This booklet will give a very good and even deep insight into the field of representations of finite groups to the non-specialist; it will be a helpful supplement for the specialist; and the list of references is a very valuable source of information to everybody related with this subject.

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Permutation groups by D. S. Passman. W. A. Benjamin, Inc., New York, 1968. 310 pages. U.S. $9.50 (clothbound); U.S. $4.95 (paperback).

Everyone who has taken an elementary course in the theory of representations and characters of finite groups has heard of Frobenius groups. These can be described as finite transitive permutation groups in which some non-trivial elements leave one letter fixed but only the identity leaves more than one letter fixed. One of the classical applications of character theory due to Frobenius is that in a Frobenius group the elements which move all letters, together with the identity, form a normal subgroup (called the Frobenius kernel) which complements each of the subgroups fixing one letter (called the Frobenius complements). Such groups occur often enough in the theory of finite groups to make the study of their structure worthwhile.

It is known that a finite group is a Frobenius kernel if and only if it has a fixed point free automorphism of prime order, and these groups were shown to be nilpotent by John Thompson in 1959. The facts about the Frobenius complements are less well known. Burnside knew that these groups must have all their Sylow p-groups either cyclic or (if $p = 2$) possibly generalized quaternion; but a theorem of his which purports to give their complete structure is false. In 1935 in his paper "Über endliche Fastkörper", Zassenhaus showed that a finite group is a Frobenius complement if and only if it has a faithful representation over the complex field which is fixed point-free in the sense that each non-trivial element corresponds to a matrix without 1 as an eigenvalue. He then described in detail the possible structure which such a group may have. Unfortunately, his paper contained some serious gaps, although it appears that he has since, in an unpublished manuscript, filled in these gaps. Recently two full accounts of the classification of Frobenius complements have been published. One is in the book "Spaces of Constant Curvature" (McGraw-Hill, 1967) by J. A. Wolf, and the other is in Passman's book. The two approaches differ slightly and to some extent complement each other; Wolf is interested for his applications in the fixed point-free representations whilst Passman looks at the permutation groups. The case of solvable Frobenius complements is straightforward (although not trivial), but I found the case of the non-solvable Frobenius complements a beautiful surprise. If K is a metacyclic group of order prime to 30, and $SL(2, 5)$ is the special linear group of order 120 (a central extension of the simple group of order 60), then $K \times SL(2, 5)$ is a non-solvable Frobenius complement; conversely, every non-solvable Frobenius complement is of this form or else has a subgroup of index 2 which is of this form. In contrast to the simple statement of this result, the proof is long and tedious. [Since I wrote this review I have been told by Dr. B. Huppert that he has found several gaps remaining in Wolf's account.]

This structure theorem on Frobenius complements is one of the principal aims of Passman's book. The second is Huppert's theorem (1957) on the structure of solvable doubly transitive permutation groups. One way to construct such a permutation group is as follows. Take a finite field $GF(q^n)$ and consider all permutations of this field of the form $x \to ax^{\sigma} + b$ ($a, b \in GF(q^n)$, $a \neq 0$, and σ a field automorphism). Huppert's theorem shows that, except for six exceptional degrees, every solvable doubly transitive permutation group is essentially a subgroup of one of these particular permutation groups.

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These theorems of Zassenhaus and Huppert, together with a result of Szuzuki on strictly doubly transitive groups and a description of the Mathieu groups, form the last third of the book. The first two-thirds deal with introductory material giving the necessary tools in permutation groups, transfer theory and linear representations, as well as giving the theorems of Frobenius and Thompson on Frobenius groups. Since so many results are needed in the last part these earlier chapters make a nice introduction into methods in the theory of finite groups.

This book is to be recommended as a valuable source on Frobenius groups and multiply transitive groups, and a useful introduction to some basic tools in finite group theory.

John D. Dixon, Carleton University

A first course in abstract algebra, by Hiram Paley and Paul M. Weichsel. Holt, Rinehart, and Winston, New York, 1966. xiii + 326 pages.

This is a textbook on Modern Algebra. The first chapter is set theory. The second chapter is on number theory, and includes axioms for the integers, the euclidean algorithm, congruences, and the fundamental theorem of arithmetic.

The next chapters include the basic properties of groups and rings one expects in a book at this level. The final chapter is on advanced ring theory, including the Artin-Wedderburn Theorems for semi-simple rings and the characterization of semi-simple rings in terms of injective and projective modules.

This textbook could be used in an undergraduate honours course at Manitoba in its entirety We normally use a book whose final topic is Galois theory rather than advanced ring theory; perhaps the ring theory would be more useful. This book has been used here twice in our general course programme with omissions. The number theory chapter is a good introduction to abstract mathematics for this group of students.

N. Losey, University of Manitoba

A first course in linear algebra, by D. Zelinsky. Academic Press, New York, 1968. viii + 266 pages. U.S. $6.50.

This book is an excellent introduction to the algebra and geometry of vectors, matrices, and linear transformations. It follows closely the recommendations of CUPM. A student is slowly introduced to the concept of vector spaces, linear transformations, determinants and quadratic forms; the job is well done. Each section in the book is followed by a wealth of examples. The book is practically free of typographical errors. I strongly recommend the book for a first course in linear algebra.

B.M. Puttaswamaiah, Carleton University

Introduction to the theory of algebraic functions and numbers, by M. Eichler. Academic Press, New York and London, 1966. xiv + 324 pages. U.S. $14.50.

The book is translated from the German. Although it is called an introduction, the book is too difficult to serve this purpose suitably for,