SHOWALTER, R. E., *Hilbert Space Methods for Partial Differential Equations* (Pitman, 1977), xii+196 pp., £14·00.

After a basic introduction to partial differential equations (e.g. G. Hellwig, *Partial Differential Equations*) there are a variety of directions in which to pursue the topic. One of the more popular, and probably the most appropriate, is the use of Hilbert space methods in partial differential equations. In this approach the basic questions of existence and uniqueness are resolved through the use and exploitation of the Riesz representation theorem and making precise the space from which the solution is obtained, the space from which the data may come, and the corresponding notion of continuity. The book under review is devoted to this topic, and in the reviewer's opinion is probably the best introduction to the subject available. The attraction of Showalter's book is that while it covers the basic material, and in fact touches on some of the more recent developments, it refrains from pursuing long and tedious generalisations which can often hide the main ideas from the prospective student. Hence only boundary value problems for second order elliptic equations are discussed, regularity of solutions to elliptic equations is shown only in the context that $u \in H^1(G)$ implies $u \in H^2(G)$, and results on distribution theory and semigroups are restricted to what is actually essential in the development of the resulting theory. In this manner Showalter is able to cover a considerable amount of material in the space of only 184 pages.

We now survey the main contents of the book. Chapter I presents all the elementary Hilbert space theory that is needed for future developments, including the Hilbert-Schmidt theorem. Chapter II is an introduction to distributions and Sobolev spaces, including Sobolev's lemma, trace theorems, and Rellich's lemma. Chapter III is an exposition of the theory of linear elliptic boundary value problems in variational form. An abstract Green's theorem is presented which permits the separation of the abstract problem into a partial differential equation on the region and a condition on the boundary. This approach has the pedagogical advantage of making the discussion of regularity theorems optional. Also included in this chapter is the problem of eigen-function expansions. Chapter IV is an exposition of the theory of semigroups and its application to solve initial-boundary value problems for evolution equations. Chapters V and VI provide the immediate extension of the results of Chapter IV to cover evolution equations of second order and implicit type. This material is not often included in an introductory text, and represents an area where the author has made substantial contributions of his own. Included here is a discussion of such topics as pseudoparabolic equations, Sobolev equations, and equations of degenerate or mixed type. Chapter VII is entitled "Optimization and Approximation Topics" and includes material on Dirichlet's principle, variational inequalities, optimal control of boundary value problems, and approximation of solutions to partial differential equations. Although there are many detailed computations throughout the text, the reviewer noted only a few printing errors, all of which can be easily corrected.

The author claims in the Preface that "the reader is assumed to have some prior acquaintance with the concepts of 'linear' and 'continuous' and also to believe L^2 is complete" and the formal prerequisite "consists of a good advanced calculus course and motivation to study partial differential equations". As the opening lines of this review indicate, the reviewer is a little doubtful if this is sufficient background to fully appreciate the material in the text. Even with the excellent introduction to Hilbert space theory in Chapter I, a reader who has never been exposed to functional analysis is liable to feel over his head quite quickly. The reviewer would furthermore consider it a mistake to learn the "modern" theory of partial differential equations without first learning such "classical" topics as the classification of partial differential equations as to type, the theory of characteristics, maximum principles, and elementary methods for proving existence theorems. This is in no way a criticism of Showalter's book, which the reviewer considers to be excellent, but merely to point out at which point in the learning process the reviewer considers this material to be most beneficial to the prospective student.

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