

## BOOK REVIEWS

*Risk Theory*. R. E. BEARD, T. PENTIKÄINEN and E. PESONEN, Methuen & Co Ltd., London 1969.

The actuarial business has since its beginning been closely associated with the research into the study of problems of managing enterprises whose business consists of longterm contracts involving quantities subject to random variation, i.e. risk processes. To disseminate knowledge of the theory of risk Pentikainen published an elementary textbook in Finnish language in 1955, primarily designed for the use of Finnish actuaries. Since participants at meetings of ASTIN expressed a wish for a concise book of this kind in English, the Finnish textbook has been brought up-to-date by Pentikäinen and Pesonen and passed to the English author, Beard, who for his part worked it into shape paying special attention to the actuarial attitudes of the English-speaking countries.

The authors start by stating their three "main problems of risk theory":

- What is the result of the business at the end of a certain period  $T$ ?
- What is the probability that ruin will occur at some point of time during a period  $T$ ?
- What is the probability that the business will never be ruined?

The mathematically exact formulation of these three questions is given with the use of the theory of random processes. The usual assumptions about the risk process are made and the reader is lead to the generalized Poisson function.

In order to get explicit numerical values for the amount of free reserves at each time point  $t$  we find a discussion of the normal approximation and—more generally—of the Edgeworth series. The Esscher approximations and Monte Carlo techniques are also presented as tools for numerical analysis in risk theory. For each approximation the problem of accuracy is treated seperately.

The practical actuary will appreciate the most interesting chapter on varying basic probabilities. The difference between the results from a model which assumes basic fluctuations versus one which disregards them is lucidly illustrated by an example on Stop Loss premiums.

We then find the definition of the probability of ruin for a finite and infinite time period respectively and the well known results and approximations for these probabilities.

Most interesting is also a short, but very well written discussion on business planning where the ideas of utility theory are introduced.

The book is very carefully designed for the requirements of an actuary. Without unnecessarily proving every formula the authors have nevertheless listed the most important results in every chapter and explain them with the help of many practical examples. In this connection the problems of reinsurance are treated extensively. Beside that the interested reader finds some exercises to test his understanding.

**This is a textbook** which should be recommended to every actuary who **wants to get a first** introduction into the vast field of risk theory and to the **student who in his undergraduate** years wishes to learn about a powerful **application of probability** theory. We ASTIN members—constituting the “**in-group**” of risk theory—must thank the authors for making our ideas **known to a wider circle** which hopefully will get interested in this fascinating **subject**.

HANS BÜHLMANN

*Remarks to Seal's review in Astin Bull VI on my paper 'A Review of the Collective Theory of Risk' (Suppl. to Astin Bull. V).*

Seal remarks i.a. that in the review a few papers dealing with the individual risk theory rather with the collective risk theory have been included, while other papers related to the former theory have not been even mentioned.

The fact is, that the border line between the individual and collective methods seems to have become rather vague with the modern development of the latter method. For example in a recent paper (Astin Bull V : 3, 1971), it has been supposed that a large group of insurances may be divided into sub-groups for which the view-points of the collective risk theory are applicable, this assumption has been tacitly made in papers dealing, particularly, with motor insurance. As a particular case, was assumed that the risk process of each such sub-group was a compound Poisson process. In this case, the main group was found to be in the same form, with the risk distribution defined by the convolution of the risk distributions in the sub-groups, and the claim distribution by a weighted average of the claim distributions in the sub-groups. If the sub-groups contain only one individual, the problem is principally the same, it means that the individual process shall be treated with the collective method. It seems, therefore, not unnatural to include some papers dealing with the individual theory without giving a complete list of such papers.

Seal remarks, further, that in mentioning papers dealing with pure mathematics rather than with collective risk theory neither with stochastic process theory [60 \*), [100], [179], [183], [184], [219] and [355] have not been included. For example [183], [184] deal with distributions generated by Poisson distributions, and with branch processes. These distributions, and processes are, however, of utmost interest for the collective risk theory, so that it does not seem unnatural to consider these items as belonging to the methods of the collective risk theory rather than to pure mathematics, which may also be said with respect to the remaining papers, in the list just given. Seal considers it a disadvantage that the literature list at the end of my review has not been divided into three parts, one referring to stochastic process theory and other pure mathematical items, one to the collective risk theory, and one to individual risk theory, where the latter should be either completed or eliminated. In my opinion, my comments on the development in collective risk theory seem to be sufficiently well illustrated by selected quotations. As the two parts not considering collective risk theory, are

---

\*) The figures within square brackets refer to the list of literature in my review.