by R&D firms.

Based on equation (8), the marginal cost of producing an intermediate good is given by:

$$MC_t\left(j\right) = \frac{w_t}{z^{q_t(j)}}; \quad j \in [0, 1]. \tag{9}$$

Following the existing studies on quality-improving R&D, we assume that the current and former industry leaders engage in a standard Bertrand price competition. In addition, in line with Li (2001), Iwaisako and Futagami (2013), Iwaisako (2020), and Furukawa et al. (2023), we introduce a policy variable, denoted by $\beta(\geq 1/z)$, to capture the extent of the patent breadth. Therefore, the profit-maximizing pricing for the industry leader can be expressed as:

$$p_{x,t}\left(j\right) = \beta z MC_t\left(j\right) = \beta z \frac{w_t}{z^{q_t(j)}}; \quad \beta \geq 1/z. \tag{10}$$

It should be noted that the industry sets its price equal to marginal cost when $\beta = 1/z$ is true. This case is thus associated with zero patent protection.[6]

Equipped with equation (8), the monopolistic profit for industry j is given by:

$$\Pi_{x,t}\left(j\right) = \left(\frac{\beta z - 1}{\beta z}\right) y_t. \tag{11}$$

Equation (11) shows that a larger patent breadth β increases the amount of monopolistic profit created by innovations.

Moreover, the wage income received by workers in industry j is given by:

$$w_t L_{x,t}\left(j\right) = \frac{1}{\beta z} y_t. \tag{12}$$

3.4. Research and Innovation

Let $v_t(j)$ denote the patent value of an industry $j \in [0, 1]$ and $I_t(j)$ denote the Poisson arrival rate of innovation. Following the standard approach of R&D growth model, such as Huang and Ji (2013), Zheng et al. (2020), and Chu et al. (2021), the familiar no-arbitrage condition for $v_t(j)$ is given by:

$$r_t v_t\left(j\right) = \Pi_{x,t}\left(j\right) + \dot{v}_t\left(j\right) - I_t\left(j\right) v_t\left(j\right). \tag{13}$$

Equation (13) indicates that the return on innovation $r_t v_t(j)$ is equal to the sum of the monopolistic profit $\Pi_{x,t}(j)$, the capital gains $\dot{v}_t(j)$, and the expected capital loss $I_t(j)v_t(j)$ stemming from creative destruction.

There is a continuum of R&D firms, indexed by $j \in [0, 1]$, and each R&D firm employs R&D labor $L_{r,t}(j)$ to create innovations and chooses the size of the innovation scale z to innovate upon the existing products. Once an R&D firm successfully innovates in industry j, it becomes the leader in industry j and produces the new version of good j, whose quality increases in z compared with the best existing version.

The expected profit of the j-th R&D firm $\Pi_{RD,t}(j)$ is:

$$\Pi_{RD,t}\left(j\right) = I_t\left(j\right) v_t\left(j\right) - \left(1 + i_t\right) w_t L_{r,t}\left(j\right). \tag{14}$$

In equation (14), $I_t(j)v_t(j)$ is the expected revenue of the R&D firm from investing in innovation and $w_tL_{r,t}(j)$ is the wage payment of the R&D firm. Similar to Christiano et al. (2005) and Neumeyer and Perri (2005), each of the R&D firms has to pay production costs before cashing its output sales. This creates the need for working capital, and the shortage of working capital is funded by the households. As a result, following Chu and Cozzi (2014), the total amount of real money balances that the household lends to the j-th R&D firm to finance R&D investment is equal to $w_tL_{r,t}(j)$, and the cost of borrowing is $i_tw_tL_{r,t}(j)$. Thus, the total production cost of R&D is $(1+i_t)w_tL_{r,t}(j)$.

Finally, the *firm-level* arrival rate of innovation $I_t(j)$ is given by:

$$I_t\left(j\right)=\frac{\eta L_{r,t}^{\psi-1}L_{r,t}\left(j\right)}{\kappa N_t^{\psi}}, \tag{15}$$

where $L_{r,t}=\int_0^1 L_{r,t}(j)dj$, the parameter $\eta>0$ is an innovation productivity parameter of R&D labor, and κ denotes the complexity of innovation. To shed light on the intuition behind equation (15), we then rewrite it as:

$$I_t\left(j\right)=\frac{\eta}{\kappa}\underbrace{\left(\frac{L_{r,t}}{N_t}\right)^{\psi-1}}_{\substack{\textit{Allocation of resrearch}\\ \textit{labor to the R\&D sector}}}\underbrace{\left(\frac{L_{r,t}\left(j\right)}{N_t}\right)}_{\substack{\textit{Allocation of resrearch}\\ \textit{labor to industry } j}}. \tag{15a}$$

Two points related to the specification in equation (15a) should be addressed. First, not only the allocation of research labor to the R&D sector $(L_{r,t}/N_t)$, but also the allocation of research labor to industry $j(L_{r,t}(j)/N_t)$ is crucially related to the arrival rate of innovation of industry j. In line with existing studies, such as Jones (1995), Jones and Williams (2000), Chu and Cozzi (2014), and Chu et al. (2019), the allocation of research labor to the R&D sector is subject to the negative externality of duplication across the industries. As a result, the allocation of research labor to the R&D sector is inversely related to the arrival rate of innovation, where the parameter $\psi\in(0,1)$ reflects the extent of the duplication of innovation. This negative linkage is referred to by Jones and Williams (2000) as the *stepping on toes effect*. By contrast, the allocation of research labor to industry j is confined to the manpower allocated to the single and specific industry j, so that it does not involve cross-industry duplication. Accordingly, a rise in the allocation of research labor to industry j tends to raise the arrival rate of innovation of industry j. One point should be noted here. Our model degenerates to the Dinopoulos and Segerstrom (2010) and Chu and Cozzi (2014) model when the *stepping on the toes effect* is absent (i.e. $\psi=1$). Armed with this specific assumption, these studies specify that the arrival rate of innovation of industry j is positively related to the allocation of research labor to industry j.

Second, the complexity of innovation leads to a negative effect on the arrival rate of innovation. The complexity of innovation is specified in the form of $\kappa=z^{\phi}$, implying that a larger innovation scale will increase the complexity of innovation, and therefore further decrease the arrival rate of innovation. This specification echoes empirical findings from Shenhar (1993) and Robertson and Gatignon (1998) mentioned in Section 2. In addition, the parameter $\phi>0$ reflects the sensitivity of an expansion in the innovation scale to the complexity of innovation. Conceptually, the parameter ϕ can be viewed as a proxy for the level of professional knowledge that an R&D firm possesses. An R&D firm with a smaller ϕ indicates that it has a high level of professional knowledge, and hence will experience less complexity in innovation when expanding the innovation scale by one unit. Therefore, the parameter ϕ inversely measures the level of the R&D firm's professional knowledge.

To maximize the expected profit, the entrepreneur faces two decisions: choosing the size of the innovation scale z and hiring the amount of R&D labor $L_{r,t}(j)$. First, we deal with the optimal choice of the innovation scale z. Differentiating the expected profit stated in (14) with respect to z yields the following result:[7]

$$\frac{\partial \Pi_{RD,t}\left(j\right)}{\partial z} = I_t\left(j\right)\underbrace{\frac{\partial v_t\left(j\right)}{\partial z}}_{+} + v_t\left(j\right)\underbrace{\frac{\partial I_t\left(j\right)}{\partial z}}_{-}. \tag{16}$$

Equation (16) indicates that raising the innovation scale generates two conflicting effects on the expected profit of the R&D firm. On the one hand, it increases the patent value of an industry $v_t(j)$, and hence is beneficial to the expected profit of the R&D firm. On the other hand, a larger size of the innovation scale leads to the higher complexity of innovation, which further decreases the arrival rate of innovation $I_t(j)$, and hence is harmful to the expected profit of the R&D firm. Accordingly, the optimal size of the innovation scale $\tilde{z}$ is set at the value where these two conflicting effects are balanced:

$$\tilde{z} = \frac{1+\phi}{\beta\phi}. \tag{17}$$

Some notable results emerge from equation (17). First, higher patent protection (a higher value of β) leads to a decline in the optimal size of the innovation scale. Intuitively, a rise in patent protection β tends to raise the value of innovation $v_t(j)$. An R&D firm will thus suffer from a larger expected loss when it chooses a larger size of innovation scale and then experiences innovation failure. To reduce such an expected loss, an R&D firm is motivated to select a smaller size.

Second, monetary policy in the form of changes in the nominal interest rate has no effect on the step size of quality improvement. The rationale behind this result can be explained intuitively. Referring to equation (14), the nominal interest rate will affect the borrowing cost for R&D (denoted as $i_t w_t L_{r,t}(j)$). However, it is clear from equation (16) that the R&D firm's optimal decision on the innovation size is solely related to its expected revenue $I_t(j)v_t(j)$ and is independent of its total production cost $(1 + i_t)w_t L_{r,t}(j)$. Accordingly, a change in the nominal interest rate does not affect the optimal innovation size.

Third, a lower level of professional knowledge regarding innovation (a higher value of ϕ) causes a decline in the optimal size of the innovation scale. The economic reasoning behind this outcome is straightforward. The R&D firm with a lower level of professional knowledge faces a higher risk of innovation failure (a lower arrival rate of innovation). Thus, it is inclined to choose a smaller size of innovation scale to avoid innovation failure.

The above discussions lead to the following proposition:

Proposition 1. *Stronger patent protection and a lower level of professional knowledge of innovation tend to reduce the optimal size of the innovation scale.*

Based on equation (17), to ensure that the equilibrium step size of the quality improvement is greater than 1 (i.e. $\tilde{z} > 1$), we impose the following restriction on the parameter of patent protection.

Condition RPP (Restriction on Patent Protection):

$$\beta < (1+\phi)/\phi. \tag{18}$$

We can then analyze how the complexity of innovation will react following changes in patent protection and professional knowledge of innovation after taking the R&D firm's optimal decision regarding the innovation scale into consideration.

By inserting equation (17) into κ, we can obtain $\tilde{\kappa}$, which is the complexity of innovation after the optimal innovation scale ($\tilde{z}$) is determined by the R&D firm:

$$\tilde{\kappa} = \tilde{z}^{\phi} = \left(\frac{1+\phi}{\beta\phi}\right)^{\phi}. \tag{19}$$

It is straightforward from equation (19) to infer the following result:

$$\frac{\partial \tilde{\kappa}}{\partial \beta} = \frac{\phi \tilde{\kappa}}{\tilde{z}} \frac{\partial \tilde{z}}{\partial \beta} < 0 \tag{20}$$

Based on equation (17), the strengthening of patent protection leads the R&D firm to choose a smaller size of innovation, and hence equation (20) reveals that higher patent protection is associated with a lower complexity of innovation.

We then deal with the effect of a change in professional knowledge on the complexity of innovation. Differentiating equation (19) with respect to ϕ yields:

$$\frac{\partial \tilde{\kappa}}{\partial \phi} = \underbrace{\tilde{z}^{\phi} \ln \tilde{z}}_{\textit{Direct Effect}(+)} + \underbrace{\phi \tilde{z}^{\phi-1} \frac{\partial \tilde{z}}{\partial \phi}}_{\textit{Indirect Effect}(-)} . \tag{21}$$

Based on Condition RPP and equation (17), we can infer the results: $\tilde{z}^{\phi} \ln \tilde{z} > 0$ and $\phi \tilde{z}^{\phi-1}(\partial \tilde{z}/\partial \phi) < 0$. Accordingly, equation (21) shows that a decline in professional knowledge leads to two conflicting effects on the complexity of innovation. First, a fall in professional knowledge has a direct effect in terms of stimulating the complexity of innovation. Second, a fall in professional knowledge leads the R&D firm to choose a smaller size of innovation scale, and this will reduce the complexity of innovation.

Equipped with equations (17) and (21), we can infer the following result concerning the signs of $\partial \tilde{\kappa}/\partial \phi$:

$$\begin{cases} \dfrac{\partial \tilde{\kappa}}{\partial \phi} > 0 \quad \textit{if} \quad \dfrac{1+\phi}{\phi} \dfrac{1}{e^{1/(1+\phi)}} > \beta \, ; \\[2ex] \dfrac{\partial \tilde{\kappa}}{\partial \phi} < 0 \quad \textit{if} \quad \dfrac{1+\phi}{\phi} > \beta > \dfrac{1+\phi}{\phi} \dfrac{1}{e^{1/(1+\phi)}} . \end{cases} \tag{22}$$

Equation (22) indicates that the extent of patent protection plays a crucial role in determining the signs of $\partial \tilde{\kappa}/\partial \phi$. More specifically, if β is relatively small, the first positive effect dominates the second negative effect in equation (21), thereby yielding the result $\partial \tilde{\kappa}/\partial \phi > 0$. On the contrary, if β is relatively large, the first effect falls short of the second effect, so that $\partial \tilde{\kappa}/\partial \phi < 0$ holds. The intuition for why the extent of patent protection β is crucial for determining the sign of $\partial \tilde{\kappa}/\partial \phi$ can be explained with the aid of both the direct effect and the indirect effect displayed in equation (21). On the one hand, stronger patent protection prompts R&D firms to opt for a smaller innovation scale. This diminishes the value of the positive direct effect indicated in equation (21), expressed as $\partial(\tilde{z}^{\phi} \ln \tilde{z})/\partial \beta < 0$. On the other hand, strengthening patent protection prompts R&D firms to reduce their innovation scale, thus boosting the value of the negative indirect effect indicated in equation (21), denoted as $\partial\left(\phi \tilde{z}^{\phi-1}\left(\partial \tilde{z}/\partial \phi\right)\right)/\partial \beta > 0$.

Considering both the positive direct and negative indirect effects in equation (21) simultaneously, we can infer the following two results. First, in response to a gradual increase in β, the positive direct effect gradually diminishes, while the negative indirect effect gradually increases. Therefore, we can find a threshold value of β (denoted as $\hat{\beta}$) at which these two conflicting direct and indirect effects are balanced, which is $\hat{\beta} = (1+\phi)/\left(\phi e^{1/(1+\phi)}\right)$. Second, when β is relatively small (i.e. $\beta < \hat{\beta}$), the former positive direct effect dominates the latter negative indirect effect, resulting in $\partial \tilde{\kappa}/\partial \phi > 0$ in equation (21). Conversely, when β is relatively large (i.e. $\beta > \hat{\beta}$), the former positive direct effect falls short of the latter negative indirect effect, resulting in $\partial \tilde{\kappa}/\partial \phi < 0$ in equation (21).

The above discussions lead us to establish the following proposition:

Proposition 2. *A change in professional knowledge of innovation has an ambiguous effect on the complexity of innovation, crucially depending upon the extent of patent protection.*

Finally, we introduce the free-entry condition in the R&D sector, which implies that the following zero-expected-profit condition is satisfied:

$$v_t\left(j\right) = \frac{\tilde{z}^{\phi}\left(1+i_t\right)w_t N_t^{\psi}}{\eta L_{t,t}^{\psi-1}} \tag{23}$$

This equation is used for pinning down the allocation of the R&D labor $L_{r,t}(j)$.

3.5. Government and monetary authority

Let M_t denote the nominal money supply, P_t denote the price of the final goods, and $m_t = (M_t/P_t)/N_t$ denote real money balances per capita. Based on the definition of $m_t N_t = M_t/P_t$, the evolution of m_t can then be expressed as: $\dot{m}_t/m_t = \dot{M}_t/M_t - n - \pi_t$, where $\pi_t \equiv \dot{P}_t/P_t$ is the inflation rate of the price of final goods. The monetary policy instrument that we consider is i_t which is exogenously chosen by the monetary authority.

The government finances its lump-sum transfer payments for each member of the household by issuing money. The balanced budget constraint faced by the government can then be expressed as: $\dot{M}_t/P_t = \tau_t N_t$. Given the definition $m_t = (M_t/P_t)/N_t$, the government's budget constraint can then be alternatively written as: $\dot{m}_t + (\pi_t + n)m_t = \tau_t$.

4. Decentralized equilibrium and equilibrium labor allocation

The equilibrium is a time path of allocation $\left\{c_t, l_t, m_t, y_t, x_t\left(j\right), z, L_{x,t}, L_{r,t}\right\}_{t=0}^{\infty}$, a time path of prices $\{w_t, p_t(j), v_t, r_t, i_t\}_{t=0}^{\infty}$ and policies $\{i_t\}_{t=0}^{\infty}$. At each instant of time:

- households maximize lifetime utility taking $\{i_t, r_t, w_t\}$ as given;
- competitive final goods firms produce $\{y_t\}$ and choose $\{x_t(j)\}$ to maximize profit taking $\{p_{x,t}(j)\}$ as given;
- monopolistic intermediate goods firms produce $\{x_t(j)\}$ and choose $\{p_{x,t}(j), L_{x,t}(j)\}$ to maximize profit taking $\{w_t\}$ as given;
- R&D firms choose $\{z, L_{r,t}\}$ to maximize the expected profit taking $\{w_t, i_t, v_t\}$ as given;
- the government budget constraint is balanced such that $\dot{m}_t + (\pi_t + n)m_t = \tau_t$.
- the final goods market clears such that $y_t = c_t N_t$;
- the labor market clears such that $N_t l_t = L_{x,t} + L_{r,t}$;
- the amount of money borrowed by R&D firms is $b_t N_t = w_t L_{r,t}$.

In Appendix A, we show that the dynamic system has one positive characteristic root coupled with one jump variable. Therefore, the economy will jump immediately to a unique and stable balanced growth path. This result can be summarized in the following lemma:

Lemma 1. *The economy always jumps immediately to a unique and stable balanced growth path.*

Proof. *See Appendix A.* □

Then, we deal with the equilibrium labor allocation along the balanced growth path. To make the analysis tractable and clear, we assume that there is no negative duplication externality (i.e. $\psi = 1$). However, this assumption will be relaxed later in the social welfare analysis section. Under this assumption, we rewrite the market-clearing condition for labor in per capita terms as follows:

$$l_t = l_x + l_r, \tag{24}$$

where $l_r = L_{r,t}/N_t$ denotes the per capita R&D labor input, and $l_x = L_{x,t}/N_t$ denotes the per capita labor input required to produce intermediate goods.

Along the balanced growth path, inserting (11), (12), (15) and (23) into (13) yields:

$$(\beta\tilde{z}-1)\,l_x=(1+i_t)\left(l_r+\frac{\tilde{z}^{\phi}\rho}{\eta}\right). \tag{25}$$

Finally, substituting equation (12) and $y_t=c_tN_t$ into (4), we obtain:

$$l_t=1-\beta\tilde{z}\theta\,(1+i_t\xi)\,l_x. \tag{26}$$

From equations (24), (25) and (26), we can solve the equilibrium labor allocation, which is described by the following lemma:

Lemma 2. *The equilibrium labor allocation is given by:*

$$\tilde{l}_r=\frac{(\beta\tilde{z}-1)-(1+i)\,\frac{\tilde{z}^{\phi}\rho}{\eta}\,[1+\beta\tilde{z}\theta\,(1+i\xi)]}{(\beta\tilde{z}-1)+(1+i)\,[1+\beta\tilde{z}\theta\,(1+i\xi)]}, \tag{27}$$

$$\tilde{l}_x=\frac{(1+i)\left(1+\frac{\tilde{z}^{\phi}\rho}{\eta}\right)}{(\beta\tilde{z}-1)+(1+i)\,[1+\beta\tilde{z}\theta\,(1+i\xi)]}, \tag{28}$$

$$\tilde{l}=\frac{(\beta\tilde{z}-1)-(1+i)\left[\frac{\tilde{z}^{\phi}\rho}{\eta}\beta\tilde{z}\theta\,(1+i\xi)-1\right]}{(\beta\tilde{z}-1)+(1+i)\,[1+\beta\tilde{z}\theta\,(1+i\xi)]}, \tag{29}$$

where $\tilde{z}=(1+\phi)/\beta\phi$ reported in equation (16), and $\tilde{l}_r$, $\tilde{l}_x$ and $\tilde{l}$ are the steady-state values of l_r, l_x, and l, respectively.

4.1. Patent protection policy and research labor allocation

This subsection discusses the effect of patent protection on the allocation of research labor in the steady state. Differentiating (27) with respect to β yields:

$$\frac{\partial\tilde{l}_r}{\partial\beta}=\frac{\frac{(1+\phi)\tilde{z}^{\phi-1}}{\beta^2}\,(1+i)\left(\frac{\rho}{\eta}\right)\left[1+\frac{\theta(1+i\xi)(1+\phi)}{\phi}\right]}{\frac{1}{\phi}+(1+i)\left[1+\frac{\theta(1+i\xi)(1+\phi)}{\phi}\right]}>0. \tag{30}$$

Equation (30) denotes the positive linkage between patent protection and research labor allocation. The result in equation (30) leads us to establish the following proposition:

Proposition 3. *Strength of patent protection is positively related to the equilibrium allocation of research labor.*

The intuition behind Proposition 3 can be explained as follows. In the steady state, strengthening patent protection has three effects on the allocation of research labor. The first effect is that strengthening patent protection (i.e. an increase in β) enables each of the intermediate firms to charge a higher markup $\beta\tilde{z}$, as exhibited in equation (10). This will increase the profit of intermediate firms, and in turn raise the patent value of R&D firms. Then, the R&D firms are inclined to employ more research labor.

The second effect is that, in response to strengthening patent protection, the R&D firms will choose a smaller size of innovation scale (i.e. a decline in $\tilde{z}$), which will reduce the markup $\beta\tilde{z}$. With the same reasoning (but just the opposite) as in the first effect, the R&D firms will tend to employ less research labor.

The third effect is related to the R&D firm's choice of a smaller size of innovation scale in response to stronger patent protection, mentioned in the second effect. A smaller innovation scale implies a reduction in the complexity of innovation, which will lead the R&D firms to have a higher expected profit. Thus, the R&D firms will have an incentive to hire more research labor.

Considering all three effects together, as demonstrated in equation (30), the two positive effects on the allocation of research labor dominate the negative effect. Thus, we can infer that strengthening patent protection will stimulate the allocation of labor to R&D.

4.2. Monetary policy and research labor allocation

In this subsection, we examine the relationship between monetary policy and the allocation of research labor. Differentiating (27) with respect to i_t yields:

$$\frac{\partial \tilde{l}_r}{\partial i} = \frac{-\left\{\frac{\tilde{z}^{\phi}\rho}{\eta}\left[F + \frac{(1+i)(1+\phi)\theta\xi}{\phi}\right]\left[\frac{2}{\phi} + (1+i)\,F\left(1 - \frac{\tilde{z}^{\phi}\rho}{\eta}\right)\right]\right\}}{\left\{\frac{1}{\phi} + (1+i)\,F\right\}^2} < 0. \tag{31}$$

where $F = 1 + [\theta(1 + i\xi)(1 + \phi)/\phi]$. To make our analysis meaningful, we impose the restriction that the allocation of labor to R&D is positive (i.e. $\tilde{l}_r > 0$). With this restriction, equation (31) indicates that an increase in the nominal interest rate leads to a reduction in the allocation of R&D labor. Intuitively, increasing the nominal interest rate raises the working capital costs for R&D firms, and thus R&D firms will hire less research labor. The above discussion leads to the following proposition:

Proposition 4. *The equilibrium allocation of research labor is decreasing in the nominal interest rate.*

4.3. Professional knowledge of innovation and research labor allocation

We now examine how the professional knowledge of R&D firms affects the allocation of research labor. Differentiating (27) with respect to ϕ yields:

$$\frac{\partial \tilde{l}_r}{\partial \phi} = \frac{-1}{\phi^2}\,(1+i)\left(1 + \frac{\rho\tilde{\kappa}}{\eta}\right)[1 + \theta\,(1 + i\xi)] - \frac{\rho\,(1+i)\,F}{\eta\left[\frac{1}{\phi} + (1+i)\,F\right]}\underbrace{\frac{\partial\tilde{\kappa}}{\partial\phi}}_{?}. \tag{32}$$

Equation (32) shows that a decline in the professional knowledge regarding the innovation of the R&D firm (a rise in ϕ) has an ambiguous effect on the allocation of labor to R&D. Intuitively, a fall in the R&D firm's professional knowledge generates two effects on the allocation of research labor. First, the R&D firm will take action to choose a smaller size of innovation scale, thereby causing the intermediate firm to charge a lower markup. This will reduce the profit of the intermediate firm, and in turn decrease the patent value of the R&D firm. Consequently, the R&D firm will employ less research labor in response. Second, a decline in the R&D firm's professional knowledge will affect the complexity of innovation ($\partial\tilde{\kappa}/\partial\phi$). As exhibited in Proposition 2, this effect is ambiguous. By combining the first and second effects mentioned above, we can infer that the relationship between the level of the R&D firm's professional knowledge and research labor allocation will be uncertain.

Proposition 5. *The level of the R&D firm's professional knowledge is ambiguously related to the research labor allocation in the steady state.*

5. Growth effect

This section examines how patent protection, monetary policy, and the professional knowledge of R&D firms affect economic growth. To deal with this issue, we start by deriving the steady-state equilibrium economic growth rate. Substituting the production function for each intermediate good firm reported in equation (8) into the production function for final goods in equation (6) yields:

$$y_t = L_{x,t}Z_t; \quad Z_t = \exp\left(\int_0^1 q_t\left(j\right) dj \ \ln\left(z\right)\right), \tag{33}$$

where Z_t is the aggregate technology. By applying the law of large numbers in the balanced growth equilibrium, the aggregate technology Z_t can be further expressed as:

$$Z_t = \exp\left(\int_0^1 q_t\left(j\right) dj \ \ln\left(\tilde{z}\right)\right) = \exp\left(\int_0^t I_s \, ds \ \ln\left(\tilde{z}\right)\right). \tag{33a}$$

Based on equations (15), (33) and (33a) as well as the market-clearing condition for the final goods market, we can obtain the equilibrium economic growth rate as:

$$\tilde{g} = \frac{\dot{Z}_t}{Z_t} = \underbrace{\frac{\eta}{\tilde{\kappa}}}_{(\text{g}1)} \underbrace{\tilde{l}_r}_{(\text{g}2)} \underbrace{\ln\left(\tilde{z}\right)}_{(\text{g}3)}. \tag{34}$$

Equation (34) indicates that the economic growth rate is composed of three channels which are dubbed (g1), (g2), and (g3), respectively. The first channel (g1) reveals the effect of the complexity of innovation on the economic growth rate. An increase in the complexity of innovation (a rise in $\tilde{\kappa}$) implies a reduction in the arrival rate of innovation, and thus is detrimental to economic growth. The second channel (g2) represents the effect of the research labor input on the economic growth rate. Increasing the research labor input (a rise in $\tilde{l}_r$) will increase the arrival rate of innovation, and will therefore stimulate economic growth. The third channel (g3) exhibits the effect of the size of the innovation scale on economic growth. An expansion in the size of the innovation scale (a rise in $\ln(\tilde{z})$) will improve the aggregate technology and, as a result, is beneficial to economic growth. The last two channels (g2 and g3) of equation (34) are already being developed in standard Schumpeterian growth models,[8] whereas the first channel (g1) is main contribution of this paper and is not put forth in the literature.

One point must be particularly emphasized here. When the innovation step size is exogenous ($\tilde{z}$ is a constant value), both channels (g1) and (g3) remain intact due to $\tilde{\kappa} = \tilde{z}^{\phi}$. Under such a situation, the relevant policies will affect the balanced growth rate only through the second channel (g2). As a result, with an endogenous innovation scale, two additional channels are brought into the picture in our analysis.

5.1. Patent protection policy and economic growth

This subsection analyzes the impact of patent protection on economic growth. Differentiating (34) with respect to β gives rise to:

$$\frac{\partial \tilde{g}}{\partial \beta} = \tilde{g}\left[\frac{-1}{\tilde{\kappa}} \underbrace{\frac{\partial \tilde{\kappa}}{\partial \beta}}_{-} + \frac{1}{\tilde{l}_r} \underbrace{\frac{\partial \tilde{l}_r}{\partial \beta}}_{+} + \frac{1}{\tilde{z}\ln\left(\tilde{z}\right)} \underbrace{\frac{\partial \tilde{z}}{\partial \beta}}_{-}\right] \gtrless 0. \tag{35}$$

Equation (35) shows that strengthening patent protection has an ambiguous effect on economic growth. Specifically, strengthening patent protection has three effects on economic growth. First, as exhibited in equations (19) and (20), strengthening patent protection tends to lower the complexity of innovation ($\partial\tilde{\kappa}/\partial\beta < 0$ with $\tilde{\kappa} = \tilde{z}^{\phi} = [(1+\phi)/\beta\phi]^{\phi}$). This in turn leads to an increase in the arrival rate of innovation, and hence will stimulate economic growth. Second, strengthening patent protection will increase the research labor input ($\partial\tilde{l}_r/\partial\beta > 0$), as shown in

Proposition 3, and will thus be beneficial to economic growth. Finally, as pointed out in equation (17) and Proposition 1, along the balanced growth path, strengthening patent protection results in a smaller size of innovation scale (i.e. $\partial\tilde{z}/\partial\beta < 0$). This will lower the aggregate technology, and hence is harmful to economic growth. Accordingly, with two positive effects and one negative effect, strengthening patent protection thus generates an ambiguous impact on economic growth. Compared to the previous studies, this paper provides a new plausible vehicle to explain this ambiguous relationship (see Section 2). The foregoing discussion leads to the following proposition:

Proposition 6. *The effect of patent protection on economic growth is uncertain.*

Furthermore, we can delve more deeply to analyze whether monetary policy will strengthen or weaken the growth effect of patent protection policy. To this end, differentiating equation (35) with respect to i gives rise to:

$$\frac{\partial}{\partial i}\left(\frac{\partial\tilde{g}}{\partial\beta}\right) = \underbrace{\frac{-\tilde{g}}{\tilde{\kappa}\tilde{l}_r}\underbrace{\frac{\partial\tilde{\kappa}}{\partial\beta}}_{-}\underbrace{\frac{\partial\tilde{l}_r}{\partial i}}_{-}}_{\frac{\partial}{\partial i}\left(\frac{\partial g1}{\partial\beta}\right)<0} + \underbrace{\frac{\tilde{g}}{\tilde{l}_r}\underbrace{\frac{\partial}{\partial i}\left(\frac{\partial\tilde{l}_r}{\partial\beta}\right)}_{+}}_{\frac{\partial}{\partial i}\left(\frac{\partial g2}{\partial\beta}\right)>0} + \underbrace{\frac{\tilde{g}}{\tilde{z}\tilde{l}_r\ln(\tilde{z})}\underbrace{\frac{\partial\tilde{z}}{\partial\beta}}_{-}\underbrace{\frac{\partial\tilde{l}_r}{\partial i}}_{-}}_{\frac{\partial}{\partial i}\left(\frac{\partial g3}{\partial\beta}\right)>0}. \tag{35a}$$

where

$$\frac{\partial}{\partial i}\left(\frac{\partial\tilde{l}_r}{\partial\beta}\right) = \{1+(1+i)\left[\phi+\theta\left(1+\phi\right)\left(1+i\xi\right)\right]\}^{-2}\left(\frac{\rho\left(1+\phi\right)\tilde{z}^{\phi-1}}{\eta\beta^2}\right)\{\phi+\theta\left(1+\phi\right)\left[\left(1+i\xi\right)\right.$$
$$\left.+\left(1+i\right)\xi\right]\}>0.$$

Based on the first term on the right-hand side of equation (35a), it can be inferred that increasing the nominal interest rate will reduce the positive effect of lowering the complexity of innovation on economic growth when patent protection is strengthened (i.e. $\partial(\partial g1/\partial\beta)/\partial i < 0$). The second term on the right-hand side of equation (35a) shows that when the nominal interest rate increases, labor will shift from the R&D sector to the production sector. From equation (12), it can be deduced that wages will decrease, thereby reducing the cost of employing labor for R&D firms. Therefore, when patent protection increases, the magnitude of the increase in R&D labor will rise (i.e. $\partial(\partial\tilde{l}_r/\partial\beta)/\partial i > 0$), leading to an increase in the magnitude of economic growth enhancement (i.e. $\partial(\partial g2/\partial\beta)/\partial i > 0$). Finally, the third term on the right-hand side of equation (35a) demonstrates that as the nominal interest rate increases, the extent to which enhancing patent protection reduces economic growth by inducing R&D firms to choose a lower innovation scale decreases (i.e. $\partial(\partial g3/\partial\beta)/\partial i > 0$). Combining the three effects mentioned above, the impact of an increase in nominal interest rates on the influence of patent protection on economic growth is uncertain. In Section 6, we will demonstrate through quantitative analysis how an increase in nominal interest rates alters the magnitude of the impact of changes in patent protection policy on economic growth.

5.2. Monetary policy and economic growth

This subsection deals with the effect of monetary policy on economic growth. As mentioned above, the monetary authority implements its monetary policy by targeting the nominal interest rate i. Then, differentiating (34) with respect to i yields:

$$\frac{\partial\tilde{g}}{\partial i} = \eta\frac{1}{\tilde{\kappa}}\ln(\tilde{z})\underbrace{\frac{\partial\tilde{l}_r}{\partial i}}_{-} < 0. \tag{36}$$

Equation (36) shows that an increase in the nominal interest rate will reduce the balanced economic growth rate. As shown in Proposition 4, a rise in the nominal interest rate will increase the working capital cost of R&D firms. Thus, R&D firms have an incentive to hire less research labor, which is detrimental to the economic growth rate. One point deserves special mention here. As indicated in equation (17), a rise in the nominal interest rate has no effect on the optimal innovation size, and thus the growth effect of monetary policy is the same as that in Chu and Cozzi (2014) in this endogenous innovation size model.[9] Even though monetary policy has no effect on the step size of quality improvement, in Section 6 below, we will further show that the endogeneity of the innovation scale is crucial to the welfare effect of monetary policy and the validity of the Friedman rule. The result in equation (36) leads us to establish the following proposition:

Proposition 7. *The balanced growth rate decreases with the nominal interest rate.*

5.3. Professional knowledge of innovation and economic growth

In this subsection, we examine how the level of professional knowledge of the R&D firm affects economic growth. Differentiating (34) with respect to ϕ gives rise to:

$$\frac{\partial \tilde{g}}{\partial \phi} = \tilde{g} \left[\frac{-1}{\tilde{\kappa}} \underbrace{\frac{\partial \tilde{\kappa}}{\partial \phi}}_{?} + \frac{1}{\tilde{l}_r} \underbrace{\frac{\partial \tilde{l}_r}{\partial \phi}}_{?} + \frac{1}{\tilde{z} \ln(\tilde{z})} \underbrace{\frac{\partial \tilde{z}}{\partial \phi}}_{-} \right] \gtrless 0. \tag{37}$$

Equation (37) shows that the relationship between the R&D firm's professional knowledge regarding innovation and economic growth is ambiguous. As indicated in equation (37), a decline in the R&D firm's professional knowledge (i.e. an increase in ϕ) will affect the balanced growth rate through three channels. The first channel is the complexity of innovation. As shown in Proposition 2, a decline in the R&D firm's professional knowledge exerts an ambiguous effect on the complexity of innovation, consequently leading to an uncertain impact on economic growth. The second channel is the research labor allocation. As exhibited in Proposition 5, a reduction in the R&D firm's professional knowledge has an ambiguous effect on research labor, thereby resulting in an uncertain impact on economic growth. The third channel is the step size of the quality improvement. Based on equation (17), the R&D firm will choose the smaller step size of the innovation scale in association with a lower degree of professional knowledge, which will stifle economic growth. Accordingly, putting these three channels together yields an ambiguous linkage between the R&D firm's professional knowledge of innovation and economic growth.

Proposition 8. *The level of the R&D firm's professional knowledge regarding innovation is ambiguously related to economic growth.*

6. Quantitative analysis

In this section, we analyze the effects of patent protection and monetary policies on social welfare. Given that the social welfare function derived later is quite complex, it is very difficult for us to provide a closed-form solution to solve how the welfare level is affected in response to patent protection and monetary policies. We thus need to resort to a numerical analysis. In addition, to make our numerical analysis more general, we return to our general theoretical setting and consider the effect of a negative duplication externality, $0 < \psi < 1$, on the arrival rate of innovation.

Substituting the optimal values of consumption and labor supply into equation (1), the social welfare function (i.e. the indirect lifetime utility of the representative household) $\tilde{U}$ is given by:

$$\tilde{U} = \frac{1}{\rho} \left[\ln(c_0) + \frac{\tilde{g}}{\rho} + \theta \ln\left(1 - \tilde{l}\right) \right], \tag{38}$$

where c_0 is the initial consumption. Using the market-clearing condition for final goods, (6), and (8), we have $c_0 = Z_0 l_x$, where $Z_0 = \exp\left(\int_0^1 q_0(j)dj \; \ln(z)\right)$. In line with Dinopoulos and Segerstrom (2010), we assume that $q_0(j) = 0$, and thus we can infer the results $Z_0 = 1$ and $c_0 = l_x$. From (6), (8), (15) and the market-clearing condition for final goods, we can show derive that, when the negative duplication externality is brought into the picture, the balanced growth rate reported in equation (34) should be modified as follows:

$$\tilde{g} = \frac{\dot{Z}_t}{Z_t} = \frac{\eta}{\tilde{\kappa}} \tilde{l}_r^{\psi} \; \ln(\tilde{z}). \tag{39}$$

Finally, inserting $c_0 = l_x$, (39), and the labor market-clearing condition into equation (38) yields:

$$\tilde{U} = \frac{1}{\rho}\left[\ln\left(\tilde{l}_x\right) + \frac{\eta}{\rho\tilde{\kappa}}\tilde{l}_r^{\psi} \ln(\tilde{z}) + \theta \ln\left(1 - \tilde{l}\right)\right]. \tag{40}$$

By inserting equations (27), (28), and (29) into (40), we can then examine the linkage between the social welfare level and the nominal interest rate and discuss whether the government can choose a positive nominal interest rate that maximizes the $\tilde{U}$ reported in (40). With this examination, we can infer whether the Friedman rule of a zero nominal interest rate may fail to be optimal.

We then offer a quantitative assessment by resorting to a numerical simulation, and use it to examine the macroeconomic effects on the optimal innovation size, the complexity of innovation, the allocation of research labor, economic growth, and social welfare. In particular, with the help of the numerical simulation, we can assess the validity of the Friedman rule and illustrate how the optimal interest rate is related to relevant parameters.

To perform the numerical analysis, we assign the following eight structural parameters $\{\rho, i, \xi, \eta, \beta, \phi, \theta, \psi\}$. The baseline parameters are chosen from the commonly used values in the existing literature or calibrated to match the US empirical data. Following Acemoglu and Akcigit (2012), the discount rate ρ is set to 0.05. In line with Chu and Cozzi (2014), the nominal interest rate i is set to 8%, and the consumption-CIA parameter ξ is set to 0.2. To make the markup lie within the reasonable range estimated across industries (e.g. Norrbin (1993), Basu (1996), and Jones and Williams (2000)), the parameter for the level of professional knowledge ϕ is chosen as 3.03, which makes the markup 1.33. In addition, in line with Acemoglu and Akcigit (2012) and Chu and Cozzi (2014), the innovation scale z is set to 1.05, which allows us to pin down the parameter for patent protection, $\beta = 1.267$.

Similar to Chu et al. (2012) and Yang (2018), the empirical long-run growth rate of GDP per capita in the USA is 1.5%, which enables us to calibrate the R&D productivity, $\eta = 0.852$. Next, the leisure parameter θ is calibrated to be 0.177 so as to match the per capita labor supply $l = 0.3$. Finally, following Jones and Williams (2000), the parameter for the negative duplication externality ψ is set to 0.5. Table 1 reports the baseline parameter values.

Fig. 1 depicts the effects of patent protection and the R&D firm's professional knowledge on the optimal innovation scale. Strengthening patent protection (i.e. a rise in β) and the lower level of the R&D firm's professional knowledge (i.e. a rise in ϕ) will cause the R&D firm to choose a smaller innovation scale, which is in line with equation (17). Strengthening patent protection raises the cost of an innovation failure. In addition, a lower level of the R&D firm's professional knowledge increases the risk of innovation failure at the same innovation scale. Accordingly, as illustrated in both the left and right panels of Fig. 1, in response to a rise in either β or ϕ, the R&D firm is motivated to choose a smaller innovation scale to reduce the risk of innovation failure.

Fig. 2 shows how patent protection and the R&D firm's professional knowledge affect the complexity of innovation. Strengthening patent protection (i.e. a rise in β) leads the R&D firm to choose a smaller innovation scale and thus reduces the complexity of innovation, as illustrated in the left panel of Fig. 2. Moreover, as mentioned in equation (21), a lower level of the R&D firm's professional knowledge (i.e. a rise in ϕ) has two effects on the complexity of innovation. On the

Table 1. Baseline parameters

Parameter	Value	Source/Target
ρ	0.05	Acemoglu and Akcigit (2012)
i	0.08	Chu and Cozzi (2014)
ξ	0.2	Chu and Cozzi (2014)
ψ	0.5	Jones and Williams (2000)
ϕ	3.03	Monopolistic markup = 1.33
β	1.267	Innovation scale = 1.05
η	0.852	Per capita output growth rate = 1.5%
θ	0.177	Per capita labor supply = 0.3

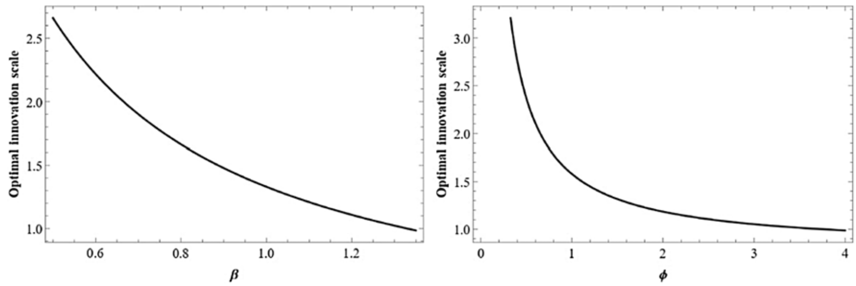

Figure 1. The innovation scale effect of patent protection and the R&D firm's professional knowledge.

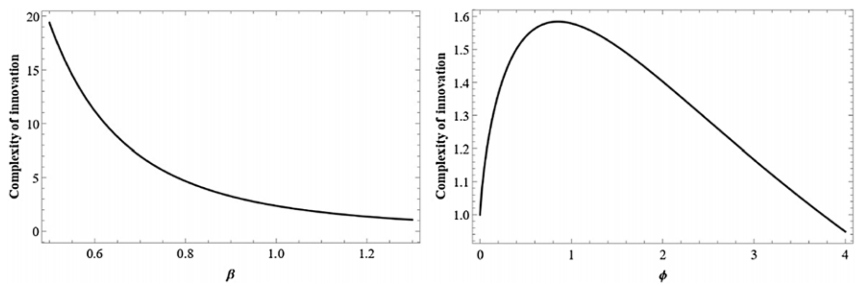

Figure 2. The complexity of innovation effect of patent protection and the R&D firm's professional knowledge.

one hand, it will directly raise the complexity of innovation at the same innovation scale. On the other hand, the R&D firm is inclined to choose a smaller innovation scale to respond to the lower level of professional knowledge, and this will reduce the complexity of innovation. Accordingly, as exhibited in the right panel of Fig. 2, the R&D firm's professional knowledge generates an inverted U effect on the complexity of innovation depending on the relative size between these two effects.

Fig. 3 illustrates the impact of patent protection and the R&D firm's professional knowledge on the allocation of research labor. Building on the earlier intuitive explanation regarding Proposition 3, strengthening patent protection (i.e. a rise in β) tends to encourage the allocation of labor to R&D. Therefore, as depicted in the left panel of Fig. 3, an increase in the extent of patent protection β is associated with a rise in the labor input of R&D firms $\tilde{l}_r$.

Moreover, as previously explained concerning Proposition 5, a reduction in the R&D firm's professional knowledge (i.e. a rise in ϕ) leads to an ambiguous effect on the allocation of research

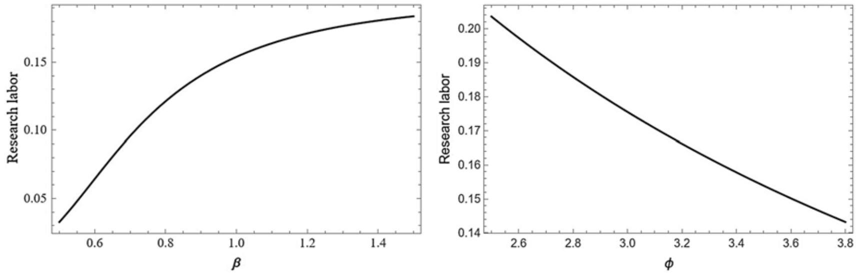

Figure 3. The research labor allocation effect of patent protection and the R&D firm's professional knowledge.

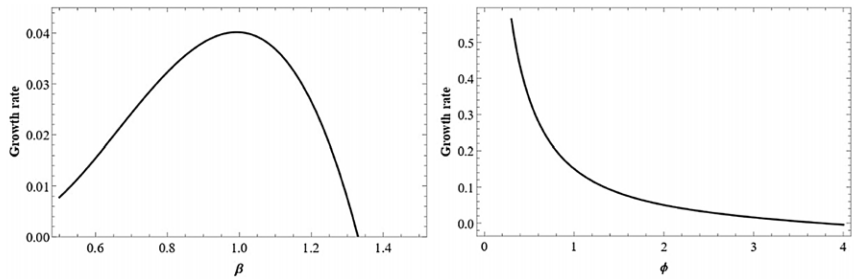

Figure 4. The growth rate effect of patent protection and the R&D firm's professional knowledge.

labor. However, according to our benchmark parameter values, the first negative effect outweighs the second ambiguous effect on the right-hand side of equation (32). Consequently, as exhibited in the right panel of Fig. 3, a decrease in the extent of patent protection (i.e. a rise in ϕ) is associated with a reduction in the labor input of R&D firms $\tilde{l}_r$.

Fig. 4 displays the effects of patent protection and the R&D firm's professional knowledge on economic growth. We first discuss the effect of patent protection on growth, as illustrated in the left panel of Fig. 4. It shows an inverted U-shaped relationship between patent protection policy and economic growth. Intuitively, strengthening patent protection (i.e. a rise in β) reduces the complexity of innovation and increases the research labor input, as shown in equations (20) and (30), both of which tend to stimulate economic growth. On the other hand, strengthening patent protection also decreases the innovation scale, and hence leads to a reduction in the technology improvement for the R&D firm, which is detrimental to economic growth. As a result, taking the channel of the endogenous innovation scale into consideration enables us to show that patent protection and economic growth exhibit an inverted U-shaped relationship.

Moreover, by referring to equation (35), we can further seek through numerical simulations that patent protection policy for maximizing the economic growth rate is 0.9925 (i.e. $\beta = 0.9925$), at which point the economic growth rate is 4%. When patent protection policy is less than 0.9925 (i.e. $\beta < 0.9925$), the positive effect of strengthening patent protection for economic growth outweighs the negative effects, thus enhancing the economic growth rate. Conversely, when patent protection policy exceeds 0.9925 (i.e. $\beta > 0.9925$), the negative effects of strengthening patent protection for economic growth outweigh the positive effect, leading to a decrease in the economic growth rate.

To complement the analysis of equation (36), we introduce a scenario with $i = 2\%$. This will facilitate our comparison of how changes in patent protection policy affect economic growth when nominal interest rates rise. We follow the approach discussed by Chu et al. (2019) to investigate the interactive effects of monetary policy and patent protection policy, and construct Table 2 accordingly. In Table 2, we delineate the impact of changes in patent protection policy on the economic

Table 2. The growth effects of patent protection policy for different nominal interest rates

	$\beta = 1.241$	$\beta = 1.254$	$\boldsymbol{\beta}$ **= 1.267**	$\beta = 1.279$	$\beta = 1.292$
$i = 8\%$					
$\Delta\tilde{g}$	–10.16%	–11.76%	**–16.67%**	–17.33%	–23.02%
$i = 2\%$					
$\Delta\tilde{g}$	–10.17%	–11.80%	**–16.69%**	–17.53%	–23.04%

Note: The numbers in bold face indicate the values under the benchmark case.

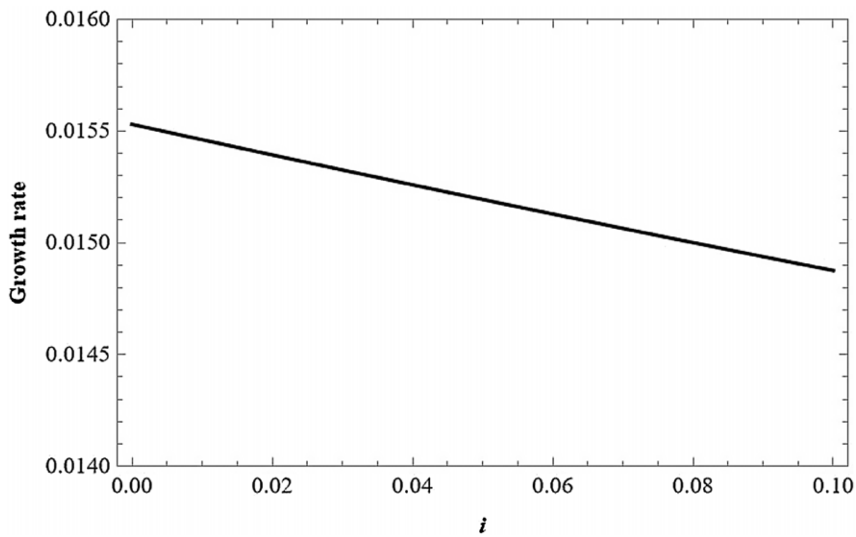

Figure 5. The effect of monetary policy on economic growth.

growth across two distinct scenarios (i.e. when $i = 8\%$ and $i = 2\%$). From Table 2, it is evident that when $i = 2\%$, the negative effects of strengthening patent protection policy on the economic growth are all greater than those for the scenario where $i = 8\%$. Consequently, an increase in the nominal interest rate reduces the negative impact of patent protection policy on economic growth.

Finally, we discuss the effect of the R&D firm's professional knowledge on growth, as illustrated in the right panel of Fig. 4. A lower level of the R&D firm's professional knowledge (i.e. a rise in ϕ) reduces the innovation scale. This in turn reduces the complexity of innovation, which is favorable to economic growth. However, a reduction in the innovation scale also decreases the step size of technology improvement and the research labor input of the R&D firm, both of which are harmful to economic growth. By taking into account all these growth effects, the lower panel of Fig. 4 reveals that the former positive effect falls short of the latter two negative effects, and hence a lower level of the R&D firm's professional knowledge impedes economic growth.[10]

Fig. 5 depicts the effect of monetary policy on economic growth, indicating that increasing the nominal interest rate reduces the economic growth rate. The economic intuition is reported in Subsection 5.2. Raising the nominal interest rate increases the borrowing costs (interest payments) in relation to R&D investment, and thus the R&D firm is inclined to reduce the research labor input. This tends to stifle economic growth.

Fig. 6 depicts the effect of monetary policy on social welfare. Based on equation (40), an increase in the nominal interest rate has two conflicting effects on social welfare, and hence has a reverse U-shaped relation with social welfare. The intuition behind this result can be explained with the aid of the Segerstrom (1998) insight regarding the linkage between R&D investment and

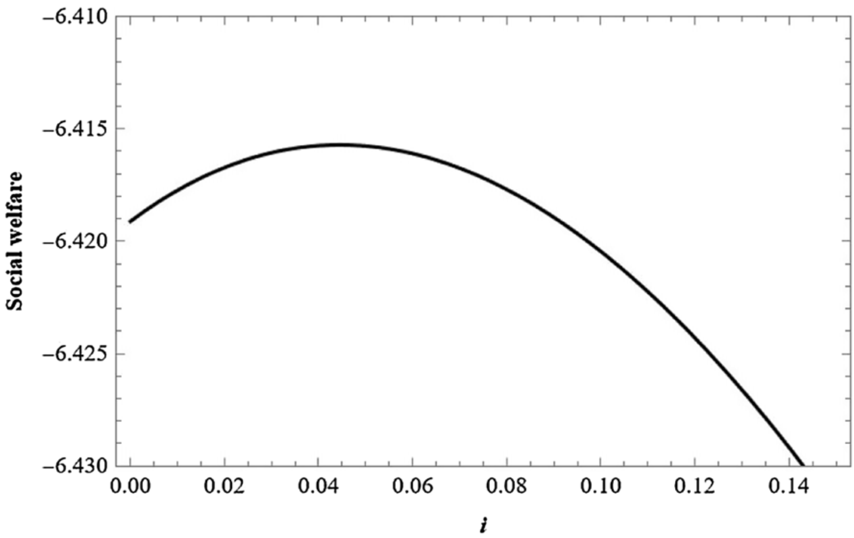

Figure 6. The effect of monetary policy on social welfare.

social welfare. Segerstrom (1998) points out that innovation success gives rise to two kinds of distortions.[11] The first distortion is the consumer surplus effect. The final goods sector benefits from improving the production technology once the R&D firm succeeds in innovation. However, the R&D firm does not take this external benefit into account in its profit-maximization decision. This will cause an under-investment in R&D compared to the social optimum. The second distortion is the business stealing effect. The existing intermediate firm will be driven out of business when innovation is successful. However, the loss of the monopolistic profit of the existing intermediate firms will not be considered in the R&D firm's profit-maximization decision. Thus, it will result in an over-investment in R&D compared to the social optimum. Based on our benchmark parameter values, the size of the first distortion (i.e. the under-investment in R&D) falls short of that of the second distortion (i.e. the over-investment in R&D), and hence the net effect leads the economy to be in a state of over-investment in R&D. Accordingly, as exhibited in Fig. 6, to correct for this unnecessary R&D investment, the monetary authority should choose a nominal interest rate of 4.44% so as to achieve social welfare maximization. This indicates that the Friedman rule fails to be optimal in view of the social welfare maximum.

In what follows in this section, we will deal with how the changes in patent protection and the R&D firm's professional knowledge will affect the optimal nominal interest rate. For expository convenience, in the subsequent analysis the situation exhibited in Fig. 6 (in association with the benchmark parameter values) is dubbed the benchmark case.

Fig. 7 shows that, in response to a higher patent protection (i.e. a rise in β), the monetary authority needs to choose a higher nominal interest rate so as to maximize social welfare. The intuition behind this result can also be explained by resorting to two kinds of distortions arising from R&D investment as mentioned in Fig. 6 (i.e. the positive consumer surplus effect and the negative business stealing effect).

Strengthening patent protection reduces the size of the innovation scale, which will affect the size of these two kinds of distortions. On the one hand, the reduction in the innovation scale implies a smaller step size of technology improvement. It will decrease the effect of the external benefit on the final goods sector, and will thus reduce the extent of the consumer surplus effect, thereby lowering the extent of the under-investment in R&D. On the other hand, the decline in the innovation scale also leads to a higher arrival innovation rate and thus raises the expected loss in the monopolistic profit of the existing intermediate firms due to creative destruction. This

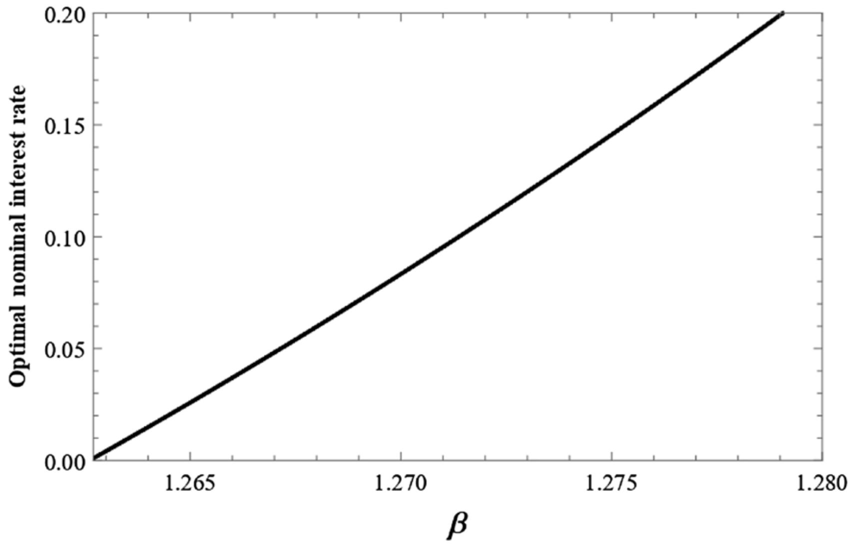

Figure 7. The effect of patent protection on the optimal nominal interest rate.

causes a reduction in the monopolistic profit of the existing intermediate firms, and hence tends to reduce the extent of the business stealing effect. As a result, the extent of the under-investment in R&D is lowered in response.

Based on our benchmark parameter values, the decline in under-investment in R&D dominates that in over-investment in R&D, and hence strengthening patent protection will enlarge the extent of over-investment in R&D compared to the benchmark case in Fig. 6. Accordingly, as illustrated in Fig. 7, in response to stronger patent protection, the monetary authority will choose a higher optimal nominal interest rate to remedy the higher over-investment in R&D.

Fig. 8 shows that, following a reduction in the R&D firm's professional knowledge (i.e. $\phi \uparrow$), the monetary authority will tend to choose a higher nominal interest rate so as to maximize social welfare. Similar to the analysis on strengthening patent protection in Fig. 7, the intuition behind this result can be grasped by resorting to two kinds of distortion arising from the R&D investment mentioned in Fig. 6 (i.e. the consumer surplus effect and the business stealing effect). On the one hand, based on equation (17), the R&D firm is motivated to choose the smaller step size of the innovation scale in association with a lower professional knowledge, thereby leading to a smaller step size of technology improvement. This in turn decreases the extent of the external benefit to the final goods sector, and therefore lessens the extent of under-investment resulting from the consumer surplus effect. On the other hand, a reduction in the R&D firm's professional knowledge also leads to a higher arrival innovation rate and thus stimulates the expected loss in the monopolistic profit of the existing intermediate firms due to creative destruction, thereby causing a fall in the monopolistic profit of the existing intermediate firms. This tends to lessen the extent of the business stealing effect, and therefore lowers the extent of the under-investment in R&D.

Equipped with our benchmark parameter values, the decline in under-investment in R&D exceeds that in over-investment in R&D, and hence a fall in the R&D firm's professional knowledge will enlarge the extent of the over-investment in R&D compared to the benchmark case in Fig. 6. Accordingly, as exhibited in Fig. 8, in response to a lower R&D firm's professional knowledge (i.e. $\phi \uparrow$), the monetary authority is inclined to choose for higher nominal interest rate to correct the higher over-investment in R&D.

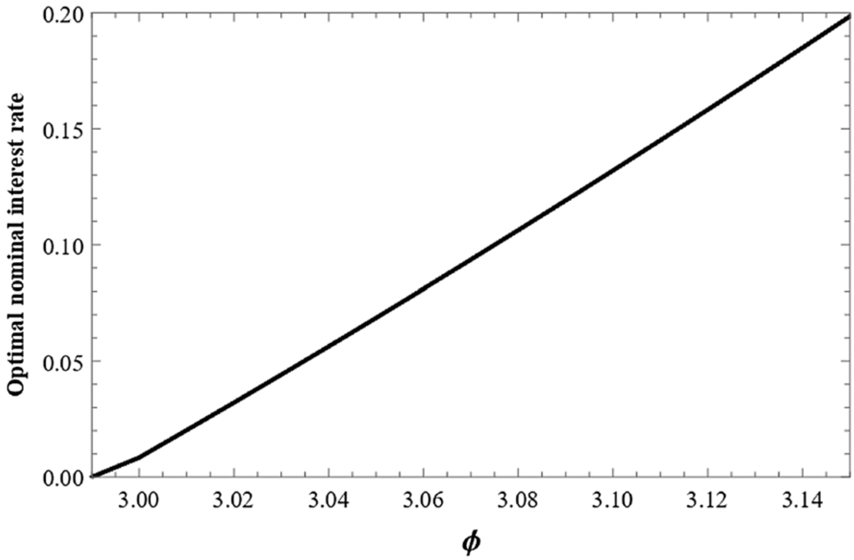

Figure 8. The effect of R&D firm's professional knowledge on the optimal nominal interest rate.

Before ending this subsection, two points deserve special mention here. First, compared with existing studies on the monetary R&D growth model (e.g. Chu and Cozzi (2014), Chu et al. (2015, 2019)), we not only implement a numerical analysis but also provide comprehensive economic intuition regarding how the changes in patent protection and the R&D firm's professional knowledge affect the optimal nominal interest rate. Second, compared with previous studies (Segerstrom (1998), Li (2003), Minniti et al. (2013)) on social welfare analysis, by proposing the channel of the endogenous innovation scale, this paper finds that patent protection and the R&D firm's professional knowledge will affect not only the consumer surplus effect, but also the business stealing effect via the endogenous innovation scale. Our analysis thus provides a new insight into the social welfare implications.

7. Conclusion

This paper builds up the monetary Schumpeterian growth model which features an endogenous innovation scale. Based on this model, we examine how the endogenous innovation scale governs the effect of patent protection and monetary policy on economic growth and social welfare.

An important finding of our analysis is that when the R&D firms have a high level of professional knowledge, they are willing to choose a high innovation challenge project (i.e. a larger innovation scale) to raise the patent value of R&D. Nevertheless, the relationship between the R&D firm's professional knowledge and economic growth is ambiguous because it depends on the tradeoff between the risk of innovation failure and the step size of the technology improvement. On the other hand, the R&D firms will choose a conservative innovation plan (i.e. a smaller innovation scale) to respond to the strengthening patent protection. To be specific, strengthening patent protection means a higher patent value of R&D, and thus these firms will be less willing to bear a higher risk of innovation failure. Interestingly, the strengthening patent protection may impede economic growth, which is quite different from existing studies such as Li (2001), Futagami and Iwaisako (2007), and Chu and Cozzi (2018).

Finally, we examine the effects of monetary policy on economic growth and social welfare. Increasing the nominal interest rate will impede economic growth. However, the effect of the nominal interest rate on social welfare is ambiguous. Hence, this paper employs a numerical simulation to evaluate the optimal nominal interest rate. We show that the optimal nominal interest rate is positive, which means that the Friedman rule is not optimal from the viewpoint of social welfare maximization. More specifically, if the R&D firm lacks professional knowledge or if the government strengthens patent protection, it will be more likely that the Friedman rule does not hold.

Although the model developed in this paper allows us to comprehend the interplay between the behavior of R&D firms and monetary policy, some issues are left open for future research. For instance, we could consider extending our closed-economy R&D-based growth model to one that is open (see Dinopoulos and Segerstrom (2010), and Iwaisako and Tanaka (2017)). This extension would enable us to discuss a case where R&D firms in different countries have distinct levels of professional knowledge. In this case, it is also worth investigating how the R&D firm's behavior in the foreign country will affect the growth and welfare effects of monetary policy and the trade policy in the domestic country.

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Notes

1 Their research indicates that the low innovation firms (i.e. those that have weaker human capital and a low knowledge stock) have the motivation to merge with high innovation firms abroad to increase their innovation knowledge, which would consequently increase their values.

2 Building on the insights from the existing literature, including Grossman and Helpman (1991, pp. 110–11), Segerstrom (1998), and Minniti et al. (2013), these two market externalities will be discussed in more detail in Section 6 below.

3 However, in the standard Schumpeterian growth model, the monopolistic intermediate firms engage in Bertrand competition in each industry. Therefore, a larger innovation scale will also result in a higher markup of intermediate goods, and thus a higher return from R&D.

4 We assume that labor is perfectly mobile across sectors. This implies that all sectors provide the same real wage w_t.

5 In our model, the step size of quality improvement z is an endogenous variable. It could be treated as an innovation plan chosen by R&D firms.

6 In the seminal work by Grossman and Helpman (1991), β is set to 1 for simplification.

7 From equations (11) and (13), we can infer the result $\partial v_t(j)/\partial z = y_t/\beta(r_t + I_t(j) - \dot{v}_t/v_t)z^2 > 0$. In addition, based on equation (15a), we can infer the result $\partial I_t(j)/\partial z = -\phi I_t(j)/z < 0$.

8 See, for instance, Segerstrom (1998), Chu and Lai (2013), Minniti et al. (2013), Chu and Cozzi (2014), and Huang et al (2023).

9 It should be noted that even though in equation (34) the second channel of the growth effect $n > 0$ is similar to Chu and Cozzi (2014), in our endogenous innovation scale model $\dot{N}_t = nN_t n > 0$ is crucially related to $U = \int_0^\infty e^{-\rho t} [\ln c_t + \theta \ln(1 - l_t)]\, dt$ and c_t, which are endogenous variables and have to do with patent protection policy and the professional knowledge of R&D firms.

10 In our numerical examinations, the negative relationship between t and l_t remains robust across various values of the nominal interest rate and different extents of patent protection. Detailed numerical results are available from the authors upon request.

11 Segerstrom (1998) indicates that his analytical framework has a third distortion brought about by the negative external effect on R&D investment. The reason for the presence of this third distortion is that, once innovation succeeds, future innovation becomes more difficult. However, this linkage is not considered in the R&D firm's decision, and will thus cause over-investment in R&D investment compared to the social optimum. This distortion is called the intertemporal R&D spillover effect. In departing from the Segerstrom (1998) analysis, our specification of the arrival rate of innovation t in equation (15) abstracts from this intertemporal R&D spillover effect.

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Appendix A

This appendix presents the proof of Lemma 1. Substituting equation (12) into equation (23) yields:

$$l_r^{1-\psi} = \frac{l_x v_t \beta \tilde{z} \eta}{y_t (1+i) \tilde{z}^{\phi}}, \tag{A1}$$

where $\tilde{z} = (1+\phi)/\beta\phi$. Equipped with equation (13), the economy's resource constraint $y_t = c_t N_t$, and equation (5), the law of motion for l_r is given by:

$$(1-\psi)\frac{\dot{l}_r}{l_r} = \frac{\dot{l}_x}{l_x} + \rho + I_t - \frac{\Pi_{x,t}}{v_t}. \tag{A2}$$

From equations (11), (15), and (23), we can rewrite equation (A2) as follows:

$$(1-\psi)\frac{\dot{l}_r}{l_r}=\frac{\dot{l}_x}{l_x}+\rho+\frac{\eta}{\tilde{z}^{\phi}}\left(l_r^{\psi}-\frac{(\beta\tilde{z}-1)}{(1+i_t)}l_x l_r^{\psi-1}\right). \tag{A3}$$

We then derive the relationship between l_x and l_r. Combining the market-clearing condition for labor, equations (4), and (12) together yields:

$$l_x=[1+\theta\,(1+i_t\xi)\,\beta\tilde{z}]^{-1}\left(1-l_r\right). \tag{A4}$$

From equation (A4), the law of motion for l_x is given by:

$$\frac{\dot{l}_x}{l_x}=\frac{-l_r}{1-l_r}\frac{\dot{l}_r}{l_r}. \tag{A5}$$

Substituting equations (14) and (A5) into equation (A3) yields:

$$\dot{l}_r=\left(1-\psi+\frac{l_r}{1-l_r}\right)^{-1}\left[\rho l_r+\frac{\eta\,(1+\Omega)\,l_r^{\psi+1}}{\tilde{z}^{\phi}}-\frac{\eta\Omega l_r^{\psi}}{\tilde{z}^{\phi}}\right], \tag{A6}$$

where $\Omega=\eta(\beta\tilde{z}-1)/\{\tilde{z}^{\phi}(1+i_t)[1+\theta(1+i_t\xi)\beta\tilde{z}]\}$. In the steady state, the $\dot{l}_r$ is equal to zero as follows:

$$\rho+\frac{\eta\,(1+\Omega)\,\tilde{l}_r^{\psi}}{\tilde{z}^{\phi}}-\frac{\eta\Omega\tilde{l}_r^{\psi-1}}{\tilde{z}^{\phi}}=0. \tag{A7}$$

From equation (A7), we can solve the equilibrium research labor allocation $\tilde{l}_r>0$. Then, we linearize the equation (A6) around the steady-state equilibrium. Differentiating the (A6) with respect to l_r yields:

$$\frac{\partial\dot{l}_r}{\partial l_r}=\left(1-\psi+\frac{\tilde{l}_r}{1-\tilde{l}_r}\right)^{-1}\left[\rho\tilde{l}_r+\frac{(1+\psi)\,(1+\Omega)\,\tilde{l}_r^{\psi}}{\tilde{z}^{\phi}}-\frac{\psi\eta\Omega\tilde{l}_r^{\psi-1}}{\tilde{z}^{\phi}}\right]>0. \tag{A8}$$

Based on equation (A7), we obtain $\partial\dot{l}_r/\partial l_r>0$. Since l_r is a jump variable and $\partial\dot{l}_r/\partial l_r>0$, l_r will jump to its steady-state value and the economy will exhibit a unique and stable balanced growth path.

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